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Geothermics 35 (2006) 44–58

GEOTHERMICS

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Potential use of particle tracking in the analysis of low-temperature geothermal developments

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Received 29 June 2005; accepted 8 November 2005

Abstract

Projects aimed at the exploitation of low-temperature geothermal systems for direct applications rarely undergo detailed analysis prior to their undertaking, and this is especially true in the case of smaller developments. This may be partly related to a lack of numerical modelling during the analysis and design of these systems, as a result of the relative inaccessibility of advective–conductive heat flow simulators. Particle tracking may provide a cost- and time-efficient alternative method for estimating production well temperatures. The results of this study indicate that particle tracking may be useful in the analysis of heat transport in injection–production doublets if pumping rates are sufficiently high relative to well spacing and aquifer thickness.

Keywords: Direct-use projects; Particle tracking; Advective heat flow; Numerical modelling; Injection wells

1. Introduction

The use of low-temperature geothermal waters for heating and other direct applications is growing in popularity in many areas and this practice is likely to increase in the future as we seek environmentally compatible sources of energy (Fridleifson, 2003). Fridleifson (2001) indicates that there were approximately 400,000 heat pumps in use in the United States in 1999 and that this number was increasing at a rate of 10% per year. Other nations, such as Austria, Canada, Germany, Sweden and Switzerland, have also experienced increasing demand for heat pump technology.

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0375-6505/$30.00 © 2005 Published by Elsevier Ltd on behalf of CNR.
doi:10.1016/j.geothermics.2005.11.001

Nomenclature

c_{sat}	heat capacity of saturated porous medium [J kg^{-1} K^{-1}]
C_{pr}	heat capacity of rock [J kg^{-1} K^{-1}]
C_{pw}	heat capacity of water [J kg^{-1} K^{-1}]
D	distance between injection and production wells [m]
D_{T}	thermal retardation factor
h	thickness of aquifer [m]
N_{aq}	number of flowpaths originating from regional groundwater flow
N_i	number of flowpaths for source i
N_{total}	total number of flowpaths
Pe_{doublet}	Peclet number for doublet
Q	pumping rate [m^3 s^{-1}]
$t_{\mathrm{T}}^{\mathrm{BT}}$	breakthrough time for heat [s]
$t_{\mathrm{W}}^{\mathrm{BT}}$	breakthrough time for water or a conservative tracer [s]
T^*	dimensionless temperature
T_{in}	injection temperature [°C]
T_{measured}	measured temperature [°C]
T_{o}	temperature of native groundwater [°C]
$T_{\mathrm{predicted}}$	temperature predicted by model [°C]
T_{prod}	production well temperature [°C]
Greek letters	
κ_{sat}	thermal conductivity of saturated porous medium [W m^{-1} K^{-1}]
ρ_{r}	density of rock [kg m^{-3}]
ρ_{sat}	density of saturated porous medium [kg m^{-3}]
ρ_{w}	density of fluid [kg m^{-3}]
φ	porosity

Many of these heat pumps are of the “open loop” style and rely on groundwater to transfer heat. Hydrogeological analysis of these systems is therefore necessary to ensure that the geothermal heat pump systems are designed in a sustainable fashion.

Hydrogeological research offers a great deal of insight into the behaviour of geothermal reservoirs and aquifer thermal energy storage (ATES) but this research has for the most part been focused on large systems. Powerful computer modelling codes can be applied to solve hydraulic head and temperature fields in a coupled manner, considering both advective and conductive heat flow. While these codes are commonly used in academia for research and in some government agencies and major consulting companies to analyze larger projects, they are not widely used in the analysis and design of small-to-intermediate geothermal heat pump systems. The reasons for the infrequent application of these codes in common practice include cost, inappropriate personnel training, and, in many codes, a lack of user-friendly graphic interfaces. However, as the demand for these technologies increases, we must be prepared to simplify the design process of these developments without sacrificing accuracy and efficiency.

Most small-scale developments of groundwater resources for heating and cooling purposes use injection–production doublets. In such systems, water is withdrawn from an aquifer in one well, passed through a heat exchanger, and then injected back into the same aquifer in a second well. The mathematics of transport in doublets has been studied in detail in terms of solutes (Grove et al., 1970) and heat (Gringarten and Sauty, 1975). The solutions derived by Gringarten and Sauty (1975) provide an estimate of the time of first arrival of colder water at the production well, or "thermal breakthrough". The analytical solution, however, applies to a homogeneous medium and boundary conditions are laterally infinite, although it is possible to analyze some more complex situations through the use of superposition techniques. Although it is possible to solve for temperatures at the injection well using an analytical solution, this approach generally requires complex mathematical methods, and a survey of the relevant literature suggests that this is rarely done in practice. The shortcomings of analytical models, combined with the relative inaccessibility of powerful numerical simulators, have led to a lack of analysis of small-scale heating and cooling projects that rely on low-temperature geothermal waters. In the current study, particle tracking is proposed as an alternate method for analyzing heat flow in a doublet system.

