Mental Math
Yearly Plan
Grade 9

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NOVA SCOTIA
Education
English Program Services

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Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

Mental Math: Grade 9 Yearly Plan

In this yearly plan for mental math in grade 9, an attempt has been made to align specific activities with the related chapter in the new grade 9 text, *Mathematics 9: Focus on Understanding*. In some areas, the mental math content is too broad to be covered in the time frame allotted for a single chapter. While it is desirable to match this content to the unit being taught, it is quite acceptable to complete some mental math topics when doing subsequent chapters that do not have obvious mental math connections. In the grade 9 mathematics program, students need to make decisions on whether an exact answer or approximate answer is required. When an exact answer is required, they need to decide whether to use mental math, pencil and paper computation, or a calculator. Keeping this in mind it is appropriate to allow practice in mental math to happen at the start or within a lesson.

The following are strategies from previous grades that students should be familiar with and can be briefly reviewed at the start of the year and reinforced through the development of appropriate questions and practice throughout the year.

Mental Math (M) (Grade 7–B2)

- Front–end (addition & multiplication)
- Compatible numbers
- Compatible factors
- Halve / Double
- Compensate, break up and bridge (Grade 4)

Estimation (E) (Grade 7 – B1)

- Rounding
- Front–end
- Special numbers
- Clustering of near compatibles
- Compatibles
- Using referents

Properties to be reviewed and used throughout the year, where appropriate

- Distributive
- Associative
- Commutative
- Identity

Chapter	Skill	Example
Number Sense	**Compare Real Numbers:** Begin with *reviewing* prior knowledge of integers, fractions, decimals, and approximating square roots. ▪ Fractions: discussions could include common denominators, common numerators, benchmarks, "one-away from making a whole", ▪ Decimals: "comparing common place values", finding a value in-between two other values. ▪ Approximating and calculating square roots	Fractions: Place the appropriate symbol (>, <, or =) between the two fractions. a) $\frac{5}{7}, \frac{3}{7}$ b) $\frac{5}{6}, \frac{5}{8}$ c) $\frac{3}{7}, \frac{5}{8}$ d) $\frac{9}{10}, \frac{5}{8}$ Decimals: Place the appropriate symbol (>, <, or =) between the two decimals. a) 0.2, 0.20 b) 0.123, 0.1234 c) 2.002, 2 .020 d) 2.5, 2.49 e) –1.3, –1.5 Write a decimal number between each pair: a) –2.3, 2.31 b) 0.54, –0.541 Calculate the square root of the following numbers: a) 36 b) 144 c) 400 d) 4900 e) $\frac{9}{100}$ f) 0.25 Approximate the square root of the following numbers: a) 29 b) 99 c) 63 d) 150
	Compare Real Numbers:	Identify the larger value: a) $\frac{1}{3}$, 0.4 d) $\sqrt{49}$, -7

Chapter	Skill	Example
		e) $\sqrt{1.21}$, 0.1 f) $\sqrt{70}$, 8.8 g) 0.40404…, 0.404004…
	Order Real Numbers: Students will use their comparing strategies to order a group of real numbers.	Order from least to greatest: a) $\frac{5}{6}$, $1\frac{3}{4}$, $\sqrt{2}$ b) –0.25, $\sqrt{0.25}$, $\frac{1}{4}$ c) π, $\frac{-10}{-3}$, 3.04
	Operations with Rationals: *Review* operations with rational numbers using strategies such as: ▪ Front End Addition (*Mental Math in the Junior High*, page 41) ▪ Compatible Addends(*Mental Math in the Junior High*, pages 51-56, 65, 99-102) ▪ Front End Multiplication (*Mental Math in the Junior High*, page 73) ▪ Compatible Factors (*Mental Math in the Junior High* page 123) ▪ Making Compatible Numbers (*Mental Math in the Junior High* page 125) ▪ Halve/ Double(*Mental Math in the Junior High* pages 117-120) ▪ Compensate: 140 – 69 can be thought of as 140 – 70 then *compensate* by adding 1. 12.5 – 4.7 can be thought of as 12.5 – 4.5 then *compensate* by subtracting 0.2. ▪ Review the four	Use the properties of numbers (Associative, Commutative, and Distributive) to calculate mentally: a) 2 × 24 × 50 b) 2 × 3.4 × 5 c) $4 \times \frac{3}{10} \times 2.5$ d) 50 × 14 e) 2.5 × 16 f) $7 \times \frac{3}{4} \times 12$ d) 0.3333… × 21 × 5 e) 0.25 × 25 × 16 f) 299 × 15 Use your compensation strategies for mentally calculating the following: a) 167 – 38 b) $3\frac{1}{2} - 2\frac{7}{8}$ c) 6.7 – 7.8

