

# A Model Slicing Method for Workflow Verification

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## Abstract

Workflow systems increase productivity and quality of service; however, defects in a workflow design may have severe consequences. While model checking techniques can be used to verify the correctness of a workflow model, these techniques typically suffer from the *state explosion* problem. We propose a model slicing algorithm with a formal proof to address this problem. The algorithm is integrated into our NOVA Workflow framework, which facilitates design, verification, execution, and error-handling. A case study of a healthcare model is used to show that the proposed algorithm makes the verification more efficient in terms of state space and hence for memory and time usage.

*Keywords:* Model slicing, formal verification, workflow modelling, model checking, linear temporal logic

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## 1 Introduction

Today business process models are frequently used to design safety critical systems which are often very complex. Model checking or other similar verification techniques are required to ensure that these process models exhibit the desired behavior. While current model checkers are much more powerful than their predecessors, they still frequently suffer from the state explosion problem [4]. Much research has been done during the past decade to handle this problem. While there are many techniques to optimize model checking algorithms, e.g., *Partial Order Reduction* (POR) [14], *Symmetry-based Reduction* [8], the category of techniques called *Model Abstraction* [4] is also very important to alleviate the state explosion problem. These techniques abstract away irrelevant details w.r.t. the formula being verified before the system model is input to the model checker.

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There are two types of model abstraction techniques: *data abstraction*, which uses a smaller set of data values to represent the actual values of the system, and *model slicing*, which eliminates model components that will not affect the truth value of the formula. In this paper, we focus on the latter technique.

While the problem of *software program slicing* [7] is a well-studied topic, there are relatively few works in slicing of formal diagrammatic languages, even fewer for workflow modeling languages. This paper develops and integrates model abstraction techniques into a workflow framework.

Our group developed NOVA Workflow, a framework for workflow design, verification, execution, and error-handling. Using the NOVA Editor one can graphically design a workflow using the *Compensable Workflow Modeling Language* (CWML) [16] and write business logic for the tasks. The workflow model is then automatically translated to DVE, the input language of the DiVinE [3] model checker, using the NOVA Translator [16]. Although we could model a large workflow and translate it into a model checking program using the NOVA Translator, the time and space required for the verification was sometimes unacceptable. Our experiments showed that even though DiVinE is equipped with POR and many other heuristics, it requires a huge amount of memory and time for the verification of a large model.

In this paper we present a model slicing algorithm for the workflows constructed using CWML. The algorithm has been implemented in the NOVA Translator. It takes a workflow model and the specification of an  $LTL_X$  formula  $\phi$  (the subset of LTL formulae not containing the  $X$  operator) and reduces the model in such a way that the truth of  $\phi$  is preserved and reflected. We give the details of the proof of the stuttering equivalence of the original models and the reduced models generated by our algorithm. Moreover, we show how effectively the proposed method reduces the size of the state space. We expect the proposed algorithm to be easily applied to any block-structured modeling language, e.g., ADEPT2 [18]. Note that the algorithm deals with the feature of data-awareness now commonly found in workflow modeling languages. We show the applicability and effectiveness of the method with a fairly big model of a healthcare workflow.

Fig. 1 shows an overview over the different steps in the proposed approach of the paper. Our workflow models  $M$  are specified by CWML. Each model  $M$  is serialised as a corresponding textual expression valid w.r.t. the BNFs in Definitions 2.2 and 2.3. We use Algorithm 1 to parse the expression and generate a Task Syntax Tree (TST)  $\lambda$ . Algorithm 3 is used to reduce  $\lambda$  to a reduced TST  $\lambda'$  w.r.t. an  $LTL_X$ -formula  $\phi$ . From the reduced TST  $\lambda'$ , Algorithm 2 creates a textual expression which in turn will be deserialised and visualised by NOVA Workflow as a CWML model  $M'$ . In Section 3.3, we prove that the models  $M$  and  $M'$  are stuttering equivalent w.r.t.  $\phi$ .

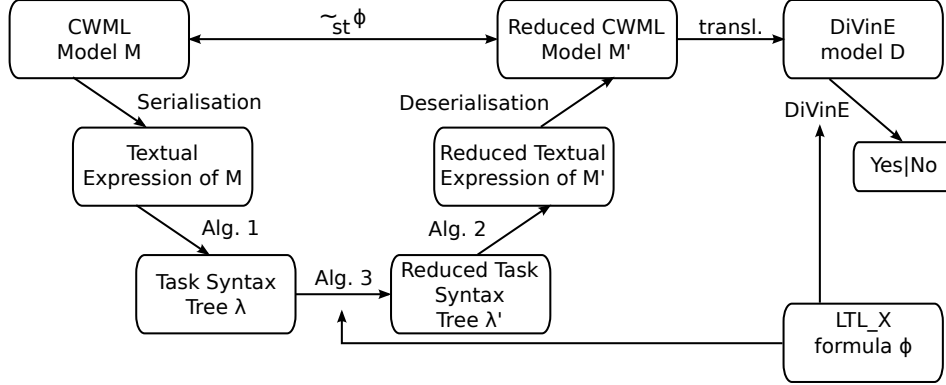


Fig. 1. Overview of the proposed approach

Section 2 provides some background information. In Section 3 we present the model slicing algorithm and the proof of the stuttering equivalence of a workflow model  $M$  and the corresponding reduced model  $M'$ . Section 4 presents a realistic case study. Section 5 relates our approach to other work, and Section 6 concludes the paper and outlines future research.

