Supplemental Midterm Exam: Electricity and Magnetism 322
November 22, 2002

Open book. Point values are given with each question. Total exam is worth 60 points.

1. (a) State Legendre's equation as a function of u. (2)

   (b) If solutions to Legendre's equation are given by power series

$$P_\ell(u) = \sum_{n=0}^{\infty} C_n u^n \tag{1}$$

   what is the relationship between the C_n for different values of ℓ. If ℓ is constrained to be a non-negative integer calculate the value of $C_{\ell+2}$ i.e. C_{n+2} when $n = \ell$. (8)

   (c) The first Legendre polynomial is given to be $P_1(\cos\theta) = \cos\theta$. If we assume separable solutions of Laplace's equation in spherical polar coordinates so that

$$V(r, \theta, \phi) = R(r)P_\ell(\cos\theta). \tag{2}$$

   What are the two solutions of $R(r)$ and $V(r, \theta, \phi)$ that solve Laplace's equation for $\ell = 1$. What are the names that we have given to these solutions? (hint: it isn't George!) (10)

   (d) Show that $V(r, \theta, \phi) = \frac{\cos\theta}{r^2}$ is a solution of Laplace's equation in spherical polar coordinates. (8)

2. Consider a spherical capacitor with concentric spheres of radius a (inner) and b (outer). The outer sphere has a potential of $V_0 a/b$ and the inner sphere has a potential of V_0.

   (a) What is the natural coordinate system to describe $V(\vec{x})$? Can you use symmetry to reduce the number of dependent variables? (5)

   (b) Use the FISHTANK method to solve the electrostatic problem between the spheres and give $V(\vec{x})$ in this region. How would your answer be modified if the outer sphere was grounded and the inner sphere were held at $V_0\frac{b-a}{b}$? Would you expect the electric field to change? Why or why not? (12)

   (c) If $\vec{E}(\vec{x}) = \frac{V_0 a}{r^2}\hat{r}$ what is σ on the inner sphere? What is the total charge on the inner sphere? (7)

   (d) Calculate the internal energy as a function of V_0 using the integral

$$U = \frac{\epsilon_0}{2}\int_{r=a}^{r=b} E^2 d^3x = \frac{\epsilon_0}{2}\int_a^b E(r)^2(4\pi r^2)\,dr \tag{3}$$

   (I have changed the volume integral to a 1-D integral involving r only.) (8)