2. Theoretical background

Particle tracking is a well-known technique in groundwater modelling and is commonly used to help visualize certain transport problems as well as to trace solute and contaminant paths (Anderson and Woessner, 1992). Recently, this technique has received attention because of its utility in delineating well capture zones in groundwater protection studies (e.g. Frind et al., 2002; Neupauer and Wilson, 2003) and the efficiency of pump and treat systems used in groundwater remediation (e.g. Luo and Kitanidis, 2004). It has, however, also been shown that particle tracking can be used to estimate solute concentrations at production wells (Javandel et al., 1984; Johnson et al., 1994). This technique assumes that each pathline represents a unit solute concentration and the concentration at the production well is the result of dilution. In the current study, it is proposed that particle tracking can be used in combination with a numerical solution to fluid flow to estimate temperatures at production wells in a similar fashion under some particular circumstances.

Advective heat transport in porous media is very similar to the transport of a solute that is affected by retardation due to sorption of a chemical species, and can be related to the transport of water or a conservative solute using a thermal retardation factor (e.g. Keys and Brown, 1978; Chevalier and Banton, 1999; Shook, 2001). The thermal retardation factor (D_{T}) can be defined as follows (Shook, 2001)

$$D_{\mathrm{T}}=\frac{(1-\varphi)\rho_{\mathrm{r}}C_{\mathrm{pr}}}{\varphi\rho_{\mathrm{w}}C_{\mathrm{pw}}} \tag{1}$$

where φ is porosity, ρ_{r} the rock density, C_{pr} the rock-specific heat, ρ_{w} the fluid density and C_{pw} the specific heat of the fluid. This factor relates the breakthrough times for temperature ($t_{\mathrm{T}}^{\mathrm{BT}}$) and water ($t_{\mathrm{W}}^{\mathrm{BT}}$) as follows:

$$t_{\mathrm{T}}^{\mathrm{BT}}=t_{\mathrm{W}}^{\mathrm{BT}}(1+D_{\mathrm{T}}) \tag{2}$$

This relationship has been known for quite some time (e.g. Gringarten and Sauty, 1975; Keys and Brown, 1978; Doughty et al., 1982). Using this relationship in conjunction with particle tracking may allow for some inferences on the behaviour of injection–production doublets by examining transport by advection along a pathline. The arrival of a thermal front at the production well will follow the same pattern as the arrival of water or a conservative solute but will be shifted in time due to thermal retardation. This assumes that the pathlines represent streamtubes carrying equal amounts of fluid and that each streamtube has a temperature equal to a pathline flowing through its centre. It should be noted that in many cases the streamlines generated by a particle-tracking program do not coincide with those produced by a flownet. However, if particles are placed appropriately, the flowpaths created by particle tracking are similar enough that they can provide a reasonable estimate in most cases. In the study of injection and production wells, the most appropriate distribution of particle placement was found to be a ring of particles around the injection well for forward tracking or around the production well for backward tracking. However, even for this placement of particles, there is not a perfect agreement between the streamlines and flowpaths because streamlines will be spaced more closely between the wells than in the regions on the outside of the doublet. This results in particles being placed too far apart on the interior of the doublet and too close together on the exterior. The flowpaths and equipotentials should be examined to verify that they generally meet the requirements of flownet construction (e.g. Fetter, 1994).

Once the results of particle tracking have been shifted in time, the temperature at a production well can be estimated by examining the origin of the pathlines arriving at a production well in cases where groundwater is used for either heating or cooling purposes. If heat transport by conduction and the effects of temperature-dependent variations in density and viscosity are considered negligible, the temperature at the production well can be estimated using the following general relationship:

$$T_{\mathrm{prod}} = T_{\mathrm{o}} \frac{N_{\mathrm{aq}}}{N_{\mathrm{total}}} + \frac{\sum_{i=1}^{n} T_i N_i}{N_{\mathrm{total}}} \quad (3)$$

where T_{prod} is the temperature at the production well at a given time, T_{o} the background temperature of the aquifer, N_{aq} the number of pathlines arriving at the production well carrying native groundwater, N_{total} the total number of pathlines considered, N_i the number of pathlines arriving at the production well from injection well i at a given time, and T_i the temperature at injection well i (there is a total of n injection wells). This equation is applicable in cases where the injection temperature is constant. If the temperature of the injectate varies with time, it may underestimate temperature changes at the production well following thermal breakthrough. Eq. (3) also assumes that there is no density-driven flow, suggesting small vertical thermal gradients. For the case of a doublet in an aquifer of infinite extent with no regional groundwater flow, the following equation is derived:

$$T_{\mathrm{prod}} = T_{\mathrm{o}} \left(\frac{N_{\mathrm{aq}}}{N_{\mathrm{total}}} \right) + T_{\mathrm{in}} \left(\frac{N_{\mathrm{prod}}}{N_{\mathrm{total}}} \right) \quad (4)$$