## 2 Preliminaries

A workflow model in CWML consists of tasks and operators connecting these tasks. The execution of some tasks are guarded by preconditions, and may perform some actions when executed. We begin by reviewing some definitions.

**Definition 2.1 (Term, Precondition and Action)** A term  $t$  is recursively defined using BNF as  $t ::= c \mid \chi \mid t \circ_A t$ , where  $\circ_A \in \{+, -, *, \div\}$ ,  $c$  is a natural number and  $\chi$  is a (natural) variable. A precondition is a formula  $\psi$  defined as  $\psi ::= t \circ_C t \mid \psi \circ_P \psi$ , where  $\circ_C \in \{<, \leq, >, \geq, ==\}$  and  $\circ_P \in \{\&\&, \|\}$ . An action  $\alpha$  is an assignment defined as  $\alpha ::= v = t$ ;  $v$  is called an *assignee* variable. We abuse the notation  $\{\alpha\}$  to denote a set of actions.

A compensable task can undo or rollback its operations at a later time after its successful execution, if required. In CWML, compensable tasks are composed with  $t$ -calculus operators [10] to provide for a variety of methods of compensation.  $t$ -calculus allows one to combine compensable transactions to set up a long running business transaction which has compensation as its main error recovery technique. A compensable task consists only of atomic compensable tasks composed with the compensable operators  $\circ_{T_c}$ .

**Definition 2.2 (Compensable Task)** A compensable task  $T_c$  is recursively defined using BNF as  $T_c ::= \tau_c \mid (\{\psi_{T_{c1}}\}T_{c1} \circ_{T_c} \{\psi_{T_{c2}}\}T_{c2})$ , where  $\tau_c ::= \langle id \rangle \{\alpha_{\tau_c}, \alpha_{\tau_c}^b\}$  is an atomic compensable task which has a set of forward and compensation actions associated to it and  $\langle id \rangle$  is the name of the task, moreover,  $\circ_{T_c} \in \{\odot, \wedge, \otimes, \textcircled{S}, \textcircled{A}\}$  is a  $t$ -calculus operator. For  $T_{c1} \otimes T_{c2}$  each of  $T_{c1}, T_{c2}$  has a precondition  $\psi_{T_{c1}}, \psi_{T_{c2}}$ , respectively.

The operators used to combine compensable tasks are explained as follows:

- $T_1 \odot T_2$  (Sequential):  $T_1$  will be executed first, then  $T_2$  will be executed.
- $T_1 \triangleleft T_2$  (Parallel):  $T_1$  and  $T_2$  will be executed in parallel. If either of them is aborted, the other one will also be aborted.
- $T_1 \otimes T_2$  (Internal choice): Exactly one of the tasks will be executed.
- $T_1 \textcircled{S} T_2$  (Speculative choice):  $T_1$  and  $T_2$  will be executed in parallel, the task that reaches the goal first will be accepted, the other one will be aborted.
- $T_1 \textcircled{A} T_2$  (Alternative forwarding):  $T_1$  will be executed first to achieve the goal, if  $T_1$  is aborted,  $T_2$  will be executed to achieve the goal.

A task may contain both compensable tasks and uncompensable tasks, connected by operators.

**Definition 2.3 (Task)** A task  $T$  is recursively defined using BNF as  $T ::= \tau \mid T_c \mid (\{\psi_{T_1}\}T_1 \circ_T \{\psi_{T_2}\}T_2) \mid \{\psi_T\}(T)^+$ , where  $\tau ::= \langle id \rangle \{\alpha\}$  is an atomic uncompensable task which has a set of actions  $\{\alpha\}$  associated to it and  $\langle id \rangle$  is the name of the task, moreover,  $\circ_T \in \{\bullet, \wedge, \times, \vee\}$  is a binary operator, and  $T^+$  is a unary operator applied to  $T$ . For  $T_1 \times T_2$ ,  $T_1 \vee T_2$  and  $T^+$ , each of  $T_1, T_2, T$  has a precondition  $\psi_{T_1}, \psi_{T_2}, \psi_T$ , respectively.

The operators used to combine tasks are explained as follows:

- $T_1 \bullet T_2$  (Sequential):  $T_1$  will be executed first, then  $T_2$  will be executed.
- $T_1 \wedge T_2$  (Parallel):  $T_1$  and  $T_2$  will be executed in parallel.
- $T_1 \times T_2$  (Exclusive choice): Exactly one of the two tasks will be executed.
- $T_1 \vee T_2$  (Choice):  $T_1$  or  $T_2$  or both will be executed in parallel.
- $T^+$  (Loop):  $T$  is executed repeatedly as long as the precondition is true.

A workflow model  $M$  in CWML is a task with one input and one output conditions. The underlying semantics of  $M$  is given by a Petri net; see [15] for the Petri nets underlying each of the tasks. The *state* of  $M$  is determined by the marking of its underlying Petri net. A path in  $M$  is defined as follows.

**Definition 2.4 (Path)** Given a workflow model  $M$ , a path  $\pi = (s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \dots)$  is a sequence of state changes caused by sequentially executing tasks in  $M$ . The length of a path  $\pi$  is the number of state changes in  $\pi$ .

Note that by  $(s_0 \rightarrow s_1 \dots s_j \xrightarrow{\tau_i} s_{j+1} \dots)$  we denote any path that involves the execution of the task  $\tau_i$ . Moreover, we denote by  $(s_0 \rightarrow s_1 \dots s_j \mid \xrightarrow{\tau_i} s_{j+1} \dots)$  any path up to the execution of  $\tau_i$ . We use  $\pi_k$  to denote a path of length  $k$ . A task or an operator which causes a change of state could be either uncompensable or compensable. Therefore, in Section 3.3, we are able to prove the stuttering equivalence by using structural induction on length of paths regardless of the tasks and or operators used.

