Victorian Essential Learning Standards

Standards and progression points

Mathematics

In Mathematics, there is one point (0.5) at Level 1 for assessing student progress towards the Level 1 standard in the *Number, Space, Measurement, chance and data* and *Working mathematically* dimensions.

Mathematics – Progressing towards Level 1

Progression Point 0.5

At 0.5, the work of a student progressing towards the standard at Level 1 demonstrates, for example:

Number

- association of number names with numerals and models of numbers (counting or subitising)
- use of drawn simple symbols in place of objects; for example, B for boy
- ordering of objects and sets; for example, largest to smallest
- placement of a variety of objects in order from first to third
- use of one-to-one correspondence and numbers 1 to 10 when counting

Space

- sorting of objects according to shape
- matching of shapes to solve puzzles; for example, jigsaw puzzles
- naming of common two-dimensional objects according to geometrical shape
- use of the terms *next to*, *beside* or *in front of* to describe their own location relative to other students
- sequences of movements following simple directions

Measurement, chance and data

Progression Point 0.5

At 0.5, the work of a student progressing towards the standard at Level 1 demonstrates, for example:

- use of descriptive terms such as *longer*, *taller* and *heavier* to compare length and mass of pairs of familiar objects
- use of *yesterday*, *today* and *tomorrow* and the ability to name the corresponding days
- awareness that a clock changes in response to time in a daily cycle
- use of a die or similar device to determine the range of outcomes in a game
- interpretation of pictographs, and collection and sorting of items or data in preparation for the creation of a pictograph

Working mathematically

- selection of appropriate materials for illustrating a mathematical problem or its solution
- use of drawing to represent problems and solutions
- verbal description of simple patterns and extension of these patterns
- recognition and use of numbers on a calculator
- recognition that addition is relevant to a task

Mathematics – Level 1

Number

At Level 1, students form small sets of objects from simple descriptions and make simple correspondences between those sets. They count the size of small sets using the numbers 0 to 20. They use one-to-one correspondence to identify when two sets are equal in size and when one set is larger than another. They form collections of sets of equal size. They use ordinal numbers to describe the position of elements in a set from first to tenth. They use materials to model addition and subtraction by the aggregation (grouping together) and disaggregation (moving apart) of objects. They add and subtract by counting forward and backward using the numbers from 0 to 20.

Space

At Level 1, students recognise, copy and draw points, lines and simple free-hand curves. They identify basic two-dimensional shapes such as triangles, circles and squares and three-dimensional solids and objects such as boxes and balls. They recognise the interior and exterior of shapes and objects. They sort

geometric objects according to simple descriptions. They place and orientate shapes according to simple descriptions such as *next to*, *beside*, *in front of*, *behind*, *over* and *under*.

They develop and follow simple instructions to move and place shapes and objects in familiar situations in relation to what they can see, and to move themselves from one place to another.

Measurement, chance and data

At Level 1, students compare length, area, capacity and mass of familiar objects using descriptive terms such as *longer*, *taller*, *larger*, *holds more* and *heavier*. They make measurements using informal units such as paces for length, handprints for area, glasses for capacity, and bricks for weight.

They recognise the continuity of time and the natural cycles such as day/night and the seasons. They correctly sequence days of the week. They use informal units such as heartbeats and hand claps at regular intervals to measure and describe the passage of time.

They recognise and respond to unpredictability and variability in events, such as getting or not getting a certain number on the roll of a die in a game or the outcome of a coin toss. They collect and display data related to their own activities using simple pictographs.

Working mathematically

At Level 1, students use diagrams and materials to investigate mathematical and real life situations. They explore patterns in number and space by manipulating objects according to simple rules (for example, turning letters to make patterns like *bqbqbq*, or flipping to make *bdbdbdbd*).

They test simple conjectures such as ‘nine is four more than five’. They make rough estimates and check their work with respect to computations and constructions in *Number*, *Space*, and *Measurement, chance and data*. They devise and follow ways of recording computations using the digit keys and +, − and = keys on a four function calculator.

They use drawing tools such as simple shape templates and geometry software to draw points, lines, shapes and simple patterns. They copy a picture of a simple composite shape such as a child’s sketch of a house.

In Mathematics, standards for the *Structure* dimension are introduced at Level 3.