where T_{prod} is the temperature at the production well, T_{in} the temperature of injected water, N_{aq} the number of particles still in the aquifer at a given time, N_{prod} the number of particles that have arrived at the production well from the injection well at given time, and N_{total} the total number of particles introduced into the system. In this simple case, all water streamlines arriving at the production well originate from the injection well and it is easy to estimate the temperature of the produced fluid using either forward or backward particle tracking. However, in cases where not all waters arriving at the production well at late times originate from the injection well, backward particle tracking is required to determine the origin of the water. In cases where groundwater flow is constant throughout the period of interest, it is possible to estimate the evolution of production well temperatures over time. However, if there are significant variations in groundwater flow

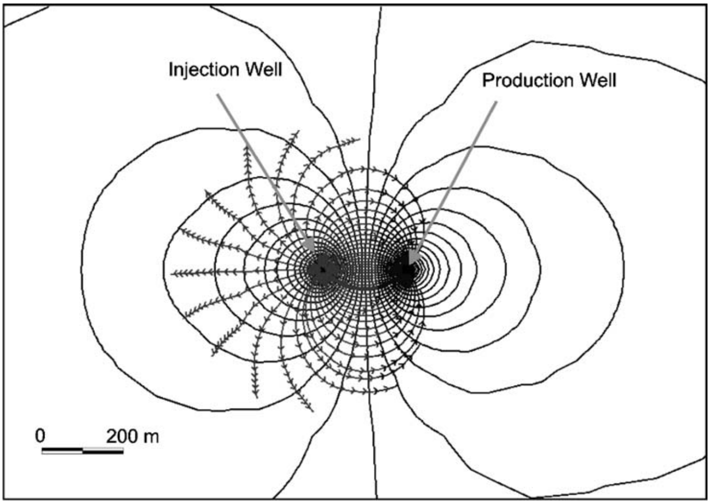

Fig. 1. Case 1, flowpaths (lines with arrows) and equipotentials (solid lines) for an injection–production doublet model pumped at a rate of 0.001 m^3/s, 1.5 years after start of operation. Equipotential interval: 0.0025 m. Note that only the central area of the modelled domain is shown.

velocities over time caused by changes in either production or injection rates, it is likely that only quasi-steady state temperatures can be estimated at late times, assuming that groundwater flow patterns will stabilize at later times. In the current study, the usefulness of particle tracking in advective heat flow analysis is demonstrated for each of the above cases.

3. Applications

3.1. Case 1: A single doublet in an area of constant head

The possibility of using particle tracking to estimate temperatures at the production well is investigated by studying a series of models with Visual MODFLOW® and MODPATH® (Waterloo Hydrogeologic, 2004)[1], and the results are compared to those obtained using METRA (Painter and Seth, 2003), an integrated finite difference code that simulates subsurface heat flow by both conduction and advection. The models used in this study utilized a grid spacing of 5 m in the vicinity of the production and injection wells but this increased up to 100 m towards the edges of the model domain where gradients are low. In all cases examined, the aquifer was discretized into five layers. In the particle-tracking model, 50 particles were released at the injection well at the beginning of the simulation by placing them in a 5-m radius circle around the injection well. The pathlines of these particles are a good, although not perfect, approximation of a flownet (Fig. 1). The pathlines meet equipotentials at right angles and when the appropriate equipotential interval is used, the curvilinear square requirement is approximately met in most cases.

The positions of these particles at various times were used to calculate production well temperatures using Eq. (4) in combination with a shift in the time of the results in accordance with Eq. (2). For the purpose of generality, these results are presented in terms of dimensionless temperature

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[1] MODPATH is a particle tracking post-processing package for MODFLOW.

Table 1
Parameters used in the particle-tracking and advective–conductive models

Model parameter	Cases 1 and 2	Case 3
Hydraulic conductivity (m/s)	10^{-3}	10^{-3}
Porosity	0.1	0.095
Pumping rate (L/s)	1.0	See Table 2
Thermal conductivity (W/mK)	2.5	2.4
Specific heat capacity of porous medium (J/K kg)	1318	1318
Specific heat capacity of fluid (J/K kg)	4184	4184
Specific heat capacity of rock (J/K kg)	1200	1200
Density of porous medium (kg/m^3)	2350	2350
Density of water (kg/m^3)	1000	1000
Density of rock (kg/m^3)	2500	2500

(T^*), which is defined as follows:

$$T^* = \frac{T - T_\text{o}}{T_\text{in} - T_\text{o}} \tag{5}$$

Since T_in is lower than T_o, an increase in T^* corresponds to a decrease in aquifer or fluid production temperature.