Chapter	Skill	Example
	operations on integers. Students should be able to mentally perform the sum, difference, product, or quotient of two "friendly" integers using strategies from prior grades. Include discussion of the zero principle as well.	
	Mentally calculate the squares of numbers	Mentally calculate the following: a) 20^2 b) 1.2^2 c) $(\frac{3}{4})^2$ d) 99^2 e) 5.5^2 f) $(\sqrt{4.3})^2$
	Laws of exponents: Mentally calculate problems using the laws of exponents: ▪ Students should be able to simplify appropriate expressions using their laws of exponents. Including problems with "friendly" or reasonable coefficients that will also allow the use of other strategies.	Express each in standard form: a) $\frac{7^2}{7^4}$ b) $(5^2 \times 3^4)^0$ c) $8^{-2} \times 8^4$ d) $\frac{2^4}{8^2}$ e) $(\frac{1}{3})^{-3} \div 9^2$ Simplify: a) $(2x^3)(-3x^5)$ b) $(\frac{1}{2}x)^2(4x^4)$ c) $(0.2x)^{-4}(40x^5)$

Chapter	Skill	Example
	Scientific Notation: Estimate: Students should be able to use estimation strategies with scientific notation. Students should first be able to identify the conditions necessary for easy mental calculations. They can then proceed to mentally calculate the answers. The starred questions are examples of those that can be done mentally.	Estimate the answer for each of the following: a) $1.78 \times 10^5 + 5.34 \times 10^5$ b) $7.01 \times 10^{-3} + 3.97 \times 10^{-3}$ c) $(2.46 \times 10^2)(3.2 \times 10^4)$ d) $9.86 \times 10^6 \div 2.21 \times 10^2$ Determine which of the following can be found mentally and calculate: a) $2.5 \times 10^4 + 3.5 \times 10^7$ b) * $3.4 \times 10^5 + 4.8 \times 10^5$ c) $1.8 \times 10^{-4} + 1.8 \times 10^4$ d) *$5.79 \times 10^{-5} - 4.01 \times 10^{-5}$ e) $6.3 \times 10^{-9} - 6.2 \times 10^{-5}$ f) *$4.4 \times 10^3 - 6.4 \times 10^3$ g) *$5.5 \times 10^{-3} (2 \times 10^{-5}$ h) $(3.48 \times 10^6)\ 2.7 \times 10^6$ i) *$4.5 \times 10^6 \div 1.5 \times 10^{-5}$ j) $3.67 \times 10^4 \div 2.81 \times 10^{-2}$
Patterns and Relations	Evaluate a single variable expression (start with whole numbers, then fractions and decimals). Progression of the types of expressions is also important (e.g.$2x + 4$, $4 + 2x$, $2x - 4$, $-2x + 4$, $4 - 2x$)	Evaluate the following expressions for the given value : (Do each evaluation separately) a) $3x + 1$ $x = 2$; $x = -6$; $x = \frac{1}{3}$; $x = -\frac{4}{3}$ $x = 0.3$ $x = -0.5$ b) $5 + 4x$ $x = 10$; $x = -4$; $x = -\frac{1}{4}$ $x = \frac{3}{2}$; $x = 0.75$; $x = 1.25$ c) $6x - 8$ $x = 0$; $x = -2$; $x = \frac{1}{3}$; $x = -\frac{1}{12}$; $x = 1.5$; $x = -2.5$ d) $\frac{1}{2}x + 10$ $x = 6$; $x = -8$; $x = \frac{4}{7}$ $x = 4\frac{2}{3}$; $x = -12.6$ e) $4 - x^2$ $x = 3$; $x = -2$; $x = \frac{1}{3}$ $x = 0.3$; $x = 0.5$ f) $2^x + 3$ $x = 0$; $x = 3$; $x = -2$ g) $(4x) \div 3$ $x = 6$; $x = 33$, $x = -15$ $x = \frac{3}{4}$; $x = 1.5$

<table>
<tr><th>Chapter</th><th>Skill</th><th>Example</th></tr>
<tr><td></td><td>Determine from a table of values if the slope of the graph will be negative or positive.

Determine from a table of values if the graph of the relationship will be linear or non-linear.</td><td>
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<tr><td>4</td><td>7</td></tr>
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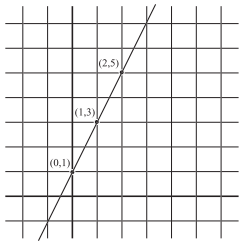
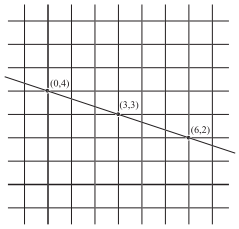
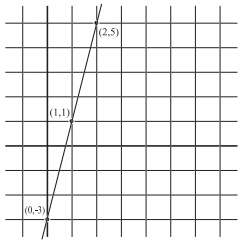
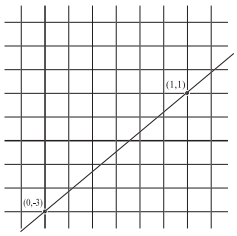
</td></tr>
<tr><td></td><td>Mentally calculate the slope of a line from a graph</td><td>