### 3 Workflow Slicing

In this section, we first present two algorithms: one to create TSTs from CWML models, and one to create CWML models from TSTs. Then we present our slicing algorithm and prove that using that algorithm to reduce a workflow model will yield a model that is stuttering equivalent to the original model.

#### 3.1 Task Syntax Tree

Workflow models specified by CWML have a graphical structure. These structures are serialised as textual expressions which are valid w.r.t. the BNFs in Definitions 2.2 and 2.3. These expressions are parsed and represented as task syntax trees (TST) where a non-leaf node represents an operator and a leaf node represents an atomic (possibly compensable) task (see Fig. 2). Algorithm 1, outlining how a TST is generated from textual expressions, is adapted from the standard parsing process [11] with assignment of preconditions and actions. Note that only non-leaf nodes (operators) of  $\times$  (XOR) or  $\vee$  (OR) or  $\otimes$  (Internal choice) or  $+$  have non-empty preconditions.

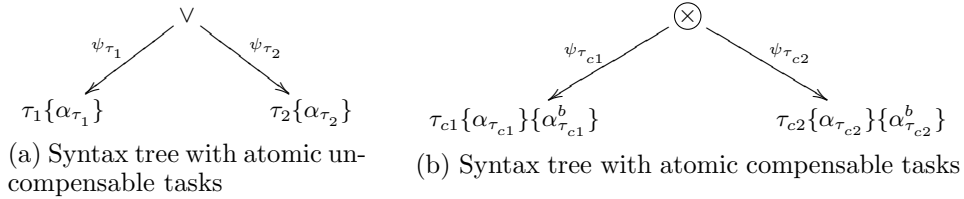


Fig. 2. Examples of workflow syntax trees

**Input:** Textual expression of workflow model  $M$

**Result:** Task Syntax Tree  $\lambda$

- (i) Create tokens using Lexical Analysis;
- (ii) Create parse tree,  $\lambda$ , using the grammar provided in Definitions 2.1, 2.2, and 2.3 (while parsing we consider parenthesis have precedence over operators);
  - (a) Assign preconditions to the immediate branch(es) of non-leaf nodes;
  - (b) Assign actions to leaf nodes (atomic tasks).

**Algorithm 1.** Text2TST: Construction of TSTs from the textual expression of workflow models

After slicing the TST, we use the Algorithm 2 to generate a textual expression from the reduced TST. These expressions are then deserialised to graphical CWML models by NOVA Workflow.

**Input:** Task Syntax Tree  $\lambda$   
**Result:** Textual expression of workflow model  $M$

```

1  $\eta_{curr} \leftarrow root(\lambda)$ ;
  if  $\eta_{curr}$  is NIL then
2    $\lfloor$  return  $strcat(\tau_{NIL}, \{\}' )$ ;
3 else if  $\eta_{curr}$  is a leaf node then
4    $\lfloor$  return  $strcat(\eta_{curr}, \{\}, \alpha_{\eta_{curr}}, \{\}' )$ ;
5 else
6    $\lfloor$  /* for internal nodes */
7     else if  $\eta_{curr}$  is a binary operator then
8        $\lfloor$   $\eta_l \leftarrow \eta_{curr}.leftChild$ ;
9          $\lfloor$   $\eta_r \leftarrow \eta_{curr}.rightChild$ ;
10          if  $\eta_{curr}$  is  $\times$  (XOR) or  $\vee$  (OR) or  $\otimes$  (Internal choice) then
11             $\lfloor$   $operand1 \leftarrow strcat(\{\}, \psi_{\eta_l}, \{\}', TST2Text(subTree(\eta_l)))$ ;
12               $\lfloor$   $operand2 \leftarrow strcat(\{\}, \psi_{\eta_r}, \{\}', TST2Text(subTree(\eta_r)))$ ;
13                 $\lfloor$  return  $strcat('(', operand1, \eta_{curr}, operand2, \{\}' )$ ;
14            else
15               $\lfloor$   $operand1 \leftarrow strcat(\{\}', TST2Text(subTree(\eta_l)))$ ;
16                 $\lfloor$   $operand2 \leftarrow strcat(\{\}', TST2Text(subTree(\eta_r)))$ ;
17                   $\lfloor$  return  $strcat('(', operand1, \eta_{curr}, operand2, \{\}' )$ ;
18            else
19               $\lfloor$  /* for + */
20                 $\lfloor$   $\eta_d \leftarrow \eta_{curr}.leftChild$ ;
21                   $\lfloor$  return  $strcat(\{\}, \psi_{\eta_d}, \{\}' (\{\}', TST2Text(subTree(\eta_d)), \{\}' ), \eta_{curr})$ ;
22             $\lfloor$ 
23    $\lfloor$ 

```

**Algorithm 2.** TST2Text: Construction of the textual expression of workflow models from TSTs

### 3.2 Slicing Algorithm

The slicing algorithm (see Algorithm 3) reduces the size of the workflow model based on the LTL formula subject to verification. Given a workflow, we reduce its TST  $\lambda$  based on the LTL formula  $\phi$  that we wish to verify. In the workflow slicing algorithm, we first determine the propositional variables used in  $\phi$  and store them in a set  $E$  (Preserved Elements). Then, the set  $E$  will be extended recursively with tasks, preconditions, variables and actions that are *visible* w.r.t. the variables in  $E$ .