The models utilized in the study consisted of an injection well and a production well, spaced 100 m apart, in the centre of a 4000 m × 4000 m × 10 m thick aquifer. The grids used in both the MODFLOW and METRA codes were identical. The entire domain was assigned a permeability of 10^{-10} m^2 and a porosity of 0.10, which is consistent with many shallow aquifers currently being examined for low-temperature geothermal applications (e.g. Allen and Bridger, 2003; Ferguson and Woodbury, 2005). Other parameters used in these models are outlined in Table 1. Fixed temperature and hydraulic head boundary conditions were assumed around the lateral edges of the model (i.e. 20 °C and 10 m at the top of the aquifer), while zero mass and heat flux boundaries were assigned at the top and bottom of the model. To examine the applicability of particle tracking, which only considers advection, to problems where both advection and conduction may be significant, injection and production rates of 10^{-4}, 10^{-3}, and 10^{-2} m^3/s were considered.

Of the three pumping rates examined, the results of the particle-tracking model most closely resembled the more accurate advective–conductive model at the highest pumping rate (i.e. 10^{-2} m^3/s; Fig. 2). Breakthrough of cool water occurs more rapidly according to the particle-tracking model, but after one year the difference between the dimensionless temperatures predicted by the particle-tracking model and those predicted by the advective–conductive model is approximately 0.05–0.1. In the models with a 10^{-3} m^3/s pumping rate, thermal breakthrough given by the particle-tracking model is slightly earlier than that in the advective–conductive model. In addition, a more rapid decrease in temperature following thermal breakthrough is predicted by the particle-tracking model. At later times, the dimensionless temperatures given by the advective–conductive model are approximately 0.1 less than those predicted by particle tracking. When pumping rate was 10^{-4} m^3/s, the results of the two models were drastically different, with dimensionless temperatures differing by approximately 0.3 at later times. This result was anticipated since heat conduction becomes more important as advection induced by fluid injection and production decreases.

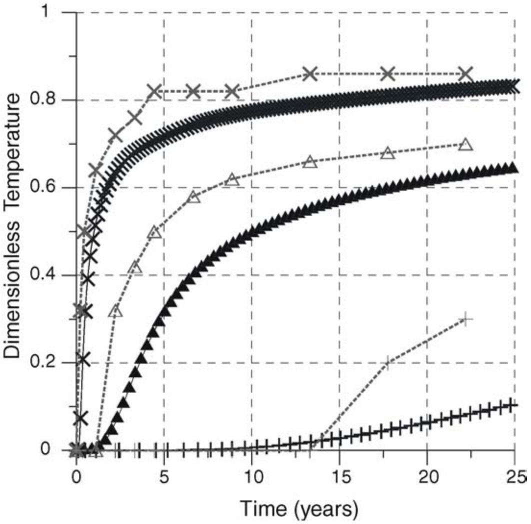

Fig. 2. Case 1, comparison of dimensionless fluid production temperatures between those predicted by the advective–conductive model and those estimated based on particle tracking for pumping rates of 10^{-4} m^3/s (×), 10^{-3} m^3/s (triangles) and 10^{-2} m^3/s (crosses). Advective–conductive model results are shown in black (solid lines) and particle-tracking-based estimates are shown in grey (dashed lines).

The effects of well spacing were also studied. It is expected that an increase in the distance between wells (i.e. longer flowpaths) will allow for more conductive heat transport perpendicular to the paths, which will reduce changes in fluid production temperature. Well spacings of 50, 100 and 200 m were examined, assuming an injection and production rate of 10^{-3} m^3/s. At 50 and 100 m spacing, the estimates of dimensionless temperatures produced by particle tracking were approximately 0.1 greater than those predicted by the advective–conductive model (Fig. 3). However, the general trends were matched in both cases. In the model with a 200-m well spacing, the dimensionless temperatures given by the particle-tracking model were approximately 0.2 larger (i.e. actual fluid temperatures lower) than those obtained with the advective–conductive model. Also, the temperature decrease predicted by this model was much more gradual than that shown by the particle-tracking model. Given that heat conduction is neglected in the particle-tracking model, this result is not unexpected.

The effect of aquifer thickness on fluid production temperature was also examined. When using a production/injection rate of 10^{-3} m^3/s, it was found that the results of the particle-tracking models and advective–conductive models most closely agreed at smaller aquifer thicknesses (Fig. 4). Dimensionless temperatures differed by approximately 0.05 in the case of the 1-m thick aquifer, and by approximately 0.1 when the thickness was assumed to be 10 m. In these two cases the general trends and thermal breakthrough times were similar. However, the thermal breakthrough time for the 100-m thick aquifer given by the particle-tracking and the advective–conductive models did not agree, and after 20 years the dimensionless temperature difference between them was more than 0.2.

