Midterm Exam: Electricity and Magnetism 322
October 9, 2002

Open book. Point values are given with each question. Total exam is worth $100 + 20$ points.

1. (a) Give the definition of a vector in Einstein summation notation. (5)
   (b) If $\vec{x} = 3\hat{\imath} - 2\hat{\jmath} + \sqrt{3}\,\hat{k}$ what are x_1, x_2, and x_3? (5)
   (c) Consider a counter clockwise rotation of $\pi/6$ about the z-axis. What is x_1'? (10)
   (d) You perform a further rotation of 20 degrees counterclockwise about the original x-axis. What is $|\vec{x}''|$? (the two primes refer to the two transformations) (5)

2. In some neighbourhood of the origin we have a surface that lies in the xy-plane. We refer to the region above the plane as region 2 and the region below as region 1. The electric field (in V/m) is found to be

$$\vec{E}(\vec{x}) = \begin{cases} 3\hat{\jmath} + 2\hat{k} & \text{for } z \geq 0 \\ 3\hat{\jmath} + \hat{k} & \text{for } z < 0 \end{cases} \tag{1}$$

   Consider a rectangular path lying in the yz-plane that runs in a counter clockwise direction through the following four verticies (distance dimensions in metres)

$$\begin{aligned} \vec{x}_1 &= -0.1\,\hat{k} \\ \vec{x}_2 &= \hat{\jmath} - 0.1\,\hat{k} \\ \vec{x}_3 &= \hat{\jmath} + 0.1\,\hat{k} \\ \vec{x}_4 &= 0.1\,\hat{k}. \end{aligned}$$

   (a) Sketch this vector field and path in the yz-plane. (5)
   (b) What is the value of $-\int \vec{E} \cdot d\vec{\ell}$ along the segment of the path from $\vec{x}_1$ to $\vec{x}_2$? Using this knowledge and one of our equivalent definitions of scalar potential what is $V(\vec{x}_2) - V(\vec{x}_1)$. (10)
   (c) Knowing that $\vec{\nabla} \times \vec{E} = 0$ what would you conclude about the value of $-\oint \vec{E} \cdot d\vec{\ell}$? Explain your reasoning using Stokes' Theorem. (5)
   (d) What is σ, the charge density in the xy-plane? (5)
   (e) Show that a scalar potential of

$$V(\vec{x}) = \begin{cases} -3y - 2z & \text{for } z \geq 0 \\ -3y - z & \text{for } z < 0 \end{cases} \tag{2}$$

   gives $\vec{E}(\vec{x})$. What value does it give for $V(\vec{x}_2) - V(\vec{x}_1)$? (10)

3. A charge q sits at the origin ($\vec{x}_1 = 0$).

   (a) According to Coulomb's law what is $\vec{E}(\vec{x})$ in spherical polar coordinates? What is $V(\vec{x})$? Show that $\vec{E}(\vec{x}) = -\vec{\nabla} V(\vec{x})$. (10)
   (b) What is the leading order dependence of $V(\vec{x})$ on r for large r? (5)

   You place a second charge of $-q$ at $\vec{x}_2 = d\hat{\imath}$.

(c) What is the leading order dependence of $V(\vec{x})$ now? (5)

(d) Calculate the dipole moment for the two charges and give $V(\vec{x})$ to leading order. Does $\vec{p}$ depend on the choice of origin? (5)

(e) Calculate the quadrupole moment tensor. (5)

(f) Calculate the monopole and dipole contributions to the scalar potential at $\vec{x} = 5d\,\hat{\imath}$. (10)

BONUS State Poisson's equation, the appropriate condition equation for Green's function, the Green's function, and Green's solution for the scalar potential $V(\vec{x})$ for a charge distribution $\rho(\vec{x}\,')$. (20)