Midterm Exam: Electricity and Magnetism 322
November 21, 2002

Open book and notes. Point values are given with each question. Total exam is worth 65 points with a 10 point bonus question. Maximum mark can exceed 100%.

1. Show that $V(r,\theta,\phi) = \frac{C\cos\theta}{r^2}$ is a solution of Laplace's equation in spherical polar coordinates where C is a constant. (15)

2. Consider a spherical capacitor with concentric spheres of radius a (inner) and b (outer). The outer sphere has a potential of $V_0 a/b$ and the inner sphere has a potential of V_0.

   (a) What is the natural coordinate system to describe $V(\vec{x})$? Can you use symmetry to reduce the number of dependent variables? (5)

   (b) Use the FISHTANK method to solve the electrostatic problem between the spheres and give $V(\vec{x})$ in this region. (10)

   (c) If $\vec{E}(\vec{x}) = \frac{V_0 a}{r^2}\hat{r}$ what is σ on the inner sphere? What is the total charge on the inner sphere? Explain how this form of $\vec{E}$ matches the boundary conditions at the surface of the conductor. (8)

   (d) Calculate the internal energy as a function of V_0 using the integral

   $$U = \frac{\epsilon_0}{2}\int_{r=a}^{r=b} E^2 d^3x = \frac{\epsilon_0}{2}\int_a^b \{E(r)\}^2(4\pi r^2)\,dr \tag{1}$$

   (I have changed the volume integral to a 1-D integral involving r only.) (7)

3. A positive charge of q sits on the z-axis distance d above a grounded conducting plate whose lateral dimension is much larger than d. The upper surface of the plate is coincident with the xy plane.

   (a) What is the appropriate image charge for this problem? Give its value and location. Where is the real charge that is induced in the conductor located? (10)

   (b) Sketch the equipotentials in dashed lines and the electric field lines as solid lines with arrows. What do they look like in the region $z < 0$? (10)

   (c) BONUS: What is the approximate electric potential in terms of distance from the origin r if $r \gg d$? (Hint: the form of the answer is on this page.) Explain some of your reasoning. (10)