There are several factors that could contribute to the discrepancies between the temperatures predicted by the two models, including the possibility that some of the water arriving at the production well does not originate at the injection well and the inability of the particle-tracking model to simulate conductive heat losses. Backward particle tracking was used to determine whether

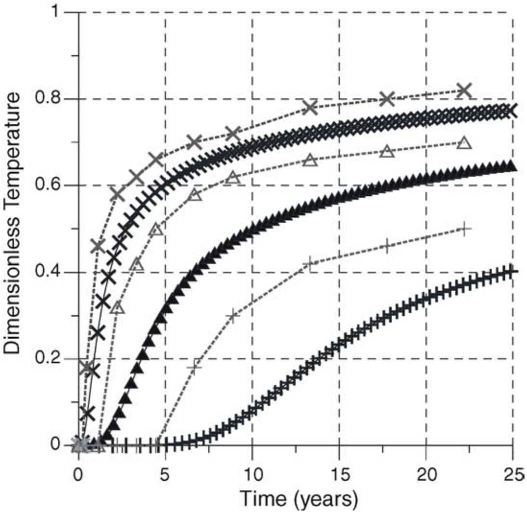

Fig. 3. Case 1, comparison of dimensionless fluid production temperatures between those predicted by the advective–conductive model and those estimated based on particle tracking for a pumping rate of 10^{-3} m^3/s and well spacings of 50 m (×), 100 m (triangles) and 200 m (crosses). Advective–conductive model results are shown in black (solid lines) and particle-tracking-based estimates in grey (dashed lines).

streamlines that did not originate at the injection well might be a factor. It was found that in each of the cases studied utilizing the particle-tracking model there was one streamline that did not begin at the injection well. However, this does not explain much of the discrepancy and it is likely that neglecting conduction is a much more important factor. The better agreement between particle-tracking-based estimates and advective–conductive model results at higher production

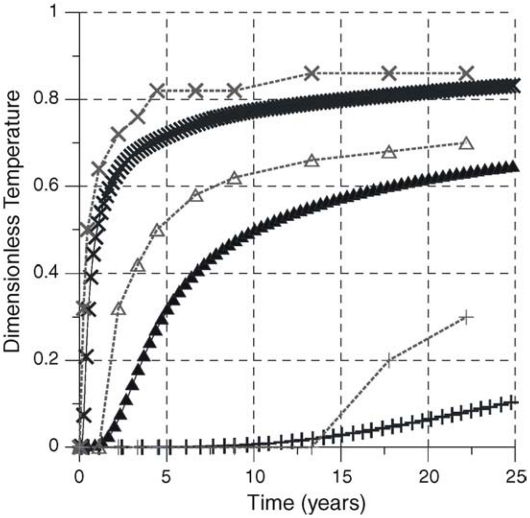

Fig. 4. Case 1, comparison of dimensionless fluid production temperatures between those predicted by the advective–conductive model and those estimated based on particle tracking for a pumping rate of 10^{-3} m^3/s and aquifer thicknesses of 1 m (×), 10 m (triangles) and 100 m (crosses). Advective–conductive model results are shown in black (solid lines) and particle-tracking-based estimates are shown in grey (dashed lines).

rates supports this theory since conduction will be less important when aquifer fluid flow velocities are high. It should also be noted that in thicker aquifers, density-driven fluid flow may become important if there are significant vertical temperature gradients. This was not considered in this study but could cause significant differences between the results of particle tracking and advective–conductive models.

3.2. Case 2: Doublet in an area with significant regional groundwater flow

Gringarten and Sauty (1975) suggest that, when the axis of an injection–production doublet is aligned correctly in an aquifer where regional groundwater flow is sufficiently high, the injected water may never reach the production well and affect the temperature of the produced water. While this may be difficult to achieve in practice, it is possible to reduce the change in fluid production temperatures by taking advantage of the regional groundwater flow. To examine this idea with particle tracking, a group of models were studied using the same grid and well placement as in case 1. A head difference was imposed at the northern and southern ends of the model, along with no-flow boundaries at the eastern and western boundaries, to create regional groundwater velocities of 1, 10 and 100 m/year perpendicular to the axis of the doublet. For each of these velocities, models were run with injection and production rates at 10^{-4}, 10^{-3} and 10^{-2} m^3/s.

For the 1 m/year regional groundwater velocity case (Fig. 5a), the temperature changes estimated by particle tracking exceeded those predicted by the advective–conductive models. The overall results were similar to those when regional flow was not considered. When the groundwater velocity was 10 m/year (Fig. 5b), the results were somewhat similar to those computed for the 1 m/year case. That is, for the two higher injection–production flow rates the relative differences and trends obtained using the particle tracking and advective–conductive models were comparable. At the lowest pumping rate (i.e. 10^{-4} m^3/s), however, there was a significant difference between the computed results after approximately 15 years. At a regional groundwater velocity of 100 m/year (Fig. 5c), the results of the particle-tracking and advective–conductive models were consistent at the lowest pumping rate, as there was no temperature change at the production well. At higher rates (i.e. 10^{-3} and 10^{-2} m^3/s), on the other hand, the estimated dimensionless temperatures based on particle tracking were lower than those predicted by the advective–conductive model after about 2.5 and 7.5 years, respectively.