**Definition 3.1 (Visible Precondition, Action, Atomic Task, and Operator)** An action  $\alpha$  is visible, i.e.  $visible(\alpha) = true$ , iff the *assignee* variable of  $\alpha$  is in  $E$ . An atomic (compensable) task  $\tau$  is visible, i.e.  $visible(\tau) = true$ , iff there exists any action  $\alpha_\tau$  of  $\tau$  which is visible. An operator is visible iff any of its operands (can be operators or tasks) is visible. Preconditions of an operator are visible iff the operator is visible.

**Input:** TST  $\lambda$ , and LTL formula  $\phi$   
**Result:** Reduced TST  $\lambda'$

```

13  $E \leftarrow \emptyset$ ;
    for each variable  $v \in \phi$  do
14      $E \leftarrow E \cup \{v\}$ ;
15  $size \leftarrow 0$ ;
    while  $size \neq E.size$  do
16      $size \leftarrow E.size$ ;
        for each leaf node  $\eta \in \lambda$  do
17         if  $\alpha_\eta$  is visible then
18              $E \leftarrow E \cup \{\eta, \alpha_\eta\}$ ;  $\eta_{curr} \leftarrow \eta.parentNode$ ;
                /* recursively add all ancestors */
19             while  $\eta_{curr}$  is not the root do
20                  $E \leftarrow E \cup \{\eta_{curr}\}$ ;
                    /* the four operators with conditions */
21                 if  $\eta_{curr}$  is  $\times$  (XOR) or  $\vee$  (OR) or  $\otimes$  (Internal choice) or  $+$ 
                    then
22                      $\eta_l \leftarrow \eta_{curr}.leftChild$ ;  $\eta_r \leftarrow \eta_{curr}.rightChild$ ;
                        /* add all branch conditions */
23                      $E \leftarrow E \cup \{\psi_{\eta_l}, \psi_{\eta_r}\}$ ;
24                      $\eta_{curr} \leftarrow \eta_{curr}.parentNode$ ;
                /* Construct the reduced syntax tree  $\lambda'$  */
25  $\lambda' \leftarrow \lambda$ ;
        for each node  $\eta \in \lambda'$  do
26         if  $\eta \notin E$  then
27              $\eta \leftarrow NIL$ ;
    
```

**Algorithm 3.** Workflow slicing

The algorithm constructs a reduced syntax tree  $\lambda'$  by eliminating all the nodes and pre-conditions which are not present in  $E$  and results in a reduced model  $M'$  from  $\lambda$ . The next example illustrates the algorithm's main steps.

**Example 3.2 (Workflow Slicing)** Fig. 3 shows a workflow model  $M_{ex}$  containing 10 atomic tasks. The formula  $\phi$  we wish to verify is:  $G((v1 == 1) \rightarrow F(v2 == 1))$ , meaning that if  $v1$  is set with value 1,  $v2$  will eventually be set with value 1. Task preconditions are shown along the edges and task actions are shown below the tasks. The textual representation of the lower portion of the workflow is as follows:  $((\{ \} Task\_2 \{ v1 = \{ 1, 2 \} \} \bullet (\{ v1 == 1 \} Task\_5 \{ v2 = 1 \} \times \{ v1 != 1 \} Task\_6 \{ v3 = 1 \} )) \bullet \{ \} Task\_9 \{ v6 = 1 \})$ .

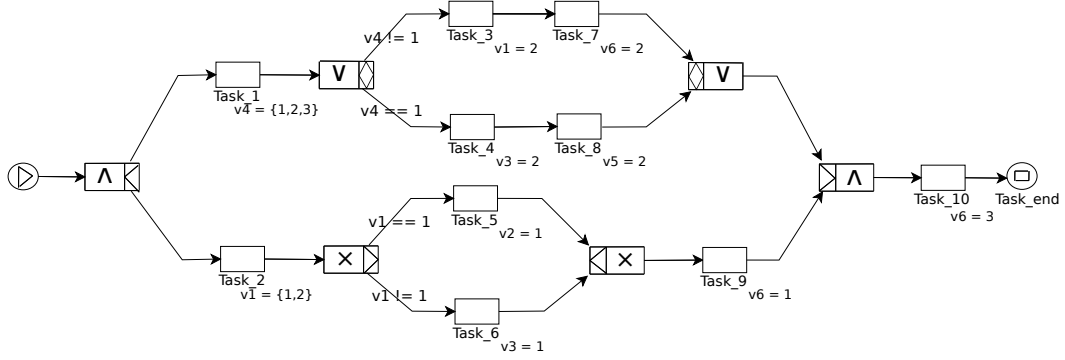
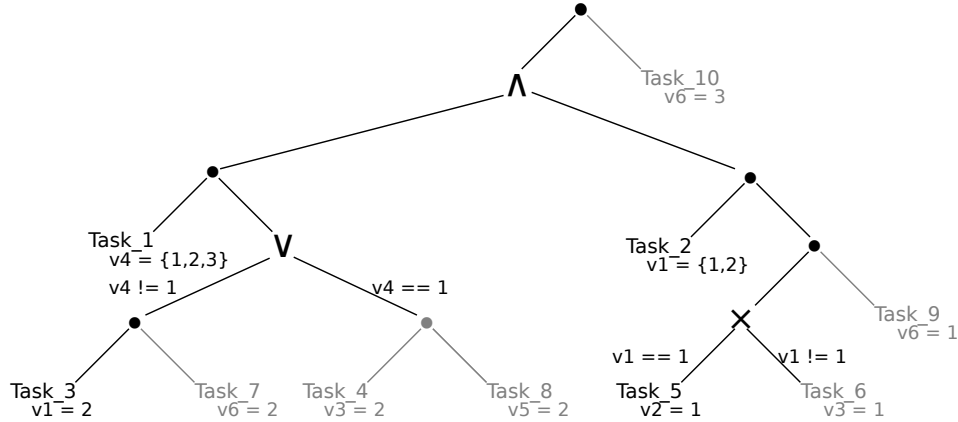
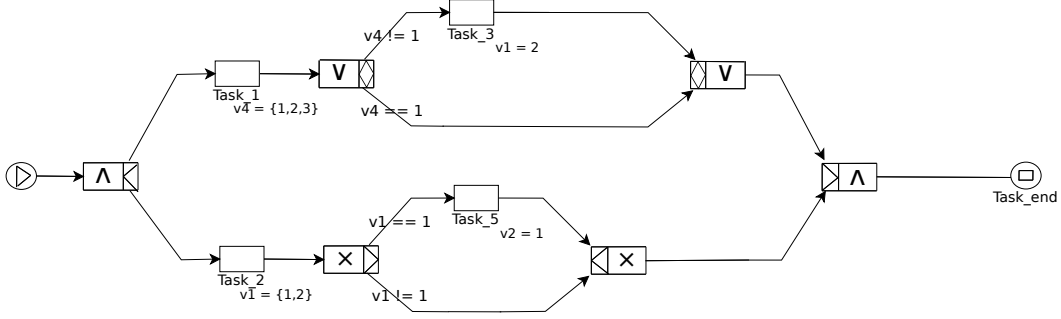