Mathematics – Progressing towards Level 2

Progression Point 1.25

At 1.25, the work of a student progressing towards the standard at Level 2 demonstrates, for example:

Number

- ordering of lists of small sets of numbers up to 20
- counting forwards and backwards by 1 from starting points between 1 and 100
- calculation of the next number when asked to add 1 or 2 to any natural number from 0 to 10
- drawing of diagrams to show sharing of up to 20 items
- drawing of diagrams to show subtraction activities

Space

- recognition of lines, corners and boundaries in two-dimensional shapes
- classification of shapes according to number of sides
- representation of squares, rectangles, circles

Progression Point 1.5

At 1.5, the work of a student progressing towards the standard at Level 2 demonstrates, for example:

Number

- ordering of money amounts in cents
- counting by 2s, 5s and 10s from 0 to a given target, and recognition of the associated number patterns; for example, 7, 9, 11 …
- use of *half* and *quarter* as a descriptor; for example, a quarter of a cake
- addition and subtraction of two-digit multiples of ten by counting on and counting back
- counting on from the larger of two collections to find their total
- use of the number properties (commutative and associative) of addition in mental computation, and recognition of complements to ten; for example, 3 + 4 + 7 + 6 = 3 + 7 + 4 + 6 = 10 + 10 = 20

Space

- recognition of congruence of two shapes
- identification of shapes with symmetry
- labelling and use of points on diagrams to specify lines, corners and boundaries

Progression Point 1.75

At 1.75, the work of a student progressing towards the standard at Level 2 demonstrates, for example:

Number

- counting by 1s, 10s and 100s from 0 to 1000
- grouping of coins of the same denomination in sets of $1
- development and use of a 'fact family' linking 25 + 5 = 30 to 5 + 25 = 30, 30 − 5 = 25 and 30 − 25 = 5
- addition and subtraction of numbers less than 10 through recall and use of number facts
- identification of half of a set of objects, including recognition of the need for $\frac{1}{2}$ when sharing an odd number of objects

Space

- recognition of whether a single transformation produces a congruent or similar shape
- identification of the important features of two-dimensional shapes and use of these distinguishing features to compare and

Progression Point 1.25	Progression Point 1.5	Progression Point 1.75
At 1.25, the work of a student progressing towards the standard at Level 2 demonstrates, for example:	*At 1.5, the work of a student progressing towards the standard at Level 2 demonstrates, for example:*	*At 1.75, the work of a student progressing towards the standard at Level 2 demonstrates, for example:*
and triangles using freehand drawing, templates and technology • recognition and naming of spheres and cubes • identification of main features on local maps	• sorting of objects onto a venn diagram labelled with shape information • production of similar figures by enlargement	contrast various shapes • production of simple patterns with transformations (flips, slides, turns) • specification of instructions for movement, including relative position and quarter turns left and right • decomposition of three-dimensional shapes into their respective nets; for example, by cutting up boxes • construction of informal local maps
Measurement, chance and data • informal measurement of length by making, describing and comparing personal units • use of a clock to determine the hour • ordering of days, weeks, months and years • understanding of distinction between cold, cool, warm, hot and boiling • awareness that some events are equally likely to occur; for example, a head or a tail showing when a coin is tossed	**Measurement, chance and data** • use of uniform units for length; for example, cm as a unit for measuring length • informal measurement of area and mass by making, describing and comparing personal units • knowledge of the relationship between analogue and digital clocks • knowledge of the outcomes of chance events such as rolling a die • interpretation of pictographs, bar and column graphs	**Measurement, chance and data** • informal measurement of capacity by making, describing and comparing personal units • construction of a time line for daily activity and use of a diary for recording daily events • drawing of an analogue clock to match a given digital time and of reading an analogue clock to the nearest half hour • ordering of familiar events in terms of their probability between *impossible* and *certain* • collection and recording of categorical and numerical data

Progression Point 1.25	Progression Point 1.5	Progression Point 1.75
At 1.25, the work of a student progressing towards the standard at Level 2 demonstrates, for example:	*At 1.5, the work of a student progressing towards the standard at Level 2 demonstrates, for example:*	*At 1.75, the work of a student progressing towards the standard at Level 2 demonstrates, for example:*
Working mathematically • development of descriptive rules for patterns • use and justification of approximations • elementary use of mathematical symbols to describe their own thought processes • reading and rewriting of numbers from a calculator • checking of calculations using technology	**Working mathematically** • formulation and testing of conjectures using models that involve, for example, objects, patterns, shapes and numbers • verification of estimation of a solution to a number sentence • understanding of how to follow a sequence of steps in a procedure • addition of numbers on a calculator and recognition of the function of calculator keys • understanding of appropriate action for responding to an incorrect calculator result	**Working mathematically** • continuation of patterns and the recognition of inconsistencies • search for alternative methods in order to verify answers • assessment of the context at hand, and an explanation of this assessment • representation of data using pictographs that are either hand-drawn or assisted by technology