The results indicate that, in cases where regional groundwater velocity is relatively low compared to the pumping rates (i.e. the 1 and 10 m/year cases), particle tracking appears to perform nearly as well as it does in the absence of regional groundwater flow. At higher regional groundwater velocities (100 m/year), however, fluid production temperature changes are severely underestimated for an injection/production rate of 10^{-3} m^3/s and slightly underestimated when the rate is 10^{-2} m^3/s. This is likely the effect of violations in flownet construction caused by the difference between particle placement and the position of flow lines around the injection well; particles had been distributed evenly in a circle surrounding the well. Regional groundwater flow caused flowlines to deviate from this circular arrangement, particularly in the case of intermediate production rates where well hydraulic effects associated with fluid production and injection have less influence on the potentiometric field.

3.3. Case 3: Multiple injection and production wells

The quasi-steady state groundwater production temperatures in an area of Winnipeg, Manitoba, Canada were studied to assess the utility of the particle-tracking method in places where

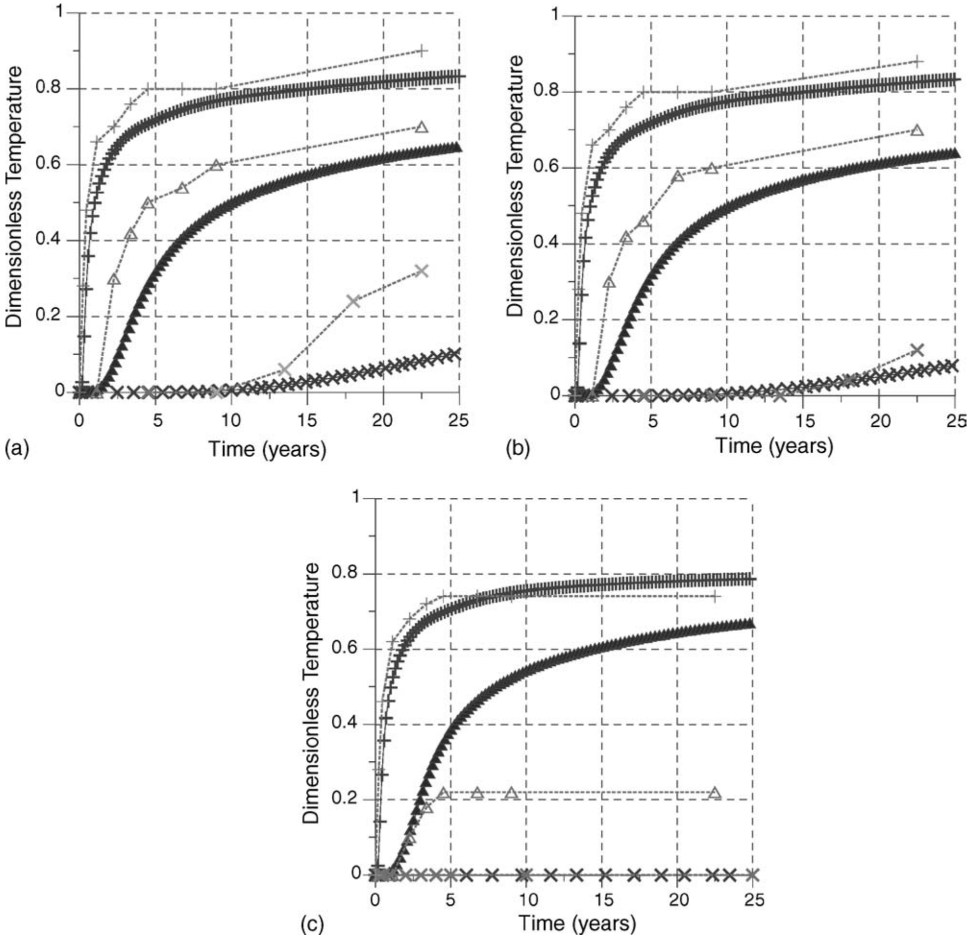

Fig. 5. Case 2, comparison of dimensionless fluid production temperatures between those predicted by the advective–conductive model and those estimated based on particle tracking for injection–production rates of 10^{-4} m^3/s (×), 10^{-3} m^3/s (triangles) and 10^{-2} m^3/s (crosses) and regional groundwater flow velocities of: (a) 1 m/year; (b) 10 m/year and (c) 100 m/year. Advective–conductive model results are shown in black (solid lines) and particle-tracking-based estimates are shown in grey (dashed lines).

