BASIC CONCEPTS OF FORMAL LOGIC

Logic is one of the oldest subjects of formal instruction. It was probably taught at Plato's Academy and at other schools in ancient Greece in the fourth century B.C., and the oldest surviving systematic treatise on logic, Aristotle's *Prior and Posterior Analytics*, was written during this period as well. The antiquity of formal logic is not surprising given that logic is the study of the correct forms of reasoning, a matter of concern to all thinking persons.

Because correct reasoning is something we try to do when we speak and think, it is important not to confuse the logical aspects of speech and thought with their psychological and linguistic properties. Logic is not concerned with the mental processes that take place in our mind when we are actually thinking. Nor is logic concerned with the linguistic properties of different languages and the ways in which words come to acquire their significance in particular linguistic communities. The standards of correct reasoning established by logic are meant to apply to the evaluation of reasoning by all persons at all times and places.

Two properties of reasoning, in particular, are studied by formal logic: consistency and valid inference. In order to understand what consistency and valid inference are, they must be clearly distinguished from something else with which they are often confused, namely truth.

TRUTH

Truth, and its opposite, falsity, are properties that belong to only one kind of thing, namely propositions, or statements. Propositions and statements are capable of being true or false, but this does not hold of all sentences. Sentences such as commands, questions, and expressions of volition are neither true nor false. For example, the sentence "Pass the salt!" is in the imperative mood and is typically used to give a command; imperative sentences, however, are neither true nor false. For our purposes, we shall restrict our discussion to statements or propositions, which are capable of being either true or false.

Law of Excluded Middle: It is characteristic of propositions that they obey the Law of Excluded Middle, also known as the Law of Bivalence, which states that every proposition must be either true or false. In other words, any middle position between truth and falsity is excluded. From the Law of Excluded Middle it follows that, for any given proposition and the negation of that proposition, it must be the case that one of them is true and one false. For if a proposition cannot be anything other than true or false, then, if a proposition is true, its negation must be false, and if a proposition is false, its negation must be true.

Law of Non-Contradiction: The propositions we are dealing with also obey what is known as the Law of Non-Contradiction, which states that it is impossible for a proposition and its negation both to be true at the same time. In other words, at one and the same time, one cannot truthfully both assert and deny that something is the case.

ARGUMENTS, VALIDITY, AND SOUNDNESS

Validity is a property of arguments alone; it does not apply to a single proposition, or even to a set of unconnected propositions. Thus, before we can define validity, we have to define what an argument is.

For the purposes of formal logic, an *argument* is defined simply as a set of propositions, one of which can be inferred from the others. The proposition that is inferred from the other propositions is the *conclusion* of the argument; the propositions from which the conclusion is inferred are the *premises* of the argument. Thus, in every argument there is one proposition that acts as a conclusion and one or more other propositions that act as premises.

Usually, we indicate that we are making an argument by using certain words or phrases to signal that certain propositions are to be understood as premises or as a conclusion; these are called *premise* and *conclusion indicators*. Typical premise indicators are words or phrases such as: because, for, since, as, inasmuch as, it follows from, as shown by, as indicated by, the reason is that. Typical conclusion indicators are words or phrases such as: hence, therefore, then, consequently, so, accordingly, it follows that, we may infer/derive/deduce that, which proves that, which shows that.

An *enthymeme* is an argument in which the conclusion or one of the premises has been left unstated.

A *sorites* is a connected series of arguments in which the conclusion of one argument also serves as a premise in another argument.

Validity is the standard of good or correct deductive arguments: an argument is said to be deductively valid if and only if whenever all the premises are true, the conclusion *must* also be true. (This is to be contrasted with an *inductive* argument in which the truth of the premises indicates only that it is *probable* that the conclusion is true.) In other words, a deductively valid argument is one that can *never* have, at one and the same time, true premises and a false conclusion. Thus, to show that a particular argument is valid, it is not sufficient to show that its premises and conclusion happen to be true here and now. Nor is it necessary that the conclusion or any of the premises of a deductively valid argument actually be true. To claim that an argument is deductively valid is to say that, whatever the truth or falsity of its premises and conclusion, it could *never* be the case that, at one and the same time, all of the premises of that argument were true and its conclusion false.