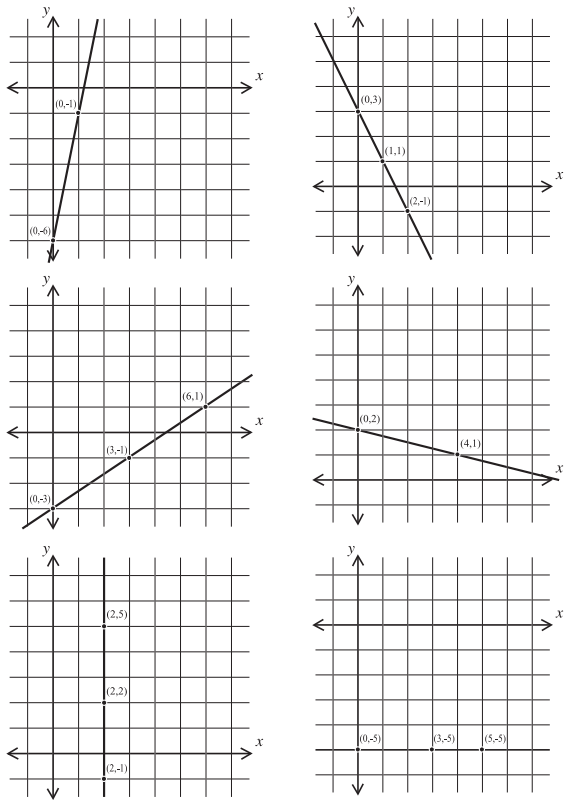


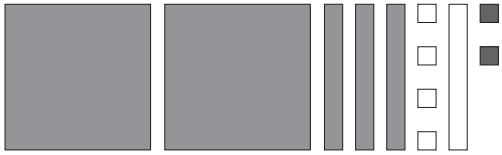
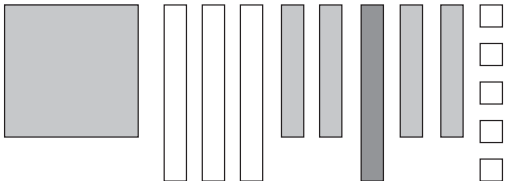
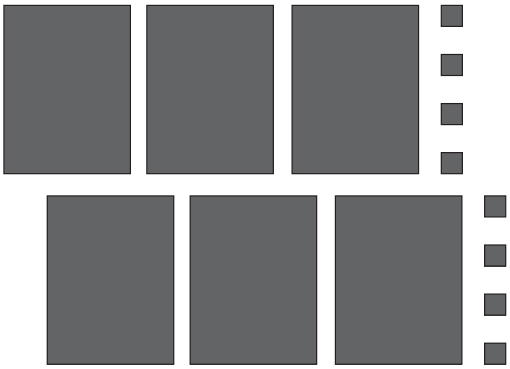


</td></tr>
<tr><td></td><td>Give a linear equation and ask to calculate the slope</td><td>a) $y = 2x + 4$,
b) $2y = 4x + 10$,
c) $5y = -15x$,
d) $\frac{1}{2}y = 4x + 3$</td></tr>
<tr><td></td><td>Give a linear equation and ask to calculate the y-intercept

Give a linear equation and ask to graph the equation of the line:

Hand out a photocopied coordinate system and a transparency sheet. The transparency should have a diagonal line drawn from corner to corner. The students are shown an equation on the board and then are asked to</td><td>a) $y= 2x +4$,
b) $2y = x -10$,
c) $\frac{1}{3}y = x + 1$</td></tr>
</table>

Chapter	Skill	Example
	place the "line" on the coordinate system to graph the equation. Teachers can easily look at the graph on the students' desks to see if the graphing was completed correctly.	
	Show graphs of various "friendly" lines including horizontal and vertical lines and ask for the equation of the line	***Determine the equations of these lines mentally.*** 

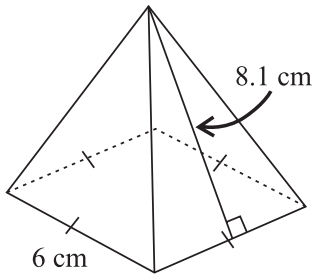
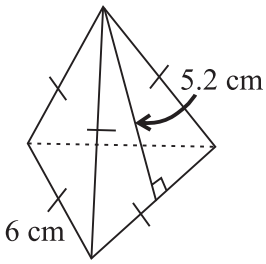
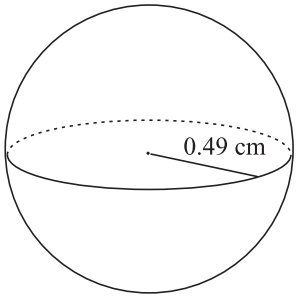
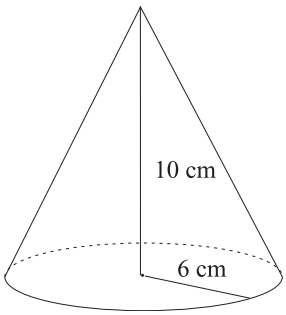
Chapter	Skill	Example
Equations and Inequations	Give an algebraic expression and ask students to model using Alge-Tiles.	Model the following expressions with Alge-Tiles : a) $4x^2 + 3x - 2$ b) $2(2y^2 - 3x - 1)$ Variations of the above: 1. Model the expression $4x^2 + 3x - 2$ using the minimum amount of tiles possible. 2. Model the expression $4x^2 + 3x - 2$ using more than 9(numbers could vary to meet your needs) tile pieces.
	Give a model and ask for the expression(The overhead or magnetic white board works well)	State in simplest form the expressions illustrated below: a)  b)  c) 