 Fig. 3. A sample workflow  $M_{ex}$ 

Fig. 4 shows the task syntax tree for  $M_{ex}$ . The preconditions of tasks Task\_6 and Task\_5,  $v1 \neq 1$  and  $v1 == 1$  respectively, are inserted into  $E$  as the variable  $v1$  occurs in  $\phi$ . Tasks Task\_2, Task\_5 and Task\_3 are visible as they have visible actions. Paths from these tasks to the root node are then made visible and all the preconditions along these paths will become visible (represented by thick lines). As  $v4$  becomes visible, the preconditions  $v4 == 1$  and  $v4 \neq 1$  become also visible. Task\_1 becomes visible as it has an action  $v4 = 1,2,3$  which changes the value of a visible variable. The rest of  $\lambda_{ex}$  will be invisible; this is indicated by gray color in Fig. 4.


 Fig. 4. The syntax tree  $\lambda_{ex}$  of the workflow  $M_{ex}$  from Fig. 3

The reduced workflow  $M'_{ex}$  is shown in Fig. 5.  $M'_{ex}$  has fewer concurrent tasks but will provide the same verification result for  $\phi$  (see the proof of stuttering equivalence in Section 3.3). In  $M'_{ex}$  the formula  $\phi$  does not hold; one of the counter-examples is the following sequence of task execution: Task\_2  $\rightarrow$  Task\_1  $\rightarrow$  Task\_3  $\rightarrow$  Task\_5  $\rightarrow$  Task\_9  $\rightarrow$  Task\_end). This counter-example shows that the variable  $v1$  is set with the value 1 in task Task\_2, and it is reset with another value 2 in task Task\_3. For this execution the formula  $G((v1 == 1) \rightarrow F(v2 == 1))$  does not hold in  $M'_{ex}$  and hence not in  $M_{ex}$ .


 Fig. 5. The reduced workflow  $M'_{ex}$  of the workflow  $M_{ex}$  from Fig. 3

### 3.3 Proof of Stuttering Equivalence

This section gives a proof for the stuttering equivalence of a workflow model and its reduced one w.r.t. an LTL formula  $\phi$ . We adapted the definition of stuttering equivalence from [4] which is based on Kripke structures. Let  $V$  be the set of system variables, and variables in  $V$  range over a finite set  $D$ , sometimes called the domain or universe of the interpretation. A *valuation* for  $V$  is a function that associates a value  $d \in D$  to each variable  $v \in V$ . *AVP* is a set of *atomic valuation propositions*, where each proposition typically has the form  $v = d$ . Note that *AVP* is a subset of the atomic propositions *AP*. The labelling function  $L : S \rightarrow 2^{AVP}$  returns the subset  $L(s) \subseteq AVP$  which are true in  $s \in S$ . Further, the visible labelling function  $L_\phi$  returns for each  $s$  the subset  $L_\phi(s) \subseteq L(s)$  whose variables occur in  $\phi$ .

**Definition 3.3 (Visible Label Function)** Let  $\phi$  be an LTL formula and let  $V_\phi$  be the set of variables occurring in  $\phi$ ; the visible label function for a state  $s$ ,  $L_\phi(s)$ , is defined as  $L_\phi(s) = \{p \mid var(p) \in V_\phi\}$ , where  $p$  is a proposition in  $L(s)$  and  $var(p)$  returns the variable of proposition  $p$ .

We now define the stuttering equivalence of paths and workflow models.