Mathematics – Level 2

Number

At Level 2, students model the place value of the natural numbers from 0 to 1000. They order numbers and count to 1000 by 1s, 10s and 100s. Students skip count by 2s, 4s and 5s from 0 to 100 starting from any natural number. They form patterns and sets of numbers based on simple criteria such as odd and even numbers. They order money amounts in dollars and cents and carry out simple money calculations. They describe simple fractions such as one half, one third and one quarter in terms of equal sized parts of a whole object, such as a quarter of a pizza, and subsets such as half of a set of 20 coloured pencils. They add and subtract one- and two-digit numbers by counting on and counting back. They mentally compute simple addition and subtraction calculations involving one- or two-digit natural numbers, using number facts such as complement to 10, doubles and near doubles. They describe and calculate simple multiplication as repeated addition, such as 3 × 5 = 5 + 5 + 5; and division as sharing, such as 8 shared between 4. They use commutative and associative properties of addition and multiplication in mental computation (for example, 3 + 4 = 4 + 3 and 3 + 4 + 5 can be done as 7 + 5 or 3 + 9).

Space

At Level 2, students recognise lines, surfaces and planes, corners and boundaries; familiar two-dimensional shapes including rectangles, rhombuses and hexagons, and three-dimensional shapes and objects including pyramids, cones, and cylinders. They arrange a collection of geometric shapes, such as a set of attribute blocks, into subsets according to simple criteria, and recognise when one set of shapes is a subset of another set of shapes. They recognise and describe symmetry, asymmetry, and congruence in these shapes and objects. They accurately draw simple two-dimensional shapes by hand and construct, copy and combine these shapes using drawing tools and geometry software. They apply simple transformations to shapes (flips, turns, slides and enlargements) and depict both the original and transformed shape together. They specify location as a relative position, including left and right, and interpret simple networks, diagrams and maps involving a small number of points, objects or locations.

Measurement, chance and data

At Level 2, students make, describe and compare measurements of length, area, volume, mass and time using informal units. They recognise the differences between non-uniform measures, such as hand-spans, to measure length, and uniform measures, such as icy-pole sticks. They judge relative capacity of familiar objects and containers by eye and make informal comparisons of weight by hefting. They describe temperature using qualitative terms (for example, cold, warm, hot). Students use formal units such as hour and minute for time, litre for capacity and the standard units of metres, kilograms and seconds.

Students recognise the key elements of the calendar and place in sequence days, weeks and months. They describe common and familiar time patterns and such as the time, duration and day of regular sport training and tell the time at hours and half-hours using an analogue clock, and to hours and minutes using a digital clock.

Students predict the outcome of chance events, such as the rolling of a die, using qualitative terms such as certain, likely, unlikely and impossible. They collect simple categorical and numerical data (count of frequency) and present this data using pictographs and simple bar graphs.

Working mathematically

At Level 2, students make and test simple conjectures by finding examples, counter-examples and special cases and informally decide whether a conjecture is likely to be true. They use place value to enter and read displayed numbers on a calculator. They use a four-function calculator, including use of the constant addition function and × key, to check the accuracy of mental and written estimations and approximations and solutions to simple number sentences and equations.

In Mathematics, standards for the *Structure* dimension are introduced at Level 3.

Mathematics – Progressing towards Level 3

Progression Point 2.25

At 2.25, the work of a student progressing towards the standard at Level 3 demonstrates, for example:

Number

- use of place value (as the idea that 'ten of these is one of those') to determine the size and order of whole numbers to hundreds
- use of money as a model for grouping and unpacking lots of 10s
- rounding of amounts of money up and down to the nearest dollar
- use of written number sentences such as 20 ÷ 4 = 5 to summarise sharing (partition) and 'how many?' (quotition) processes
- use of fractions with numerators other than one; for example, $\frac{3}{4}$ of a block of chocolate