more complex aquifer development patterns exist. In this particular area, groundwater is used for air conditioning, and water with elevated temperatures relative to native groundwater is injected back into the aquifer. The design and analysis methods for both applications are essentially the same. Results of pump tests in the area reported by Render (1981) suggest that aquifer transmissivities ranged from 228 to 1022 m^2/day. Drilling conducted by Render (1981) and numerical modelling performed by Ferguson (2004) indicate that the hydraulic conductivity in the carbonate aquifer is not uniform and that the majority of water is produced from the upper 5 m, which is extensively fractured. Numerical modelling suggests that the permeability of this upper fractured carbonate rock is approximately 10^{-10} m^2, and that the permeability of the surrounding intact carbonate rock is likely to be several orders of magnitude lower (Ferguson, 2004). For this reason, only the permeable zone was included in the model, although it is likely that further

Table 2
Records available from Manitoba Water Branch for groundwater use in an area of western Winnipeg, Manitoba (after Render, 1981)

System	Estimated production rate (m^3/s)	Year system was installed	Measured injection temperature (approximate) (°C)	Measured quasi-steady state temperature (°C)	Estimated quasi-steady state temperature (°C)
A	19×10^{-3}	1965	N/A[a]	10.0	9.2
B	16×10^{-3}	1973	7.2	7.2	7.2
C	19×10^{-3}	1973	13.5	13.0	13.5
D	28.5×10^{-3}[b]	1977	14.0	10.0	9.5

A background temperature of 6.5 °C was assumed for the entire aquifer.

[a] System A discharges to a nearby surface water body instead of injecting the produced water back into the aquifer.

[b] System D recharges only a portion of the water withdrawn; the estimated re-injection rate is only 22.2×10^{-3} m^3/s.

discrepancies could arise between the estimates produced by particle tracking and those produced by advective–conductive modelling if heat conduction to the surrounding less permeable rock was considered. The permeable zone was assumed to be homogeneous in the model developed for case 3; the hydraulic and thermal properties for the zone used in the models are given in Table 1.

Fixed-head boundary conditions were placed at the southern and northern boundaries of the model, as well as along the southern portion of its western boundary. The assumed boundary conditions in the south and west correspond to the position of a river, while those in the north represent an area far enough removed from the wells that it would not be affected by their operation. The eastern boundary and the northern section of the western boundary were assumed to be impermeable based on the observation that regional groundwater flow is from north to south. The upper and lower surfaces of the fractured carbonate rocks were also treated as impermeable boundaries considering the relatively low permeability of the overlying till and underlying non-fractured rocks. Production and injection rates and temperatures are provided in Table 2.

It was not possible to examine transient temperature changes through conventional particle tracking without a great deal of difficulty because the four geothermal heat pump systems in the area were installed at different times (Table 2). This situation is not easy to deal with, due to the retarded nature of heat flow in advective systems relative to the flow of water (or a conservative solute). MODPATH and other particle-tracking codes utilize the velocities of a previously solved fluid flow problem. The fluid flow velocities given by MODPATH are reduced to “thermal velocities”, or thermal front velocities, taking into consideration that the thermal and hydraulic changes have different response times. Using a particle-tracking code that directly applies a retardation factor could streamline the data processing, but this is not possible with MODPATH or most other commonly used particle-tracking codes. Despite this limitation, it is still possible to estimate what the quasi-steady state fluid production temperatures will be by considering the origin of the particles arriving at a given well and using Eq. (3). In the model examined, 30 particles were released at each production well and were then tracked backwards in time (Fig. 6). For the studied model, it was found that this number of particles were sufficient to obtain a reasonably accurate solution, although higher resolution could be achieved using more particles. The temperature of the fluid produced by the wells was the weight-average of the temperature of the place of origin of the particles reaching the production well (i.e. the temperature of the fluid injected and/or the background temperature of the aquifer).

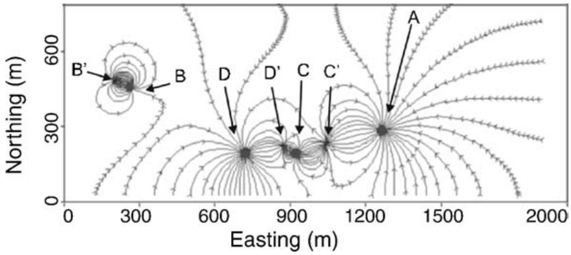

Fig. 6. Computed backward particle tracks (i.e. backward flowpaths) for an aquifer in southern Manitoba; A, B, C and D denote production wells; B′, C′ and D′, injection wells (see text).

The results of this exercise produced reasonable results, having an R^2-correlation coefficient of 0.963 between measured and predicted temperatures, which is significant at the $p = 0.01$ level for the following regressed best-fit line:

$$T_{\text{measured}} = 0.874T_{\text{predicted}} - 1.42 \tag{6}$$

This line is significantly different from that with a slope of one passing through the origin at the $p = 0.05$ level. While this indicates that there may be some difficulty in estimating fluid production temperatures accurately with the particle-tracking approach, it may be used to obtain reasonable estimates.

