Final Exam: Electricity and Magnetism 322
April 20, 2002

Open book. Point values are given with each question. Total exam is worth $140 + 25$ points.

1. (a) What is the relationship between the force, electric field, and charge? (5)
   (b) Setup the problem and then state the Green's function solution for Poisson's equation. (5)
   (c) What are the expressions for electric field and potential for a point charge at the origin? Demonstrate that $\vec{E} = -\nabla V$ in Cartesian and spherical polar coordinates for this example. (10)
   (d) State the equation that allows you to express $\vec{E}$ as the gradient of a scalar function. (5)

2. Show that the monopole, dipole, and quadrupole expressions for the potential

$$V(r,\theta,\phi) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{1}$$

$$V(r,\theta,\phi) = \frac{1}{4\pi\epsilon_0}\frac{p_0\cos\theta}{r^2} \tag{2}$$

$$V(r,\theta,\phi) = \frac{Q_2}{4\pi\epsilon_0}\frac{(3\cos^2\theta - 1)}{2r^3} \tag{3}$$

satisfy Laplace's equation for $r \neq 0$. If Laplace's equation just comes out to zero why do we have all of the prefactors in these expressions? (25)

3. Consider an infinitely long square pipe whose central axis is coincident with the z-axis with sides at $x = \pm a/2$ and $y = \pm a/2$. The conducting sides at $x = \pm a/2$ are grounded and the sides at $y = \pm a/2$ have a potential that depends on x

$$V(x, \pm a/2) = 2V_0 x/a. \tag{4}$$

   (a) Based on the symmetry of the problem what do you conclude about the dependence of V on x, y, and z. (even, odd, constant, zero?) inside of the pipe. (5)
   (b) Consider solving Laplace's equation by separation of variables. Follow the arguments that lead you to the conclusion (7)

$$V(x,y,z) = \sum_{n=0}^{\infty} C_n \sin[(2n+1)\frac{\pi x}{a}]\cosh[(2n+1)\frac{\pi y}{a}]. \tag{5}$$

   (c) Use the fact that the Fourier sine series of a linear function is

$$\frac{2V_0}{a}x = \sum_{n=0}^{\infty} \frac{8V_0\pi^2(-1)^n}{(2n+1)^2}\sin[(2n+1)\frac{\pi x}{a}] \tag{6}$$

   to determine C_n. (8)
   (d) Can you just continue this solution outside of the pipe to find $V(x,y,z)$ in that region? Why or why not? Does the expression as written still solve Laplace's equation outside of the pipe? (5)

(e) How is the problem modified for side plates with potentials $-V_0$ and $+V_0$? Write the solution for Laplace's equation that satisfies all of the boundary conditions. (I assume you have the mental "capacity" to guess an appropriate solution.) (5)

4. A coaxial cable is composed of concentric cylindrical conductors. One has radius of a and the other has a larger radius b. Suppose that the outer conductor is grounded and the inner is held at potential V_0.

   (a) Use Gauss Law in integral form along with symmetry arguments to determine the electric field in the region between the conductors if the inner conductor has a charge per unit length of λ. (5)

   (b) The path integral of the electric field from inner conductor to the outer conductor should allow you to find the relationship between λ and V_0. Give the capacitance per unit length $C' = \lambda/V_0$. (7)

   (c) Look at the discontinuity in $\vec{E}(r)$ to find σ on the inner surface. (3)

   (d) Suppose that the space between the conductors is filled with a dielectric with dielectric constant $\kappa = r/a$. The charge density on the inner conductor is unchanged. Calculate $\vec{D}(r)$ and $\vec{E}(r)$. (10)

   (e) Perform the path integral to find V between the conductors and again calculate C'. (5)

   (f) What is the bound charge density as a function of r? (Remember that you are in cylindrical coordinates.) (5)

5. A dielectric with $\kappa = 8$ sits in the $z < 0$ region of space. There is no free charge density. Just above the dielectric

$$\vec{E} = 3\hat{\imath} - \hat{k} \tag{7}$$

   in units of V/m.

   (a) State the boundary conditions on $\vec{E}$ and $\vec{D}$ and the equations that form their basis. (5)

   (b) What is $\vec{E}$ immmediately below the surface? (10)

   (c) What is σ_b? (5)

   (d) If the dielectric is uniform, is there any ρ_b in the bulk? (5)

6. By popular request BONUS: Outside of a hollow sphere we find a quadrupole potential

$$V(r, \theta, \phi) = \frac{Q_2}{4\pi\epsilon_0} \frac{(3\cos^2\theta - 1)}{2r^3} \tag{8}$$

   Just off the top of your head what do you expect the total charge on the sphere to be. (5) Why? (5) What is the expression for the surface charge density assuming $\vec{E}$ inside is zero? (5) Find the total charge from $\theta = 0$ to $\pi/4$. (10)