Space

- use of instructions to create tessellations such as in paving and brickwork patterns
- recognition and naming of familiar three-dimensional shapes
- use of 'vertical' and 'horizontal' to describe orientation

Progression Point 2.5

At 2.5, the work of a student progressing towards the standard at Level 3 demonstrates, for example:

Number

- addition and subtraction of amounts of money including calculation of change from $10
- automatic recall of number facts from 2, 5 and 10 multiplication tables
- use of strategies such as 'near doubles', 'adding 9' and 'build to next 10' to solve addition and subtraction problems
- use of written methods for whole number problems of addition and subtraction involving numbers up to 99
- development and use of fraction notation and recognition of equivalent fractions such as $\frac{1}{2} = \frac{4}{8}$, including the ordering of fractions using physical models

Space

- identification of shapes in terms of faces, edges and vertices
- representation of angles formed dynamically; for example, between the hands of a clock or between their own limbs, and explanations of these angles in terms of simple fractions of a

Progression Point 2.75

At 2.75, the work of a student progressing towards the standard at Level 3 demonstrates, for example:

Number

- use of place value (as the idea that 'ten of these is one of those') to determine the size and order of decimals to hundredths
- use of algorithms for the addition and subtraction of numbers to two decimal places
- representation of multiplication as a rectangular array and as the area of a rectangle
- use of fact families (5 × 7 = 35, 35 ÷ 7 = 5) to solve division problems

Space

- knowledge of the names of polygons using Greek prefixes; for example, *hex*agon
- construction of balls (for example, tennis balls, footballs, soccer balls and beach balls) from flexible, two-dimensional pieces of material, and a template

Progression Point 2.25	Progression Point 2.5	Progression Point 2.75
At 2.25, the work of a student progressing towards the standard at Level 3 demonstrates, for example:	*At 2.5, the work of a student progressing towards the standard at Level 3 demonstrates, for example:*	*At 2.75, the work of a student progressing towards the standard at Level 3 demonstrates, for example:*
• determination of approximate north by considering the position of the sun • use of a grid to refer to objects on a map	complete revolution • construction of prisms and pyramids from their two-dimensional nets • orientation of north, south, east and west from their own current position	• representation of an object by drawing its plan • construction of a model for an angle using rotation of lines
Measurement, chance and data • use of formal units of measurement; for example, metres to measure length, and hour, minute and second for time • application of estimations using personal units, such as pace length and arm span, and comparison with measures using formal units, such as metres and centimetres • use of ruler and tape measure (linear scale) and trundle wheel (circular scale) to validate estimates of length • setting of temperature in Celsius on a circular scale; for example, on an oven, and estimation of temperature in degrees Celsius • displays of data as a column or bar graph	**Measurement, chance and data** • estimation and measurement of mass, volume and capacity of common objects; for example, kilogram of flour, litre of soft drink • reading of analogue clocks to the nearest quarter of an hour • construction and interpretation of a daily timetable • identification of events which are equally likely • construction of an appropriately labelled bar graph	**Measurement, chance and data** • calculation of area through multiplication of the length of a rectangle by its width • estimation of angle in terms of quarter turns and half turns • investigation of the fairness of events such as gambling and games through experimentation • comparison of the likelihood of everyday events and linking of events with statements about how likely they are to occur • understanding of the distinction between discrete and continuous scales
Structure • knowledge of the effect of multiplying by ten on the location of the decimal point in a number • use of lists, venn diagrams and grids to record	**Structure** • variation of order and grouping of addition (commutative and associative property) to facilitate computations; for example,	**Structure** • use of distributive property in calculations; for example, 6 × 37 = 6 × 30 + 6 × 7 • construction of lists, venn diagrams and grids

Progression Point 2.25	Progression Point 2.5	Progression Point 2.75
At 2.25, the work of a student progressing towards the standard at Level 3 demonstrates, for example:	*At 2.5, the work of a student progressing towards the standard at Level 3 demonstrates, for example:*	*At 2.75, the work of a student progressing towards the standard at Level 3 demonstrates, for example:*
items that have a certain attribute • selection of a sample from a population • recognition that one set is or is not a subset of another • use of '=' to indicate equivalence or the result of a computation	3 + 5 + 7 + 5 = 3 + 7 + 5 + 5 = 10 +10 = 20 • specification of all possible outcomes of a simple chance event • construction of number sentences • calculations using notation such as '3 + 5 − 2 ='	to be used for recording combinations of two attributes
Working mathematically • identification of pattern and similarity in data sets and shapes, and use of pattern, often by observing a set of simpler situations, as a problem solving strategy • use of materials and models to solve problems and explain answers • checking of accuracy of calculations with a calculator • use of technology to create and manipulate shapes and simple maps	**Working mathematically** • selection of appropriate situations for the use of a guess–check–improve strategy • explanation and comparison of alternative computation methods	**Working mathematically** • use of materials and models to illustrate and test generalisations • rephrasing of a problem or representing it using a physical model, diagram, list or table as a problem solving strategy • selection of multiplication and division as more efficient processes than repeated addition and subtraction • application of number skills to solve routine problems from everyday contexts • partitioning of a task into smaller sub-tasks