**Definition 3.4 (Stuttering Equivalence of Paths)** Two (possibly) infinite paths  $\pi = (s_0 \rightarrow s_2 \rightarrow s_3 \dots)$  and  $\pi' = (s'_0 \rightarrow s'_2 \rightarrow s'_3 \dots)$  are stuttering equivalent w.r.t. an LTL formula  $\phi$ , written  $\pi \sim_{st\phi} \pi'$ , if there are two infinite sequences of positive integers  $0 = i_0 < i_1 < i_2 < \dots$  and  $0 = j_0 < j_1 < j_2 < \dots$  such that for every  $k \geq 0$ ,  $L_\phi(s_{i_k}) = L_\phi(s_{i_{k+1}}) = \dots = L_\phi(s_{i_{k+1}-1}) = L_\phi(s'_{j_k}) = L_\phi(s'_{j_{k+1}}) = \dots = L_\phi(s'_{j_{k+1}-1})$ .

Thus  $\pi \sim_{st\phi} \pi'$  iff the paths can be partitioned into (possibly) infinitely many blocks, such that the states in the  $k$ th block of  $\pi$  are labeled (w.r.t.  $\phi$ ) the same as the states in the  $k$ th block of  $\pi'$ .

**Definition 3.5 (Stuttering Equivalence of Workflow Models)** Two workflow models  $M$  and  $M'$  are stuttering equivalent ( $M \sim_{st\phi} M'$ ) w.r.t. an LTL formula  $\phi$  iff:

- $L_\phi(s_0) = L_\phi(s'_0)$ , where  $s_0, s'_0$  are the initial states of  $M$  and  $M'$ , respectively, i.e.,  $M$  and  $M'$  have the same set of initial states (one each);
- for each path  $\pi$  of  $M$  there exists a path  $\pi'$  of  $M'$  such that  $\pi \sim_{st\phi} \pi'$ ; and,
- for each path  $\pi'$  of  $M'$  there exists a path  $\pi$  of  $M$  such that  $\pi' \sim_{st\phi} \pi$ .

The following theorem indicates that to prove that  $\phi$  is invariant under stuttering, it is sufficient to show that  $M$  and  $M'$  are stuttering equivalent. The proof of the following theorem may be found in [4]. Note that  $M, s_0 \models A\phi$  denotes that all paths in  $M$  starting at  $s_0$  satisfy  $\phi$ .

**Theorem 3.6** *Any  $LTL_{-X}$  formula is invariant under stuttering; that is, if  $\phi$  is an  $LTL_{-X}$  formula and  $M \sim_{st\phi} M'$  then  $M, s_0 \models A\phi$  iff  $M', s'_0 \models A\phi$ .*

**Lemma 3.7** *Given a workflow model  $M$ , an  $LTL_{-X}$  formula  $\phi$  and a reduced model  $M'$  generated by Algorithm 3, for any two paths  $\pi = s_i \xrightarrow{\tau_i} s_{i+1}$  and  $\pi' = s'_j$ , if  $\tau_i$  is invisible and  $L_\phi(s_i) = L_\phi(s'_j)$ , then  $L_\phi(s_i) = L_\phi(s_{i+1}) = L_\phi(s'_j)$  and hence  $\pi \sim_{st\phi} \pi'$ .*

**Proof.** The proof follows from the fact that the invisible task  $\tau_i$  will not change the truth values of all propositions in  $L_\phi(s_i)$ .  $\square$

**Lemma 3.8** *Given a workflow model  $M$ , an  $LTL_{-X}$  formula  $\phi$  and a reduced model  $M'$  generated by Algorithm 3, for any two paths  $\pi = s_i \xrightarrow{\tau_i} s_{i+1}$  and  $\pi' = s'_j \xrightarrow{\tau_i} s'_{j+1}$ , if  $\tau_i$  is visible and  $L_\phi(s_i) = L_\phi(s'_j)$ , then  $L_\phi(s_{i+1}) = L_\phi(s'_{j+1})$  and hence  $\pi \sim_{st\phi} \pi'$ .*

**Proof.** The proof follows from the fact that the visible task  $\tau_i$  will have the same effect on both  $L_\phi(s_i)$  and  $L_\phi(s'_j)$ .  $\square$

**Theorem 3.9** *Given a workflow model  $M$ , an  $LTL_{-X}$  formula  $\phi$  and the reduced model  $M'$  generated by Algorithm 3 from  $M$ , then  $M \sim_{st\phi} M'$ .*

**Proof.** Let  $\phi$  be an  $LTL_{-X}$  formula. The workflow model  $M$  is reduced to  $M'$  according to Algorithm 3 w.r.t. the formula  $\phi$ . We will prove this theorem using structural induction on the length of paths in  $M$  and  $M'$ .

**Base Case:** Let  $\pi_1$  be a path in  $M$ ; i.e.,  $\pi_1 = s_0 \xrightarrow{\tau_i} s_1$ . We have to show that there exists a  $\pi'$  in  $M'$  such that  $\pi_1 \sim_{st\phi} \pi'$ . There are two possibilities:

1.  $\tau_i$  is invisible According to Algorithm 3,  $\tau_i$  is not present in  $M'$ . Thus we have  $\pi_1 \sim_{st\phi} \pi' = s'_0$  (Lemma 3.7).
2.  $\tau_i$  is visible According to Algorithm 3,  $\tau_i$  is still present in  $M'$ . Thus we have  $\pi_1 \sim_{st\phi} \pi' = s'_0 \xrightarrow{\tau_i} s'$  (Lemma 3.8).

