Midterm Exam: Electricity and Magnetism 322
March 7, 2002

Open book. Point values are given with each question. Total exam is worth $70 + 6$ points.

1. Problem 2.9 in Pollack and Stump. Do the sketch of the vector field in the xy plane ($z = 0$). (5 points for each section)

   Also: Given your answer in (c) for $\nabla \times \vec{F}(\vec{x})$ can you find a scalar function $f(\vec{x})$ such that $\vec{F}(\vec{x}) = \nabla f(\vec{x})$? (5)

2. A charge q sits on the z-axis distance d above the xy plane.

   (a) What is $\vec{x}\,'$? What is $r\,'$? (3)

   (b) You are interested in the potential at points along the x-axis distance x from the origin. What is $\vec{x} - \vec{x}\,'$? What is $|\vec{x} - \vec{x}\,'|$? (4)

   (c) What is the exact expression for $V(x, 0, 0)$ assuming $V(\infty) = 0$? Calculate V for $q = 10^{-9}$ C, $d = 0.01$ m, $x = 0.03$ m. (5)

   (d) Now consider the multipole expansion of the potential. Don't be concerned that $N = 1$; this just makes the sums simpler! So $q_1 = q$ and $\vec{x}_1 = \vec{x}\,'$. Using equation 3.102 give the expressions for Q, $\vec{p}$, $\mathcal{Q}_2$ in terms of q, d, and the appropriate vectors and tensors. (6)

   (e) Using 3.91 and the expressions and the values above calculate the monopole, dipole, and quadrupole contributions to $V(\vec{x})$ at $\vec{x} = 0.03\,\hat{\imath}$ (metres). You should obtain 300, 0, and -17 V respectively. WARNING: r in the multipole expansion is not $|\vec{x} - \vec{x}\,'|$! (10)

   (f) What is the percentage difference between the sum of the first three terms of multipole expansion and the exact result? How do you expect this difference to change at larger r? (Even if you got stuck you should be able to answer this one using the numbers in the previous section.) (4)

3. A charge of -10^{-11} C sits on the z-axis 4 cm above a grounded conducting plate whose lateral dimension is much larger than 4 cm. The upper surface of the plate is coincident with the xy plane.

   (a) Using the appropriate image charge for this symmetry calculate $\vec{E}$ at the surface of the plate directly below the charge i.e. at the origin. (don't forget the vector) (8)

   (b) What is $\vec{E}(0, 0, z)$ with $z < 0$? Based on this answer and the value of $\vec{E}$ in part (a) what is σ at the origin? (6)

   (c) Suppose that you move away from the origin to $(x, 0, 0)$ breaking the azimuthal symmetry of the problem. Does $\vec{E}(x, 0, 0)$ have any x or y component? Why or why not? (4)

4. BONUS: Show that $V(r, \theta) = Dr\cos\theta$ satisfies Laplace's equation for constant D. (6)