

BASIC CONCEPTS OF FORMAL LOGIC

Logic is one of the oldest subjects of formal instruction. It was probably taught at Plato's Academy and at other schools in ancient Greece in the fourth century B.C., and the oldest surviving systematic treatise on logic, Aristotle's *Prior and Posterior Analytics*, was written during this period as well. The antiquity of formal logic is not surprising given that logic is the study of the correct forms of reasoning, a matter of concern to all thinking persons.

Because correct reasoning is something we try to do when we speak and think, it is important not to confuse the logical aspects of speech and thought with their psychological and linguistic properties. Logic is not concerned with the mental processes that take place in our mind when we are actually thinking. Nor is logic concerned with the linguistic properties of different languages and the ways in which words come to acquire their significance in particular linguistic communities. The standards of correct reasoning established by logic are meant to apply to the evaluation of reasoning by all persons at all times and places.

Two properties of reasoning, in particular, are studied by formal logic: consistency and valid inference. In order to understand what consistency and valid inference are, they must be clearly distinguished from something else with which they are often confused, namely truth.

TRUTH

Truth, and its opposite, falsity, are properties that belong to only one kind of thing, namely propositions, or statements. Propositions and statements are capable of being true or false, but this does not hold of all sentences. Sentences such as commands, questions, and expressions of volition are neither true nor false. For example, the sentence "Pass the salt!" is in the imperative mood and is typically used to give a command; imperative sentences, however, are neither true nor false. For our purposes, we shall restrict our discussion to statements or propositions, which are capable of being either true or false.

Law of Excluded Middle: It is characteristic of propositions that they obey the Law of Excluded Middle, also known as the Law of Bivalence, which states that every proposition must be either true or false. In other words, any middle position between truth and falsity is excluded. From the Law of Excluded Middle it follows that, for any given proposition and the negation of that proposition, it must be the case that one of them is true and one false. For if a proposition cannot be anything other than true or false, then, if a proposition is true, its negation must be false, and if a proposition is false, its negation must be true.

Law of Non-Contradiction: The propositions we are dealing with also obey what is known as the Law of Non-Contradiction, which states that it is impossible for a proposition and its negation both to be true at the same time. In other words, at one and the same time, one cannot truthfully both assert and deny that something is the case.

ARGUMENTS, VALIDITY, AND SOUNDNESS

Validity is a property of arguments alone; it does not apply to a single proposition, or

even to a set of unconnected propositions. Thus, before we can define validity, we have to define what an argument is.

For the purposes of formal logic, an *argument* is defined simply as a set of propositions, one of which can be inferred from the others. The proposition that is inferred from the other propositions is the *conclusion* of the argument; the propositions from which the conclusion is inferred are the *premises* of the argument. Thus, in every argument there is one proposition that acts as a conclusion and one or more other propositions that act as premises.

Usually, we indicate that we are making an argument by using certain words or phrases to signal that certain propositions are to be understood as premises or as a conclusion; these are called *premise* and *conclusion indicators*. Typical premise indicators are words or phrases such as: because, for, since, as, inasmuch as, it follows from, as shown by, as indicated by, the reason is that. Typical conclusion indicators are words or phrases such as: hence, therefore, then, consequently, so, accordingly, it follows that, we may infer/derive/deduce that, which proves that, which shows that.

An *enthymeme* is an argument in which the conclusion or one of the premises has been left unstated.

A *sorites* is a connected series of arguments in which the conclusion of one argument also serves as a premise in another argument.

Validity is the standard of good or correct deductive arguments: an argument is said to be deductively valid if and only if whenever all the premises are true, the conclusion *must* also be true. (This is to be contrasted with an *inductive* argument in which the truth of the premises indicates only that it is *probable* that the conclusion is true.) In other words, a deductively valid argument is one that can *never* have, at one and the same time, true premises and a false conclusion. Thus, to show that a particular argument is valid, it is not sufficient to show that its premises and conclusion happen to be true here and now. Nor is it necessary that the conclusion or any of the premises of a deductively valid argument actually be true. To claim that an argument is deductively valid is to say that, whatever the truth or falsity of its premises and conclusion, it could *never* be the case that, at one and the same time, all of the premises of that argument were true and its conclusion false.