A *sound* argument is a valid deductive argument all of whose premises in fact are true. Thus, the soundness of an argument requires two things: deductive validity and true premises. These two requirements are independent of each other; an argument might have true premises, but not be valid, or it might be valid, but not all of its premises be true.

ARGUMENT TYPES

While there are many different kinds of arguments, in this introduction we will be concerned primarily with the basic vocabulary of truth-functional logic. All sentences can be divided into two classes. Compound sentences are those which contain another sentence as a part, and simple sentences are those which do not. For example, "He is sad and I am glad" is compound, while "He is sad" is simple. Truth-functional logic is based on the fact that there are many compound sentences whose truth-value (i.e. whose truth or falsity) is determined by the truth-values of their constituent sentences. Thus "He is sad and I am glad" is true when "He is sad" and "I am glad" are both true. The sentence "Either it will rain or it will snow" is false when it will neither rain nor snow. Compound sentences whose truth value is determined in this way are called truth-functions.

The valid argument forms listed below are based on truth-functions. In each case the premises and the conclusion are truth-functionally related in such a way that if the premises are all true, then the conclusions must also be true. This means that every argument which has one

of these forms will necessarily be valid.

TRUTH-FUNCTIONAL VOCABULARY AND SYMBOLISM

Vocabulary	Symbolism	Meaning
1) Negation:	~P	"P" is false
2) Conjunction:	P^Q	Both "P" and "Q" are true
3) Disjunction:	PvQ	Either "P" is true, or "Q" is true, or both are true
4) Material Implication:	P>Q	If "P" is true, then "Q" is also true
5) Assertion	P	"P" is true

VALID ARGUMENT FORMS

1) Modus Ponens

$$P>Q$$
$$\underline{P}$$
$$Q$$

2) Modus Tollens

$$P>Q$$
$$\underline{\sim Q}$$
$$\sim P$$

3) Disjunctive Syllogism

$$PvQ \qquad\qquad PvQ$$
$$\underline{\sim P} \qquad\qquad \underline{\sim Q}$$
$$Q \qquad\qquad P$$

4) Chain Argument

$$P>Q$$
$$\underline{Q>R}$$
$$P>R$$

EXERCISE 1: Symbolize the following arguments, name the argument form, and supply the missing conclusion.

<div align="right">
<u>Symbolic</u>
<u>Notation & Name</u>
</div>

1)   If Mario collects 100 star bits, then he will get a free life.
        <u>Mario has collected 100 coins.</u>

2)   If Mario defeats Bowser, then Peach will be freed.
        <u>Peach will not be freed</u>.

3)   If Mario is collecting green stars, then Mario has got all the prankster stars.
        <u>Mario is collecting green stars.</u>

4)   Either Mario can do a sideways jump, or Mario will not be collecting any green stars.
        <u>Mario will be collecting green stars.</u>

5)   If Mario saves Peach, then Mario will watch a boring cut scene.
        <u>If Mario will watch a boring cut scene, then he does it to get to the green stars.</u>

6)   Either Mario will defeat Flip Flop, or Mario will die trying.
        <u>Apparently he will not defeat Flip Flop.</u>

7)   If Mario has a white suit on, then he can walk on clouds.
        <u>Mario cannot walk on clouds.</u>

8)   If Mario can ride dragons, then Alice can follow rabbits down holes.
        <u>Mario can ride dragons</u>.

9)   Either Mario is the mascot here, or Luigi is.
        <u>Luigi is most definitely not the mascot here.</u>

10)   If Yoshi can eat everything, then he shouldn't be so scared all the time.
        <u>If he shouldn't be so scared all the time, then he should stick around longer.</u>

Excerize #2: Fill in the conclusion, give the symbolic notation, and name the kind of argument.

1)   If he has an alibi, then he is not a suspect.
        But he is a suspect.

2)   If you buy a new coat, you won't be able to buy your textbooks for next term.
        I see you've bought a new coat.

3)   Ellen would never do anything that wasn't in her own self-interest.
        Violating confidentiality would not be in Ellen's self-interest.

4)   Either Tom is home or he is at work.
        Tom is not at home.

5)   If Philosophy is the love of wisdom, then it studies thinking.
        If it studies thinking, then it studies logic.