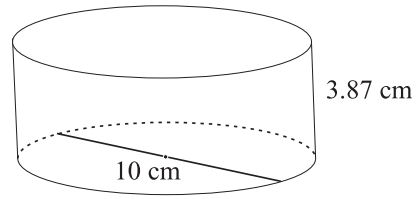
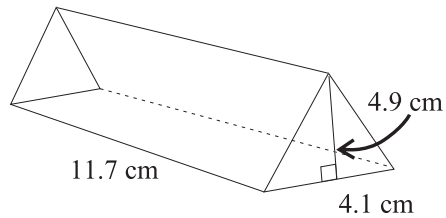
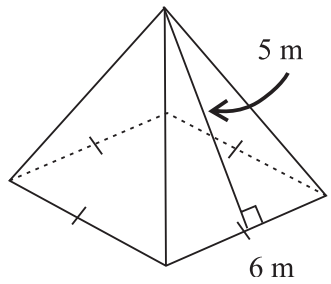
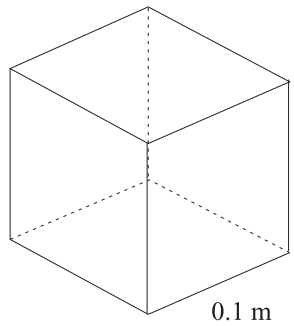
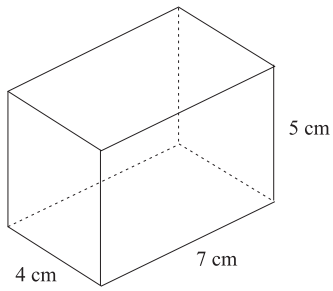
Chapter	Skill	Example
	Make decisions on what single variable equations and inequations could be solved mentally	Determine which of these equations could be solved mentally : a) $2x + 1 = 9$ b) $3x + 13 = -1.8x$ c) $\frac{x}{4} = -3$ d) $2(x + 6) = 5(2x - 1)$ e) $5(x - 4) = -10$ f) $3x - 1 = -7$ g) $\frac{2}{3}x + 6 = \frac{4}{5}x - 2$ h) $4x + 5 = 3x + 10$ i) $x^2 + 1 = 101$ j) $11 = \frac{1}{3}x - 1$ k) $4.6x + 3.4x = -24$ l) $\frac{3}{4}x - 16 = -\frac{1}{4}x$ m) $1\ \frac{7}{8}x + 2x + \frac{1}{8}x = 32$ n) $3x + 2.3 = x - 5.7$ o) $5x - 2.25 - \frac{3}{4} > 4x - 1$ p) $2x - + 10.7 < -5x + 4.23$
	Solve appropriate single variable equations and inequations by inspection. Create equations that reinforce the strategies identified in chapter 1. Again use the real number progression from whole numbers to fractions to decimals and then give a mixture.	Use the equations and inequations in a, c, e, f, h, i, j , k , l, m ,n , and o as examples of ones that can be solved mentally. Work through these with students. Many of the equations above use the 'make one' strategy developed in early grades. Discuss the many ways 'make one' can be disguised in an equation.
Probability	Benchmarks: Students have had a lot of exposure to benchmarks. They have had to determine which benchmark a number is closer to, if it is greater than that benchmark, or if it is less than that benchmark.	Determine which benchmark $(0, \frac{1}{2}, 1)$ the following numbers are closer to. a) $\frac{2}{7}, \frac{11}{36}, \frac{36}{40}, \frac{27}{52}, \frac{1}{15}$ b) 0.9, 0.09, 0.125, 0.65, 0.659