A *sound* argument is a valid deductive argument all of whose premises in fact are true. Thus, the soundness of an argument requires two things: deductive validity and true premises. These two requirements are independent of each other; an argument might have true premises, but not be valid, or it might be valid, but not all of its premises be true.

ARGUMENT TYPES

While there are many different kinds of arguments, in this introduction we will be concerned primarily with the basic vocabulary of truth-functional logic. All sentences can be divided into two classes. Compound sentences are those which contain another sentence as a part, and simple sentences are those which do not. For example, “He is sad and I am glad” is compound, while “He is sad” is simple. Truth-functional logic is based on the fact that there are many compound sentences whose truth-value (i.e. whose truth or falsity) is determined by the truth-values of their constituent sentences. Thus “He is sad and I am glad” is true when “He is sad” and “I am glad” are both true. The sentence “Either it will rain or it will snow” is false when it will neither rain nor snow. Compound sentences whose truth value is determined in this way are called truth-

functions.

The valid argument forms listed below are based on truth-functions. In each case the premises and the conclusion are truth-functionally related in such a way that if the premises are all true, then the conclusions must also be true. This means that every argument which has one of these forms will necessarily be valid.

TRUTH-FUNCTIONAL VOCABULARY AND SYMBOLISM

<u>Vocabulary</u>	<u>Symbolism</u>	<u>Meaning</u>
1) Negation:	$\sim P$	“P” is false
2) Conjunction:	$P \wedge Q$	Both “P” and “Q” are true
3) Disjunction:	$P \vee Q$	Either “P” is true, or “Q” is true, or both are true
4) Material Implication:	$P \supset Q$	If “P” is true, then “Q” is also true

VALID ARGUMENT FORMS

1) Modus Ponens	$\begin{array}{l} P \supset Q \\ \underline{P} \\ Q \end{array}$	
2) Modus Tollens	$\begin{array}{l} P \supset Q \\ \underline{\sim Q} \\ \sim P \end{array}$	
3) Disjunctive Syllogism	$\begin{array}{l} P \vee Q \\ \underline{\sim P} \\ Q \end{array}$	$\begin{array}{l} P \vee Q \\ \underline{\sim Q} \\ P \end{array}$
4) Chain Argument	$\begin{array}{l} P \supset Q \\ \underline{Q \supset R} \\ P \supset R \end{array}$	

SAMPLE EXERCIZES

To gain facility in the conscious use of these argument forms, complete the following exercises.

EXERCISE 1: Symbolize the following arguments, name the argument form, and supply the missing conclusion.

Symbolic
Notation & Name

- 1) If he has an alibi, then he is not a suspect.
But he is a suspect.

- 2) If you buy a new coat, you won't be able to buy your textbooks for next term.
I see you've bought a new coat.

- 3) Ellen would never do anything that wasn't in her own self-interest.
Violating confidentiality would not be in Ellen's self-interest.

- 4) Either Tom is home or he is at work.
Tom is not at home.

- 5) If Philosophy is the love of wisdom, then it studies thinking.
If it studies thinking, then it studies logic.

- 6) Either the Minoans lived on Crete or on Cyprus.
The Minoans did not live on Cyprus.

- 7) If the Minoans lived in Crete and the Mycenaeans lived in the Peloponnese, then there was bound to be trouble.
The Minoans lived in Crete and the Mycenaeans lived in the
Peloponnese.

- 8) If Achilles goes to Troy, Odysseus will get the armour.
If Odysseus gets the armour, Ajax will go mad.

- 9) Either Achilles will go to Troy, or Odysseus and Agamemnon will die.
Odysseus will not die.
- 10) If Odysseus eats the lotus plant, he will never see Penelope again.
Odysseus will see Penelope again.

EXERCIZE #2: Fill in the missing premise, name the argument form, and give the symbolic notation.

- 1) If Odysseus goes to Hades, he will meet Achilles' dead soul.