**Induction:** Assume that for any path  $\pi_k$  in  $M$ , there exists a path  $\pi'_l$  in  $M'$ , for an  $l \leq k$ , such that  $\pi_k \sim_{st\phi} \pi'_l$ ; that is,

$$\underbrace{s_0 \rightarrow s_1 \dots s_{k-1} \rightarrow s_k}_{\sim_{st\phi}} \underbrace{s'_0 \rightarrow s'_1 \dots s'_{l-1} \rightarrow s'_l}$$

We have to show that, for any path  $\pi_{k+1} = s_0 \rightarrow s_1 \dots s_k \xrightarrow{\tau_i} s_{k+1}$  in  $M$ , there exists in  $M'$  a path stuttering equivalent to  $\pi_{k+1}$ . There are two possibilities:

**1.  $\tau_i$  is invisible** According to Algorithm 3,  $\tau_i$  is not present in  $M'$ . Thus

$$\begin{array}{c|c} \underbrace{s_0 \rightarrow s_1 \dots s_{k-1} \rightarrow s_k} & \underbrace{\xrightarrow{\tau_i} s_{k+1}} \\ \sim_{st\phi} \text{ (Induction hypothesis)} & \text{(Lemma 3.7)} \\ \hline \underbrace{s'_0 \rightarrow s'_1 \dots s'_{l-1} \rightarrow s'_l} & \end{array}$$

**2.  $\tau_i$  is visible** According to Algorithm 3,  $\tau_i$  is still present in  $M'$ . Thus

$$\begin{array}{c|c} \underbrace{s_0 \rightarrow s_1 \dots s_{k-1} \rightarrow s_k} & \underbrace{\xrightarrow{\tau_i} s_{k+1}} \\ \sim_{st\phi} \text{ (Induction hypothesis)} & \text{(Lemma 3.8)} \\ \hline \underbrace{s'_0 \rightarrow s'_1 \dots s'_{l-1} \rightarrow s'_l} & \underbrace{\xrightarrow{\tau_i} s'_{l+1}} \end{array}$$

□

We conclude that for any path in  $M$  there is a stuttering equivalent path in  $M'$ . Similarly we can prove that for any path in  $M'$  there is a stuttering equivalent path in  $M$ .

## 4 Case Study

The Canadian Hospice Palliative Care Association National Model (CHPCA 2002) [6] was built on an understanding of health, the illness and bereavement experiences, and the role hospice palliative care plays in relieving suffering and improving quality of life. We developed a Hospice Palliative Care (HPC) workflow, in collaboration with the local health authority the Guysborough Antigonish Strait Health Authority (GASHA), following the CHPCA 2002 model. This model contains general guidelines, called *Norms of Care*. We used the NOVA Workflow to model a HPC workflow and developed LTL formulae that the workflow must satisfy to comply with the norms.

After the patient's referral is received, her eligibility is checked for HPC. If eligible, the patient is sent for a set of therapeutic encounters which contains six essential steps – each is represented as a composite task in the “Overall” workflow (Fig. 6) – that guide the interaction between care givers, the patient and family. Fig. 6 also zooms into two composite tasks, namely PC\_CONSULT and CARE\_PLANNING. The palliative workflow has approximately 250 atomic

tasks and 40 decision points. The **PC\_CONSULT** task contains uncompensable tasks and the **CARE\_PLANNING** task contains compensable tasks. Table 1 shows some preconditions and actions of tasks.

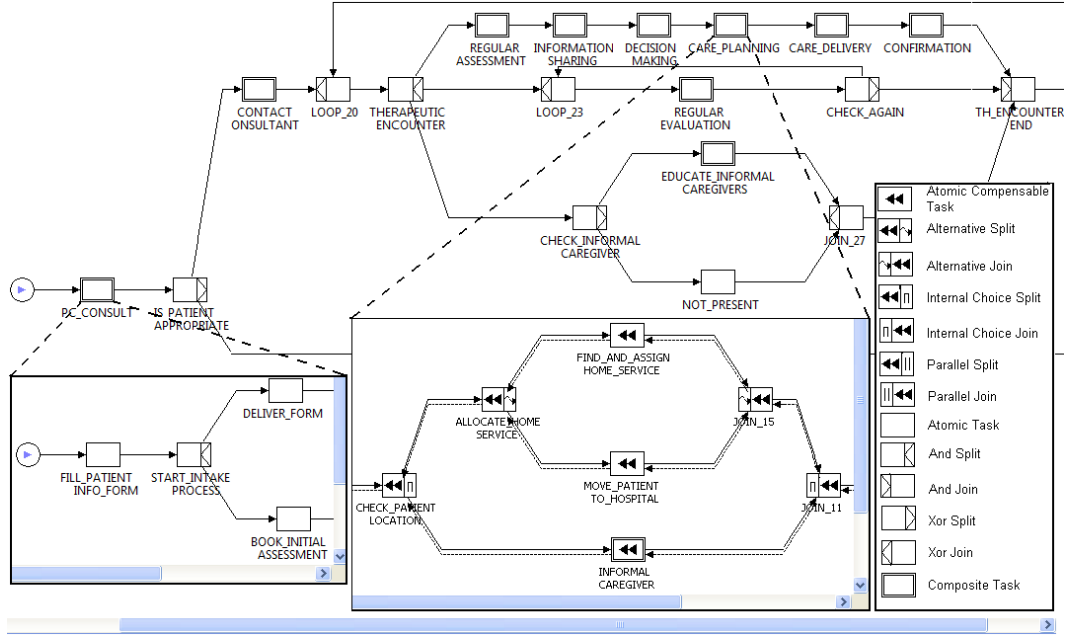


Fig. 6. Overview of the palliative care process model

**Prop1** If patient is at home, then home service must be provided. Otherwise, the patient must move to the hospital (compensation via alternative choice). In LTL:  $G ((location == 1) \rightarrow F ((home\_service == 1) \parallel (location == 2)))$ . In Fig. 6, patient's location is set with either 1 (representing home) or 2 (representing hospital) at the time of registration (**FILL\_PATIENT\_INFO\_FORM**); This information is accessed inside **CARE\_PLANNING** workflow.