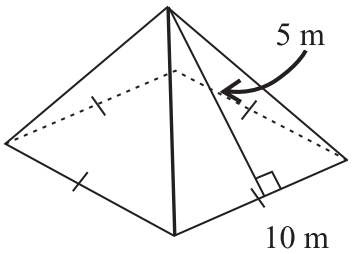
Chapter	Skill	Example
	For probability, the most common benchmarks are 0, $\frac{1}{2}$, and 1.	Complete the fraction so that it is close to $\frac{1}{2}$ but a bit larger: a) $\frac{x}{33}$ b) $\frac{42}{x}$ c) $\frac{x}{98}$ d) $\frac{60}{x}$
	Estimate probabilities	There are 3600 fish in the pond and 889 are speckled trout. Estimate the probability that you will catch a trout when you go fishing. (Students should see that 889 is close to 900 and so the probability is about $\frac{1}{4}$ or 0.25 or 25%)
	Compare and order numbers	Can reinforce work found in the Compare and Order Real Numbers sections in chapter 1
	Translating between the various forms of fractions, decimals, and percents Progression of this may be: ▪ familiar fractions to decimal form and/or percent form(eg: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$) 'Familiar fractions' may vary according to the experience of the students. ▪ less familiar fractions/decimals to percent form(1/3, 2/3, 1/8, 1/6, 5/9) ▪ fraction/decimal percents to decimal form.	When given the following fraction, decimal or percent, supply the other two equivalent forms. a) $\frac{1}{4}$ b) $\frac{2}{3}$ c) $\frac{3}{8}$ d) 0.75 e) 0.125 f) 32% g) 9% h) $\frac{7}{100}$ i) $\frac{12}{20}$ (take the opportunity to discuss strategies for these translations; i.e. $\frac{12}{20}$ may be translated easily by expressing it in equivalent forms such as $\frac{60}{100}$, $\frac{6}{10}$, $\frac{3}{5}$)

Chapter	Skill	Example
	Review multiplying fractions and decimals, by whole numbers as well as fractions by fractions symbolically. The following are concepts students should recognize to shorten their computation time: ▪ Discussion of multiplying reciprocals ▪ 'Dividing out ones' Example: In question c) students should notice that the denominator 3 and the numerator 6 have a common factor of 3. This common factor should be divided out. ▪ Whole number by a fraction, where the demoninator is a factor of the whole number.	Perform the indicated operations mentally: a) $\frac{3}{4} \times 12$ b) $-10 \times \frac{2}{5}$ c) $\frac{2}{3} \times \frac{6}{7}$ d) $\frac{3}{5} \times \frac{4}{5}$ e) $\frac{4}{9} \times \frac{9}{4}$ f) 0.25×16 g) 20×0.3
	Review calculating percents of a whole number using various strategies.	Mentally calculate each: a) 25 % × 44 (think $\frac{1}{4}$ of 44) b) 33 $\frac{1}{3}$ % of 93 (think $\frac{1}{3}$ of 93) c) 50 % of 248 (think $\frac{1}{2}$ of 248 or halve/double strategy) d) 75% of 16 (think $\frac{3}{4}$ of 16) e) 12 % of 300 (think 10% × 300 + 2% × 300) or (1% × 300) × 12 f) 54 % of 600 (think $\frac{1}{2}$ × 600 + 4% × 600) g) $\frac{1}{2}$ % of 84 (think halve/double) h) $\frac{1}{4}$ % of 200 (think halve/double twice) i) 9% of 500 (think 10% × 500 – 1% of 500)

Chapter	Skill	Example
	Review multiplying fractions, decimals, and/or percents in context Finding approx. percents in context.	$\frac{3}{4}$, (75%), **or** 0.75 of a class of 28 students are girls. How many are girls? 20% of the grade nine students at your school like rap music. If 30 students like rap music, how many students are in your school? 17 out of 24 students in your class have brown hair. Approximately, what percent of the students in your class have brown hair?
Measurement	Conversions can become memorizations of meaningless information for the students if they do not have practice in estimation of the measurements. Give students opportunities to practice their estimation skills. **Example:** Have the students work in partners, one with the metre stick and the other to do the estimating. Have the student put their hands together in a 'prayer-like' fashion and they will pull them apart according to the length given by the teacher. The student with the metre stick will measure the distance between the hands and record the amount the estimate was over or under the required distance. **Variation:** Have the students draw lines on their paper that are approximately the distance given by the teacher. They will then measure the distance and record the number of units over estimated or under estimated. This activity can be adapted to area, volume, and capacity by using familiar benchmarks of textbook covers, bottles of water, etc. Simple SI conversions (length, area, volume, volume to capacity)	Show the approximate length of a) 15 cm b) 70 mm c) 100 mm d) 100cm e) 50 cm f) 50mm Convert each of the following: g) 120 cm = _____m h) 1.2 km = _____m i) 0.3km = _____cm j) 2 m^2 = _____cm^2 k) 500 mm^2 = _____ cm^2 l) 3.5 km^2 = _____ m^2 m) 2 m^3 = _____cm^3 n) 5 000 mm^3 = _____ cm^3 o) 500 cm^3 = _____ L p) 300 L = _____ m^3

