Midterm Quiz: Electricity and Magnetism 322
March 30, 2005

Closed book, one double-sided "anything goes" formula sheet, photocopies of front and back covers. Point values are given with each question. Total quiz is worth 60 points.

1. You know that the $P_\ell(\cos\theta)$ give the θ dependence for separable solutions of Laplace's equation in spherical polar coordinates so that

$$V(r,\theta,\phi) = R(r)P_\ell(\cos\theta). \tag{1}$$

   (a) What are the $R(r)$ functions that solve Laplace's equation for a particular value of ℓ (5)

   (b) State the $V(r,\theta,\phi)$ solutions explicitly for $\ell = 1$. What are the names we give to these two solutions? (hint: it isn't George!) (10)

2. (a) Show that $V(x,y,z) = C_0\cos(\frac{\pi x}{a})\sinh(\frac{\pi y}{a})$ is a solution of Laplace's equation (do it quickly if you wish). (8)

   (b) If this was a partial solution to the electrostatic problem in a long rectangular pipe where would the "grounded" sides be? (answer isn't unique but base it on what we have done in class) (3)

   (c) If the other sides are solid conductors why do we know this is only a partial solution? (Hint: consider the variation of $V(x,y,z)$ on the other sides.) (5)

3. Consider a parallel plate capacitor with a plate separation of d much less than its lateral size. The lower plate sits in the xy-plane and is grounded. The upper plate has a potential of V_0.

   (a) What is the natural coordinate system to describe $V(\vec{x})$? Can you use symmetry to reduce the number of dependent variables? Briefly justify. (6)

   (b) Use the FISHTANK method to solve the electrostatic problem between the plates and give $V(\vec{x})$ in this region. (9)

4. Consider a grounded infinite plate with a surface in the $x-y$ plane. A charge of q sits a distance d above the plate on the z-axis.

   (a) What is the size and location of the image charge used to solve this problem? (5)

   (b) What is the shape of the equipotential near the surface? The direction of the electric field? (5)

   (c) Where are the real charges in this problem (aside from the charge at $z = d$)? (5)