**Prop2** If patient is distressed, then a social worker must be assigned in the care team. LTL:  $G ((distressed == 1) \rightarrow F (social\_worker == 1))$

**Prop3** If the patient is assigned a PPS of 50% or lower, s/he must be moved to the hospital. LTL:  $G ((pps \leq 50) \rightarrow F ((location == 2)))$

**Prop4** If the patient is with priority level of 3 or lower, s/he must be moved to the hospital. LTL:  $G ((patients\_level \leq 3) \rightarrow F ((location == 2)))$

**Prop5** If the patient's mobility change is identified, a Physiotherapist is notified. LTL:  $G ((change\_in\_mobility == 1) \rightarrow F (ack\_physiotherapist == 1))$

All experiments were executed with 64 CPU's and 3GB memory (per CPU) on the **Mahone2** cluster of ACEnet ([www.ace-net.ca](http://www.ace-net.ca)). The results are shown in Table 2. Time and memory are reduced using WS (workflow slicing) + POR (partial order reduction) compared to using POR alone. More details of this (and a larger) case study may be found in [15].

Table 1  
Tasks and their preconditions and actions

Task	Preconditions, $\psi$	Actions, $\alpha$
FILL_PATIENT_INFO_FORM		location = {1, 2}
ALLOCATE_HOME_SERVICE	location == 1	
INFORMAL_CAREGIVER	location == 2	informal_caregiver = 1
FIND_AND_ASSIGN_HOME_SERVICE		home_service = 1
ASSIGN_SOCIAL_WORKER	distressed == 1	social_worker = 1

Table 2  
Verification results for the DiVinE model checker

Property	Acc Cycle	WS + POR			POR		
		States	Mem (MB)	Time (s)	States	Mem (MB)	Time (s)
Prop1	No	126188210	88619.1	384.3	236576621	143836.2	1860
Prop2	No	107167421	83315.3	305.3	Unknown	Overflow	> hour
Prop3	No	128013744	88920.0	397.9	251323543	153290.3	1931
Prop4	No	127934841	88894.5	396.1	213254702	140215.0	1854
Prop5	No	13479	230.1	9.7	202233451	125804.1	1803

## 5 Related Work

Program slicing [20] is a well-studied technique. The basic idea is to abstract away variables and statements that do not influence the “point of interest”, called the *slicing criterion*. Program slicing can be applied to debugging, testing, software maintenance, and formal verification. Hatcliff et al. [7] extract slicing criteria using primitive propositions in LTL formulae and more importantly, define and prove formally the *correctness* of program slicing. Sloane and Holdsworth [19] propose the *generalized slicing* that considers different kinds of software entities and constructs. More importantly (w.r.t. the relevance of our work), they use the program syntax tree as the vehicle for the slicing algorithm. Unfortunately they have not included formal verification in their framework. Millett and Teitelbaum [13] propose a slicing algorithm for Promela (the input language of the model checker SPIN). Barbuti et al. [2] present, from the model checking point of view, a general theoretical result of an equivalence between a transition system model and the reduced one based on formulae represented in their proposed temporal logic called the *selective mu-calculus*. All these works strongly suggest that slicing can be applied to modeling languages at different abstraction levels.

Slicing techniques have been applied to Petri nets. Evangelista et al. [5] present a reduction technique for Coloured Petri nets (CPN). This technique only preserves the liveness of the net and only those LTL formulae that do not observe the reduced transitions of the net. Rakow [17] presents a Petri net

slicing algorithm and applies it to the verification of LTL formulae, the case study in the paper is based on a small textbook workflow example. Note that CWML can be deemed as an abstraction of CPN. We required two distinct CPNs one for the atomic uncompensable tasks and another for compensable tasks [16] as the two basic building blocks of the language and built up more complex Petri nets for each composite task. This type of abstraction is needed as real world workflows are generally complex and Petri net (including CPN) models of them can easily grow to be too large to be manageable.

There are several works that reduce the size of a workflow model. Wynn et al. [22] present reduction rules for YAWL [21] workflows with *Cancellation* regions and *OR-joins* to reduce the size of the workflow, while preserving its essential formulae w.r.t. a particular analysis problem. There, the authors only focused on the reachability analysis, whereas our slicing method works for any LTL<sub>X</sub> formula. Awad et al. [1] present a reduction procedure for BPMN graphs, but formal studies on the model equivalence are lacking. An ADEPT2 workflow can be verified using the SeaFlows compliance checker [9]. The authors discuss a data abstraction technique. As combining slicing and data abstraction is common practice in this area, we expect their technique and our slicing algorithm should complement each other well.

## 6 Conclusion and Future Work

This paper presents a model slicing algorithm. We prove the stuttering equivalence of the original models and the reduced models generated by the algorithm. This technique has been integrated into NOVA Workflow framework. Our experiments show that the technique greatly reduces the amount of memory and time for verification and makes verification of real world models of compensable systems possible.

We expect that translation of CWML to other model checkers is straightforward. Once other automated translation methods are developed, other model checkers can be used to verify large workflow models. In the future, we will consider time [12] in the model slicing algorithm as many specifications in a safety critical system such as healthcare are time sensitive.

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