Chapter	Skill	Example
	Review square numbers and square roots.	Can reinforce work found in chapter one.
	Have students estimate the solution to problems involving volume and surface area using appropriate estimation strategies. *Suitable diagrams for estimation of surface area are rectangular and triangular prisms, square and triangle based pyramids, cones, and spheres. For estimating volume, suitable diagrams may include rectangular and triangular prisms, square and triangle based pyramids, cylinders, and cones.*	1. Estimate the surface area of each figure: a) square based pyramid  b) tetrahedron (4 equal triangles)  c) sphere with a radius of 0.49 cm  2. Estimate the Volume of these figures a) cone 

Chapter	Skill	Example
		b) cylinder  c) prism 
	Make decisions on what problems about volume and surface area can be solved mentally. Recognize those that can use previously learned strategies (front-end, compatible factors, halve/double etc) Solve volume and surface area problems for prisms, pyramids, cylinders and cones mentally when appropriate. *Suitable diagrams are rectangular and triangular prisms, and pyramids.*	1. a) Find the volume of a square based pyramid with base of 11m and height of 6 m  b) Find the Surface Area of this cube  c) Find the Volume of 

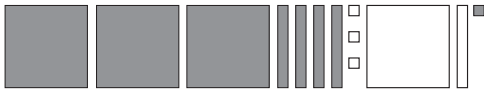
Chapter	Skill	Example
		d) Find the Surface Area of a square based pyramid with a base of 10 cm and a slant height of 5 cm 
	Have students create problems that can be estimated and/or solved mentally. These can then be presented to the class followed by a discussion of the use of efficient strategies.	
Geometry	You may wish to continue working on mental math topics from previous chapters.	
	Reinforce equivalent fractions. b) Determine equivalent fractions. Students are given a fraction that is not expressed in simplest form and asked to identify equivalent fractions from a list.	a) State three fractions equivalent to each of the following: i) $\frac{2}{3}$ ii) $\frac{3}{5}$ iii) $\frac{7}{8}$ iv) $\frac{11}{12}$ v) $\frac{13}{15}$ b) In the list identify equivalent fractions for each given fraction (or you can also ask to identify ones that are not equivalent) and discuss why. i) $\frac{9}{12}$ $(\frac{3}{4}, \frac{8}{9}, \frac{15}{20}, \frac{2}{3})$ ii) $\frac{10}{15}$ $(\frac{15}{20}, \frac{24}{36}, \frac{6}{9}, \frac{1}{5})$

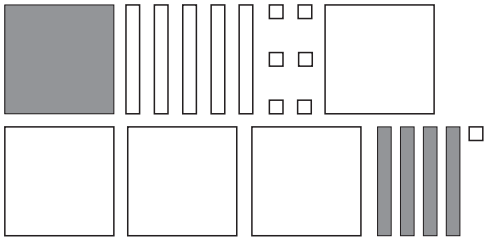
<table>
<tr><th>Chapter</th><th>Skill</th><th>Example</th></tr>
<tr><td></td><td>Determine the missing value in a proportion. Use “friendly” numbers, whole at first, so the students can easily work between or within the ratios.

This provides an opportunity for the students to focus on the multiplicative relationship of the ratios in the proportion.

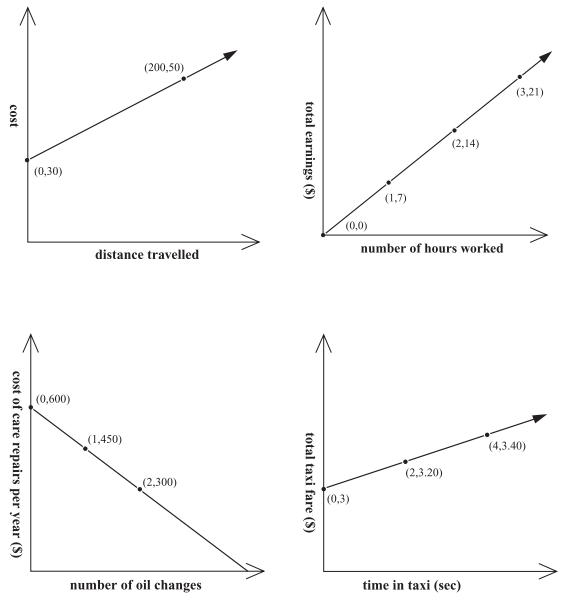
Progress then to other rational numbers.</td><td>Calculate the missing value :
$\frac{x}{6} = \frac{8}{12}$
$\frac{10}{x} = \frac{6}{18}$
$\frac{0.5}{2} = \frac{2}{x}$
$\frac{2}{6} = \frac{x}{27}$
$\frac{4.5}{x} = \frac{1}{4}$
$\frac{x}{4} = \frac{9}{x}$</td></tr>
<tr><td></td><td>Students will be able to determine and use mapping rules. Given two of the three pieces of data(pre-image coordinates, image coordinates, and mapping rule) determine the third.</td><td>Complete the table : (present each row of the table as an individual problem)
<table>
<tr><th>Pre-image</th><th>Mapping rule</th><th>Image</th></tr>
<tr><td>(-4,7)</td><td>$(x,y)\bullet(x+3,y-8)$</td><td></td></tr>
<tr><td>(-3,2)</td><td></td><td>(-3,-2)</td></tr>
<tr><td></td><td>$(x,y)\bullet(2x, 2y)$
$(x,y)\bullet(x-1,y+6)$</td><td>$(\frac{6}{8}, \frac{10}{12})$
(5,0)</td></tr>
<tr><td>(3,4)</td><td></td><td>(-4,-3)</td></tr>
</table></td></tr>
<tr><td>Polynomials</td><td>Use Alge-Tiles to model or represent expressions and have the students simplify these expressions. Overhead or magnetic tiles work well for this.</td><td>This is reinforcing the work done in chapter 3.</td></tr>
<tr><td></td><td>Show several different arrangements of Alge-tiles (A, B, …E)