6)   Either the Minoans lived on Crete or on Cyprus.
        The Minoans did not live on Cyprus.

7)   If the Minoans lived in Crete and the Mycenaeans lived in the
        Peloponnese, then there was bound to be trouble.
        The Minoans lived in Crete and the Mycenaeans lived in the
        Peloponnese.

8)   If Achilles goes to Troy, Odysseus will get the armour.
        If Odysseus gets the armour, Ajax will go mad.

9)   Either Achilles will go to Troy, or Odysseus and Agamemnon will die.
        Odysseus will not die.

10)   If Odysseus eats the lotus plant, he will never see Penelope again.
        Odysseus will see Penelope again.

EXERCIZE #3: Fill in the missing premise, name the argument form, and give the symbolic notation.

1)   If Odysseus goes to Hades, he will meet Achilles' dead soul.

        C: Odysseus will meet Achilles' dead soul.

2)   If Clytemnestra remains faithful, Agamemnon will have a happy return.

        C: Clytemnestra does not remain faithful.

3)   Clytemnestra does not remain faithful.

        C: Orestes will have some mother issues.

4)   Either Orestes will get his revenge and Electra will help or Clytemnestra will live.

        C: Electra will help.

5)   If Orestes gets revenge then the Furies will show up and be put in their place.

        C: If Clytemnestra dies the Furies will show up and be put in their place.

6)   If Oedipus doesn't go to Thebes, there will be no problems.

        C: Oedipus goes to Thebes.

7)   If Oedipus goes the Thebes and Theseus stays from Athens, then the Minotaur will live.

       C: Either Oedipus will not go to Thebes or Theseus will not stay from Athens.

8)   Theseus will not stay from Athens.

       C: Theseus will go to Crete.

9)   If Theseus would bring Ariadne to Athens and marry her, he would be a real hero.

       C: Theseus would not bring Ariadne to Athens.

10)   Theseus does not have a developed sense of filial piety.

       C: Theseus would not raise the white sail before falling asleep.

Exercise #4: Just for practice.
1)   If Socrates has no concept of justice, then he cannot undermine the others.
       But he can undermine the others.
       _____

2)   Either Socrates is a mean man, or there's a point to showing the ignorance of others.
       Socrates is not a mean man.
       _____

3)   If Polemarchus is a real warrior, he cannot accept Socrates' concept of justice.
       Polemarchus is a real warrior.
       _____

4)   If Cephalus knows justice, then we should all be truth tellers all the time.
       If we should all be truth tellers all the time, then no one would be in a relationship.
       _____

5)   If Thrasymachus should be paid, then he should say something new.
        Thrasymachus should say something new.
        _____

6)   If Glaucon is a good brother, then he should not try to outdo Adeimantus.
        If he should not try to outdo Adeimantus, then he should keep quiet.
        _____

7)   If Glaucon were injust, then he would use the Ring of Gyges.
        But he would not use the Ring of Gyges.
        _____

8)   If Socrates were just, then he would not pick on others.
        But Socrates is not just.
        _____

9)   If Lysias is as smart as his reputation, then he should say something in the *Republic*.
        Lysias is as smart as his reputation.
        _____

10)   Either the *Republic* has an ultimate definition of justice, or it doesn't.
        If it doesn't, then there is no point to reading it.
        But there is a point to reading it.
        _____

Answers to Exercizes:

Exercise #2
1) He does not have an alibi.
2) You won't be able to buy a new textbook for next term.
3) Ellen would not violate confidentiality.
4) Tom is at work.
5) If Philosophy is the love of wisdom, then it studies logic.
6) The Minoans lived on Crete.

7) There was bound to be trouble.
8) If Achilles goes to Troy, then Ajax will go mad.
9) Achilles will go to Troy.
10) Odysseus will not eat the lotus plant.

Exercise #3
1) Odysseus goes to Hades.
2) Agammemnon will not have a happy return.
3) If Clytemnestra does not remain faithful, then Orestes will have some mother issues.
(also correct: If Orestes will not have mother issues, then Clytemnestra wil remain faithful.)
4) Clytemnestra will not live.
5) If Clytemnestra dies, then Orestes gets his revenge.
6) There wil be problems.
7) The Minotaur will not live.
8) Either Theseus will stay from Athens or Theseus will go to Crete.
9) Theseus would not be a real hero.
10) If Theseus would raise the white sail before going to sleep, then Theseus has a developed sense of filial piety.