C: Odysseus will meet Achilles' dead soul.

- 2) If Clytemnestra remains faithful, Agamemnon will have a happy return.

C: Clytemnestra does not remain faithful.

- 3) Clytemnestra does not remain faithful.

C: Orestes will have some mother issues.

- 4) Either Orestes will get his revenge and Electra will help or Clytemnestra will live.

C: Electra will help.

- 5) If Orestes gets revenge then the Furies will show up and be put in their place.

C: If Clytemnestra dies the Furies will show up and be put in their place.

- 6) If Oedipus doesn't go to Thebes, there will be no problems.

C: Oedipus goes to Thebes.

- 7) If Oedipus goes the Thebes and Theseus stays from Athens, then the Minotaur will live.

- 8) C: Either Oedipus will not go to Thebes or Theseus will not stay from Athens.
Theseus will not stay from Athens.

C: Theseus will go to Crete.

- 9) If Theseus would bring Ariadne to Athens and marry her, he would be a real hero.

C: Theseus would not bring Ariadne to Athens.

- 10) Theseus does not have a developed sense of filial piety.

C: Theseus would not raise the white sail before falling asleep.

EXERCISE #3: ENTHYMEMES

Rewrite the following arguments in propositional form, supplying the missing premises. Name the argument, and symbolize it.

1) Many folk-oriented songs are popular, so this song will be popular.

2) He is a leprous man, for he is unclean. (Leviticus 13)

3) "He would not take the crown. Therefore, 'tis certain he was not ambitious." (*Julius Caesar* III, ii)

4) "Man tends to increase at a rate greater than his means of subsistence. Consequently, man is occasionally subject to a severe struggle for existence." (Charles Darwin)

5) "Yond' Cassius has a lean and hungry look . . . such men are dangerous." (*Julius Caesar* I, ii)

6) "If the glove doesn't fit, you must acquit." Johnny Cochrane

7) "I think, therefore I am." Rene Descartes

8) "I'm not your lover, I'm not your friend. I am something you can never comprehend." Prince, "I Would Die 4 U"

And with that, you're probably ready to start writing your own arguments. What follows is a template designed to facilitate writing essays. The template gives the structure for a simple chain of arguments that you will fill out for the next essay you are to write in this course. The argument outline must be completed and handed in two weeks before the essay is due; I'll need a week to evaluate them all, and once you get it back, you'll need a week to write the essay. It must also be handed in again with the essay when the essay is due so that I may compare the outline with what you actually write. If the outline is not handed in with the essay, you will receive a zero grade for the outline. The essay outline will be marked out of ten, and it will count for ten percent of your essay grade.

I strongly suggest printing off a few copies of the template and using some of them for rough work. Writing a good argument outline is, I believe, the hardest thing you'll be asked to do in the humanities, or indeed in any subject; leave yourself some time to work at it and don't be afraid to edit it extensively before coming up with a finalized version. For your next essay, you must stick to the basic structure in the outline. In future essays, you will be given more freedom to modify how the arguments of your essay fit together, which premises will receive further justification and which will not, etc. But for now, show me that you can control the basic structure provided.

The breakdown of marks for the essay outline is as follows:

1 mark per correctly formulated proposition, to a maximum of seven (no proposition gets counted twice). A "correctly formulated proposition" will have one of the forms given above in the "Truth Functional Vocabulary and Symbolism" section, and will work properly with other propositions in the outline to form an argument.

1 mark for using three different kinds of the truth functional arguments covered so far.

1 mark for interesting content. If your arguments are as plain as day, even though the argument outline may be valid and sound, no one will want to read the essay or care about your point.

1 mark for properly symbolizing and naming the arguments you use in your outline.

Argument Outline

My overall argument is (NAME):

**SYMBOLIC
NOTATION**

P1 _____

P2 _____

START HERE! My overall conclusion (thesis) is:

C1 _____

My first sub-argument is (NAME):

P3 _____

P4 _____

C2
(P1) _____

My second sub-argument is (NAME):

P5 _____

P6 _____

C3
(P2) _____

Argument Diagram:

