# Physics 201: Lab on Experimental Uncertainties Using a Pendulum 

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## Purpose

To determine the gravitational acceleration $g$ in Room 1023 of the Physical Science Complex using a series of simple pendulums. We will pay particular attention to the experimental uncertainty.

## Apparatus

stopwatches, pendulum bobs or appropriate substitutes, thin brass wire

## References

- Physics for Scientists and Engineers, Randall D. Knight, p. 429-31.
- handout on uncertainties


## Theory/Operation

The pendulum is one of the canonical examples of a simple harmonic oscillator. A simple pendulum has all of its mass concentrated in one location. This distance from this mass concentration to the point of support defines the length $\ell$. (A pendulum where the mass is distributed is known as a compound pendulum and you need to include the centre of mass and the centre of gyration in the theory.) The restoring force is provided by gravity whose strength is given by the gravitational acceleration.

Suppose that the support wire makes an angle of $\theta$ with respect to the vertical direction (as defined by the direction of the gravitional field). A free body diagram shows that the forces on the bob are the tension $F_{r}$ along the wire and the force of gravity mg . (I am using the $r$ subscript to indicate a radial force.) Both of these force vectors lie in one plane and so the motion is restricted to 2 dimensions (if you are in an inertial frame!). If we resolve the weight vector into components along and perpendicular to the wire we find that if the bob follows a circular path then $F_{r}=-m g \cos \theta$ and that there remains a tangential force of $F_{t}=-m g \sin \theta$. The negative signs mean that the forces are opposite to the directions of increasing $r$ and $\theta$. The tangential component of the acceleration $a_{t}$ may be given in terms of the second time derivative of $\theta$ (sometimes called the angular acceleration)

$$
\begin{equation*}
a_{t}=\ell \frac{d^{2} \theta}{d t^{2}}=\ell \ddot{\theta} \tag{1}
\end{equation*}
$$

(The "dot" notation refers to time derivatives. So a "double dot" means a second order derivative.) Applying Newton's second law to the tangential componet

$$
\begin{equation*}
F_{t}=m a_{t} \tag{2}
\end{equation*}
$$

$$
\begin{align*}
-m g \sin \theta & =m \ell \frac{d^{2} \theta}{d t^{2}}  \tag{3}\\
-\frac{g}{\ell} \theta & \approx \ddot{\theta} \tag{4}
\end{align*}
$$

The approximation refers to the small angle approximation for sine

$$
\begin{equation*}
\sin \theta=\theta-\frac{\theta^{3}}{3!}+\ldots \tag{5}
\end{equation*}
$$

which comes from a Taylor series (or since it is expanded around $\theta=0$ a Maclaurin series). This expansion is only valid for $\theta$ in radians and we are only keeping the first term here.

The differential equation describing $\theta$ then has the general solution

$$
\begin{equation*}
\theta(t)=A \sin (\omega t+\phi) \tag{6}
\end{equation*}
$$

This is known as simple harmonic motion and the motion is periodic. The constants $A$ and $\phi$ are the amplitude and phase respectively and depend on initial conditions. $\omega$ is the frequency or sometimes the radial frequency. In order to solve the differential equation $\omega$ and the period $T$ depend on $g$ and $\ell$.

$$
\begin{gather*}
\omega=\sqrt{\frac{g}{\ell}}  \tag{7}\\
T=\frac{2 \pi}{\omega}=2 \pi \sqrt{\frac{\ell}{g}}  \tag{8}\\
T^{2}=\frac{4 \pi^{2}}{g} \ell \tag{9}
\end{gather*}
$$

So if we have a set of simple pendulums with different lengths a plot of $T^{2}$ versus $\ell$ should yield a straight line and from the slope we can determine $g$. Our big question here is how accurately can we determine $g$ with fairly simple apparatus and where are the limits to our accuracy?

## Procedure

1. Break into groups around each of the pendulums in the lab. The jobs can be spread around the group.
2. Measure the length of each pendulum and make a reasonable estimate of the error. Calculate the percentage error. Even if you had a way of very accurately measuring the length why would you be limited in your determination of $\ell$ ?
3. Be careful with the analog stop watches! In particular they don't need a lot of winding. Measure the period in the following ways: (1) just one oscillation measured once, (2) just one oscillation but repeat the measurement 5 times, (3) measure 10 oscillations and divide by 10 to find $T$, (4) measure 10 oscillations, 5 times. In each case estimate the uncertainty. Is the fact your stop watch is analog a big problem? What type of measurement do you suggest to reduce the percentage error to roughly one half of that for the length measurement measurement? Is it possible or impossible?
4. Make a quick estimate of $g$ using your best result for $T$ so far and estimate the uncertainty. Write the result for $T, \ell$, and $g$ on the front board.
5. What about your technique? Are you timing from the first oscillation or letting the bob go back and forth a few times? Are you timing from the end points or from the centre of oscillation?
6. Proceed with your planned measurement. Be on the lookout for "miscounts" which easily creep in if you are measuring $20+$ oscillations.
7. Report your updated $T$ to the front board.
8. Cyclically move one station in the increasing $\ell$ direction and repeat the $\ell$ and high quality $T$ measurement for another group's pendulum. Report the result.

## Data Analysis

1. Make a plot of "best" $T^{2}$ versus $\ell$ from the collected data on the front board with error bars on $T^{2}$. You may do this by hand or by using a computer. One plot is fine for your whole group.
2. Determine $g$ and its uncertainty and report the result.
3. Make sure relevant info is included in your lab book.