Exercise #4
1) Socrates has a concept of justice.
2) There's a point to showing the ignorance of others.
3) He cannot accept Socrates' concept of justice.
4) If Cephalus knows justice, then no one would be in a relationship.
5) INVALID: there is nothing that can be concluded from these two sentences. This is an example of the lead up to committing the fallacy of "Affirming the Consequent".
6) If Glaucon is a good brother, then he should keep quiet.
7) Glaucon is not injust.
8) INVALID: there is nothing that can be concluded from these sentences. This is an example of the lead up to committing the fallacy of "Denying the Antecedent".
9) He should say something in the *Republic*
10) The *Republic* has a definition of justice.

And with that, you're probably ready to start writing your own arguments. What follows is a template designed to facilitate writing essays. The template gives the structure for a simple chain of arguments that you will fill out for the assignments you have in this course. The argument outline must be completed and handed in before essays are due; I'll need time to evaluate them all, and once you get it back, you'll need time to write the essay. It must also be handed in again with the essay when the essay is due so that I may compare the outline with what you actually write. If the outline is not handed in with the essay, you will receive a zero grade for the outline. The essay outline will be marked out of ten, and it will count for ten percent of your essay grade.

I strongly suggest printing off a few copies of the template and using some of them for rough work. Writing a good argument outline is, I believe, the hardest thing you'll be asked to do in the humanities, or indeed in any subject; leave yourself some time to work at it and don't be afraid to edit it extensively before coming up with a finalized version.

The breakdown of marks for the essay outline is as follows:

1 mark per correctly formulated proposition, to a maximum of seven (no proposition gets counted twice). A "correctly formulated proposition" will have one of the forms given above in the "Truth Functional Vocabulary and Symbolism" section, and will work properly with other propositions in the outline to form an argument.

1 mark for using three different kinds of the truth functional arguments covered so far. The disjunctive syllogism formulated may not use a proposition of the type, "Either x is y, or x is not y"; it must use a proposition of the type, "Either x is y, or x is z".

1 mark for interesting content. If your arguments are as plain as day, even though the argument outline may be valid and sound, no one will want to read the essay or care about your point.

1 mark for properly symbolizing and naming the arguments you use in your outline.

Some problems to watch for as you create your argument outline:

1) Your sentences become too long. It seems to me there are two separate problems here. The first is that you spend a lot of unnecessary verbiage hemming and hawing in expressing your point. Simplest is best. Cut out anything you don't need to get your point across.

2) The second problem is cramming too many ideas into one sentence. Here the problem isn't with the clarity of your expressions, but that you're simply trying to describe too many ideas and connections. Again, keep it simple.

3) In a similar vein, do not try to add things to your conclusion that are not called for in the premises. If I see extra clauses in the conclusions that are not in the premises, I know you're trying to get something from nothing.

4) Name dropping. It's all very well in the essay itself, but keep names out of the outline. Sticking names in is a quick way to distract yourself and get off topic; what we really want to deal with are the ideas themselves.

5) Repetition. This is a big problem that everyone has at some point or another. It's the problem of the same phrase showing up again and again in your outline, and it is very tough to avoid once you fall into the habit of it. My best advice is to try to make sure each of your arguments is qualitatively different from each other; each argument should express an idea or connection that is very different from that expressed in the other arguments.

6) Vagueness. Again, another tough one to avoid, especially when you're consumed with the search for simplicity. A good idea should be clear as well as simple; if things are getting to

vague, then you haven't finished thinking through the connection you're trying to express.

Argument Outline

The question I am answering is:

My overall argument is (NAME):   **SYMBOLIC NOTATION**

P1_____

P2_____

START HERE! My overall conclusion (thesis) is:

C1_____

My first sub-argument is (NAME):

P3_____

P4_____

C2
(P1)_____

My second sub-argument is (NAME):

P5_____

P6_____

C3
(P2)_____

Argument Diagram:

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        P3-----------------P4              P5-----------------P6
                |                                  |
                |                                  |
        P1 (C2)  --------------------------------------------P2 (C3)
                              |
                              |
                        C1 (Thesis)
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