4. Discussion

In many situations, particle tracking can be used to estimate thermal breakthrough times in low-temperature geothermal systems, as well as to determine the magnitude of the temperature changes in the produced water. However, this only appears to be possible when heat advection in the aquifer is much stronger than heat conduction. To assess the importance of these two thermal processes, the following dimensionless number, which is essentially a version of a Peclet number (i.e. the ratio between advection and conduction; see van der Kamp and Bachu, 1989, for other examples), may provide some insight:

$$Pe_{\text{doublet}} = \frac{\rho_{\text{sat}} c_{\text{sat}}}{\kappa_{\text{sat}}} \frac{Q}{\sqrt{hD}} \tag{7}$$

where ρ_{sat}, is the density of the saturated porous medium, c_{sat} the specific heat capacity of the rock, κ_{sat} the thermal conductivity of the saturated porous medium, Q pumping rate, h aquifer thickness and D the spacing between the injection and production wells.The values of Pe_{doublet} were greater than 30 in cases 1 and 2, where the applicability of the particle-tracking method to the analysis of heat transport in an individual injection–production doublet was examined (Table 3). In case 3, where multiple wells were examined, the Pe_{doublet} values exceeded 800 in each of the wells examined. In general, particle tracking appears to be most effective at higher values of $Pe_{\text{doublet.}}$

It should be noted that Pe_{doublet} does not consider regional groundwater flow. Larger regional groundwater flow will mean higher advective heat flow, which, in general, will increase the value of other Peclet numbers (e.g. see van der Kamp and Bachu, 1989). In this application, however, increasing regional groundwater flow might lead to violations in flownet construction and will likely decrease the accuracy of the estimated temperatures if a circular arrangement of particles

Table 3
Calculated Pe_{doublet} for cases 1 and 2

Model parameter varied	New parameter value	Pe_{doublet}
None	–	39.2
Pumping rate (m^3/s)	10^{-4}	3.9
Pumping rate (m^3/s)	10^{-2}	392.9
Well spacing (m)	50	55.4
Well spacing (m)	200	27.1
Aquifer thickness (m)	1	123.9
Aquifer thickness (m)	100	20.7

Note that the case where no parameters were varied (i.e. the base case) had a pumping rate of 10^{-3} m^3/s, a well spacing of 100 m and an aquifer thickness of 10 m.

is used. Careful placement of particles may help to address this issue but this would have to be an iterative process and would likely be quite time-consuming.

Particle tracking represents an excellent tool for determining the origin of the water arriving at a production well. It can prove useful in cases where moderate regional groundwater flow occurs and in systems that have several operating injection wells. By examining the origin of the particles arriving at a production well, it is possible to estimate the temperature of the produced water in cases where pumping rates are sufficiently high. It should be noted, however, that, as a consequence of conduction, and in some cases, hydrodynamic dispersion, the temperature of the injected water may be quite different when it arrives at the production well.

The method proposed here is difficult to apply to cases with transient groundwater flow patterns and may only be of use in areas with multiple injection and production wells once a quasi-steady state thermal equilibrium has been reached. Different systems will attain that equilibrium at different times. If re-equilibrium occurs very slowly, these estimates may be of little value. For systems that re-equilibrate quickly, it could prove a practical method for determining the long-term sustainability of a geothermal development.

Particle tracking is relatively fast compared to some other methods for determining the changes in produced fluid temperatures under different geothermal development patterns. In this study, all the particle-tracking models required less than 30 s to run on a personal computer with a 2.8 GHz processor and 512 MB of RAM. On the same computer, a variable density, variable viscosity, advective–conductive model used from 2 to 25 min of computing time. The latter is probably the worst-case scenario for the advective–conductive model because the time for each run could be reduced by assuming constant fluid density and viscosity and also by assuming a plane of symmetry. Even with these modifications, it is likely that advective–conductive modelling will still be more computationally expensive than the particle-tracking approach.

5. Conclusions

Particle tracking is a method of estimating the timing and magnitude of temperature changes in injection–production doublets in cases where Pe_{doublet} values are sufficiently high. In the studied models, particle tracking results were reasonable when Pe_{doublet} exceeded 30. Transient production well temperature changes estimated with the particle-tracking method are generally greater than those predicted with the more accurate advective–conductive models and as such can be taken as conservative estimates of the temperature change. Late time, or approximately steady-state, temperatures were also predicted with a reasonable degree of success for more complex situations involving regional groundwater flow and multiple injection and production wells. While there are limitations to the use of particle tracking in the analysis of advective heat flow problems, it is an easily accessible tool that may aid in the sustainable development of systems (including low-temperature geothermal reservoirs) for heating and cooling applications.

Acknowledgements

This study benefited from insightful conversations with Allan Woodbury of the University of Manitoba and helpful reviews from Norio Temma, Marcelo Lippmann and one anonymous reviewer. Names of commercially available software mentioned in this manuscript are purely for identification purposes and do not indicate endorsement by the author. This research was supported by a Natural Sciences and Engineering Research Council of Canada Discovery Grant to the author.

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