Mathematics – Level 3

Number

At Level 3, students use place value (as the idea that 'ten of these is one of those') to determine the size and order of whole numbers to tens of thousands, and decimals to hundredths. They round numbers up and down to the nearest unit, ten, hundred, or thousand. They develop fraction notation and compare simple common fractions such as $\frac{3}{4} > \frac{2}{3}$ using physical models. They skip count forwards and backwards, from various starting points using multiples of 2, 3, 4, 5, 10 and 100.

They estimate the results of computations and recognise whether these are likely to be over-estimates or under-estimates. They compute with numbers up to 30 using all four operations. They provide automatic recall of multiplication facts up to 10 × 10.

They devise and use written methods for:

- whole number problems of addition and subtraction involving numbers up to 999
- multiplication by single digits (using recall of multiplication tables) and multiples and powers of ten (for example, 5 × 100, 5 × 70)
- division by a single-digit divisor (based on inverse relations in multiplication tables).

They devise and use algorithms for the addition and subtraction of numbers to two decimal places, including situations involving money. They add and subtract simple common fractions with the assistance of physical models.

Space

At Level 3, students recognise and describe the directions of lines as vertical, horizontal or diagonal. They recognise angles are the result of rotation of lines with a common end-point. They recognise and describe polygons. They recognise and name common three-dimensional shapes such as spheres, prisms and pyramids. They identify edges, vertices and faces. They use two-dimensional nets, cross-sections and simple projections to represent simple three-dimensional shapes. They follow instructions to produce simple tessellations (for example, with triangles, rectangles, hexagons) and puzzles such as tangrams**.** They locate and identify places on maps and diagrams. They give travel directions and describe positions using simple compass directions (for example, N for North) and grid references on a street directory**.**

Measurement, chance and data

At Level 3, students estimate and measure length, area, volume, capacity, mass and time using appropriate instruments. They recognise and use different units of measurement including informal (for example, paces), formal (for example, centimetres) and standard metric measures (for example, metre) in appropriate contexts. They read linear scales (for example, tape measures) and circular scales (for example, bathroom scales) in measurement contexts. They read digital time displays and analogue clock times at five-minute intervals. They interpret timetables and calendars in relation to familiar events. They compare the likelihood of everyday events (for example, the chances of rain and snow). They describe the fairness of events in qualitative terms. They plan and conduct chance experiments (for example, using colours on a spinner) and display the results of these experiments. They recognise different types of data: non-

numerical (categories), separate numbers (discrete), or points on an unbroken number line (continuous). They use a column or bar graph to display the results of an experiment (for example, the frequencies of possible categories).

Structure

At Level 3, students recognise that the sharing of a collection into equal-sized parts (division) frequently leaves a remainder. They investigate sequences of decimal numbers generated using multiplication or division by 10. They understand the meaning of the '=' in mathematical statements and technology displays (for example, to indicate either the result of a computation or equivalence). They use number properties in combination to facilitate computations (for example, 7 + 10 + 13 = 10 + 7 + 13 = 10 + 20). They multiply using the distributive property of multiplication over addition (for example, 13 × 5 = (10 + 3) × 5 = 10 × 5 + 3 × 5). They list all possible outcomes of a simple chance event. They use lists, venn diagrams and grids to show the possible combinations of two attributes. They recognise samples as subsets of the population under consideration (for example, pets owned by class members as a subset of pets owned by all children). They construct number sentences with missing numbers and solve them.

Working mathematically

At Level 3, students apply number skills to everyday contexts such as shopping, with appropriate rounding to the nearest five cents. They recognise the mathematical structure of problems and use appropriate strategies (for example, recognition of sameness, difference and repetition) to find solutions.