a) Ask students to simplify and record the simplified expressions.

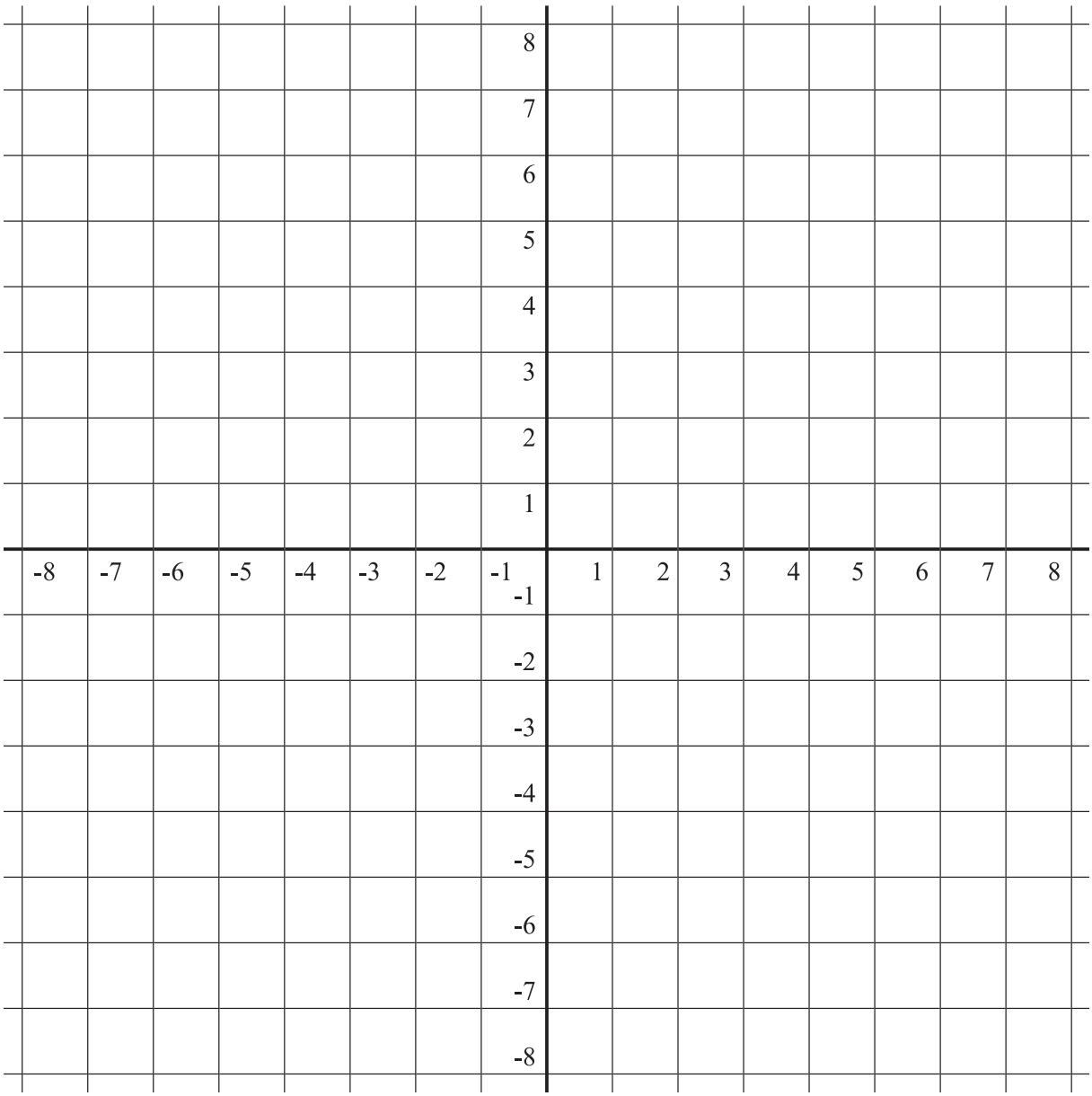
b) Have them determine the resultant expression for such things as A + B, A – B, 2C.</td><td>a)
 
