Midterm Exam: Electricity and Magnetism 322
February 16, 2005

One double sided formula sheet plus photocopies of the front and back covers of the book. Unit vector transforms available on request. Point values are given with each question. Total midterm is worth 70 points but I will mark it out of 60 with highest possible mark still 100%..

1. Consider the vector field
$$\vec{F}(\vec{x}) = \hat{r} = \frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}} \tag{1}$$

   (a) Calculate $\nabla \cdot \vec{F}$ in both spherical polar and Cartesian coordinates. (10)
   (b) Calculate $\oint \vec{F} \cdot d\vec{A}$ for a sphere of radius a centred at the origin using the method of your choice. (5)
   (c) Can we express $\vec{F}$ as the negative gradient of some scalar function $f(\vec{x})$ (i.e. $\vec{F} = -\nabla f$)? What is the relevant property of $\vec{F}$ that determines this? (5)
   (d) From the Helmholtz theorem or your knowledge of Green's solution of Poisson's equation give the answer for the form of $f(\vec{x})$. (don't need to calculate it). In this particular case I don't think the boundary conditions are strictly satisfied but a solution still exists. (5)

2. Suppose you have an infinitely long charged cylindrical shell with its axis along the z-axis (like the outer sheath of a coaxial cable). The radius of the cylinder is a and its surface charge density is σ.

   (a) What is the form of $\vec{E}$ based on the symmetry of the charge distribution and properties of $\vec{E}$? (Choose the most meaningful coordinate system and eliminate independent variables and vector components.) Give short justifying statements. (7)
   (b) Use Gauss Law in integral form to calculate $\vec{E}$ in the regions inside and outside the cylinder. (10)
   (c) State the boundary conditions for $\vec{E}$ and show that they are satisfied at the cylindrical shell. (8)

3. Consider an array of 4 charges
$$\begin{aligned} \vec{x}_1 &= d\hat{\imath} \\ \vec{x}_2 &= d\hat{\jmath} \\ \vec{x}_3 &= -d\hat{\imath} \\ \vec{x}_4 &= -d\hat{\jmath}. \end{aligned}$$
   Charges 1 and 3 are q and charges 2 and 4 are $-q$.

   (a) Calculate Q_T, $\vec{p}$, and $\mathcal{Q}_2$ the quadrupole moment tensor. What is the leading order dependence of $V(\vec{x})$ on r for large r? (10)
   (b) Give the approximate expression (i.e. keep the lowest order multipole) for $V(r, \theta, \phi)$ knowing that $\hat{r} \cdot \hat{\imath} = \sin\theta\cos\phi$ and $\hat{r} \cdot \hat{\jmath} = \sin\theta\sin\phi$. (5)
   (c) Make a sketch of the equipotential surfaces in the $x - y$ plane indicating regions of positive and negative potential. (hint: it is not quite the same as the linear quadrupole but instead looks like a different hydrogen d orbital) (5)