Midterm Exam: Electricity and Magnetism 322
November 20, 2002

Open book. Point values are given with each question. Total exam is worth 60 points.

1. (a) State and sketch the first three Legendre polynomials as a function of u. (10)

   (b) You know that the $P_\ell(\cos\theta)$ give the θ dependence for separable solutions of Laplace's equation in spherical polar coordinates so that

   $$V(r,\theta,\phi) = R(r)P_\ell(\cos\theta). \tag{1}$$

   What are the solutions of $R(r)$ that solve Laplace's equation in this case? State the $V(r,\theta,\phi)$ solutions explicitly for $\ell = 1$. What are the names we give to these two solutions? (hint: it isn't George!) (12)

   (c) Show that $V(r,\theta,\phi) = r\cos\theta$ is a solution of Laplace's equation in spherical polar coordinates. (8)

2. Consider a parallel plate capacitor with a plate separation of d much less than its lateral size. The lower plate sits in the xy-plane and is grounded. The upper plate has a potential of V_0.

   (a) What is the natural coordinate system to describe $V(\vec{x})$? Can you use symmetry to reduce the number of dependent variables? (5)

   (b) Use the FISHTANK method to solve the electrostatic problem between the plates and give $V(\vec{x})$ in this region. How would you answer be modified if the lower plate were held at $-V_0/2$ and the upper plate were held at $V_0/2$? Would you expect the electric field to change? Why or why not? (10)

   (c) If $\vec{E}(\vec{x}) = -\frac{V_0}{d}\hat{k}$ what is σ on the upper and lower plates? (5)

   (d) Calculate the internal energy as a function of V_0 using the integral

   $$U = \frac{1}{2}\int \rho V d^3x = \frac{1}{2}\int \sigma V dA \tag{2}$$

   evaluated on the upper plate. The upper plate has a total area of A. (5)

   (e) What is the capacitance of a parallel plate capacitor? Show that it agrees with $C = 2U/V_0^2$ (5)