b)
  </td></tr>
</table>

Chapter	Skill	Example
	c) Give students a resultant expression and an operation, and ask students which expression(s) underwent the operation to produce the resultant. Progression for this may begin with monomials, to binomials to trinomials and then to a mixture of polynomials. The progression depends on the experience of the students.	c)  a) answers : A. $2x^2 + 3x - 2$ B. $-x^2 + 3x + 1$ C. $-3x^2 - x - 7$ b) Simplify A + C B – C 2A c) Which two expressions will give the sum of $-4x^2 + 2x - 6$? The product of 2 and what expression will yield $-2x^2 + 6x + 2$?
	Review laws of exponents	See Chapter 1 for examples
	Identify the GCF of a binomial mentally. The progression may be from a numerical factor, to a variable and then both as GCF's.	Identify the GCF of each pair: a) 24, 36 b) 44, -88 c) -56, -63 d) x, 2x e) $3x^2$, x^2 f) $-4x^2$, 6x g) $45y^3$, $36y^2$ Identify the GCF in each expression: $6x + 9$ $-4y - 10$ $12k - 30$ $3x^2 + 4x$ $13y^4 - 4y^2$ $x^2y^6 - 7y^2$ $10x + 15x^3$ $2m^6 - 4m^3$

<table>
<tr><th>Chapter</th><th>Skill</th><th>Example</th></tr>
<tr><td></td><td>Students will be able to multiply a monomial by polynomials using “friendly” numbers with previously learned strategies.</td><td>Determine the product of each:
$(3x)\ (4x^5)$
$(4x^2)\ (13x^3)\ (25x^{-1})$
$(\frac{1}{4}x^3)\ (12x^{-2})$
$(0.5x^9)\ (86x^2)$
$(22x)\ (15x)$
$(4x^6)\ (99x^{-2})$
$(25y^{-5})(36y^{-7})$think $25\times(4 \times 9) = (25 \times 4) \times 9$</td></tr>
<tr><td></td><td>Have students solve problems such as “find two numbers that multiply to give _____ and add to give _____” and visa versa</td><td>Find two numbers that multiply to give you _____and add to give you _____
<table><tr><th>Multiply to give</th><th>Add to give</th></tr><tr><td>6</td><td>5</td></tr><tr><td>-20</td><td>8</td></tr><tr><td>30</td><td>-11</td></tr></table></td></tr>
<tr><td></td><td>Multiply two binomials, and factor expressions into two binomials using “friendly” numbers.</td><td>Determine the product of each:
$(x + 3)\ (x + 8)$
$(y – 4)\ (y + 5)$
$(z – 12)\ (z – 3)$
$(b + 7)\ (b – 7)$
Factor each expression into two binomials
$x^2 + 4x + 3$
$x^2 + 7x – 18$
$y^2 - 10y + 24$
$x^2 + x + 0.25$
$x^2 - 36$
$x^2 - 1.44$
$x^2 + \frac{9}{2}x + 2$
$x^2 + \frac{37}{3}x + 4$</td></tr>
<tr><td></td><td>Make decisions on which polynomials can be factored into two binomials.</td><td>Determine which expressions can be factored and discuss why.
a) $x^2 +16$
b) $x^2 -16$
c) $x^2 +7x+10$
d) $x^2 -9x+18$
e) $x^2 +4x-5$
f) $x^2 +8x-7$
g) $x^2 +x$</td></tr>
</table>

Chapter	Skill	Example
	Divide a polynomial by a monomial.	Simplify a) $\frac{36x^6}{4x^4}$ b) $\frac{2.4x^3}{1.2x^2}$ c) $\frac{\left(12x^2+4x+20\right)}{4}$ d) $\frac{15x^6}{0.5x}$
	Have students create expressions that can be added, subtracted, multiplied or divided mentally.	
Data Management	Create sets of data and have students mentally calculate the mean using prior strategies.	Calculate the mean of each. 28 + 36 + 22 + 34 (compatibles) 75 + 29 + 46 + 54(break up and bridge) 4.6 + 3.5 + 8.4 + 1.5 + 2 (make one) 410 + 120 + 330 + 140 (front end addition or break up and bridge)
	Review work on slope. This can be extended to students calculating and describing the meaning of the slope and y-intercept in context when presented with labeled graphs.	Reinforcement of work done in chapter two where students determined the slope from a graph and from an equation. 

Chapter	Skill	Example
	Present scenarios and ask students to determine if the data would represent continuous or discrete data.	Determine if the data collected would be an example of continuous or discrete data: a) Filling up the gas tank of a car. Cost vs number of litres bought b) buying a quantity of newspapers. Cost vs number of papers purchased.
	Review work on evaluating expressions	Reinforcement of work done in chapter two where students evaluated various expressions using whole numbers, fractions, and decimals.


8
7
6
5
4
3
2
1
-8 -7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8
-1
-2
-3
-4
-5
-6
-7
-8