Students test the truth of mathematical statements and generalisations. For example, in:

- number (which shapes can be easily used to show fractions)
- computations (whether products will be odd or even, the patterns of remainders from division)
- number patterns (the patterns of ones digits of multiples, terminating or repeating decimals resulting from division)
- shape properties (which shapes have symmetry, which solids can be stacked)
- transformations (the effects of slides, reflections and turns on a shape)
- measurement (the relationship between size and capacity of a container).

Students use calculators to explore number patterns and check the accuracy of estimations. They use a variety of computer software to create diagrams, shapes, tessellations and to organise and present data.

Mathematics – Progressing towards Level 4

Progression Point 3.25

At 3.25, the work of a student progressing towards the standard at Level 4 demonstrates, for example:

Number

- use of large number multiples of ten to approximate common quantities; for example, 100 000 people in a major sports venue
- representation of square numbers using a power of 2; for example, $9 = 3^2$
- use of ratios to describe relative sizes
- appropriate selection and use of mental and written algorithms to add, subtract, multiply and divide (by single digits) natural numbers
- multiplication of fractions by fractions through use of the rectangle area model (grid)
- use of brackets to determine order of operations

Space

- recognition of angles between lines, particularly when lines are parallel or perpendicular
- use of scaled grids to draw similar figures (enlarged or reduced)
- use of a graphical scale to determine actual size and distance from a map

Progression Point 3.5

At 3.5, the work of a student progressing towards the standard at Level 4 demonstrates, for example:

Number

- listing of objects and their size, where size varies from thousandths to thousands of a unit
- addition, subtraction and multiplication of fractions and decimals (to one decimal place) using approximations such as whole number estimates and technology to confirm accuracy
- representation of simple ratios as percentages, fractions and decimals
- identification of calculation errors resulting in unreasonable results
- ordering of integers (for example, positive and negative temperatures), positive fractions and decimals

Space

- classification and sorting of two-dimensional shapes using the properties of lines (curvature, orientation and length) and angles (less than, equal to, or greater than 90°)
- construction or selection of possible objects given a plan (bird's eye view) or an elevation (side view)

Progression Point 3.75

At 3.75, the work of a student progressing towards the standard at Level 4 demonstrates, for example:

Number

- multiplication by increasing and decreasing by a factor of two; for example, 24 × 16 = 48 × 8 = 96 × 4 = 192 × 2 = 384 × 1 = 384
- recognition of equivalent rates expressed as percentages, fractions and decimals
- recognition that multiplication can either enlarge or reduce the magnitude of a number (multiplication by fractions or decimals)
- use of inverse relationship between multiplication and division to validate calculations
- creation of sets of multiples of numbers and their representation in index form; for example, 3, 9, 27 written as 3^1, 3^2, 3^3 respectively

Space

- construction of a copy of a shape, given details about side lengths and angles
- use of two-dimensional isometric drawings of three-dimensional objects, noting how shapes are not always preserved; for example, squares become parallelograms
- knowledge that the sum of angles at a point on

Progression Point 3.25	Progression Point 3.5	Progression Point 3.75
At 3.25, the work of a student progressing towards the standard at Level 4 demonstrates, for example:	*At 3.5, the work of a student progressing towards the standard at Level 4 demonstrates, for example:*	*At 3.75, the work of a student progressing towards the standard at Level 4 demonstrates, for example:*
	• construction of transformed shapes and patterns by stamping, folding and rotating	a straight line is 180°
• interpretation of maps of their own immediate environment using various scales; for example, school ground, suburb, state, country • description of a path by a set of coordinates	• representation of relationships within a family (people or animals) through use of a tree diagram (network)	• use of a compass and compass directions to describe orientation in the school ground
Measurement, chance and data	**Measurement, chance and data**	**Measurement, chance and data**
• estimation and measurement of perimeter of polygons • conversion between metric measurements for length; for example, 0.27m = 27cm • estimation and measurement of angles in degrees to the nearest 10° • use of fractions to assign probability values between 0 and 1 to probabilities based on symmetry; for example, Pr(six on a die) = $\frac{1}{6}$ • identification of mode and range for a set of data	• estimation and measurement of surface area; for example, use of square metres, and area of land; for example, use of hectares • awareness of the accuracy of measurement required and the appropriate tools and units • subdivision of a circle into two sectors according to a given proportion for arc length • design of questionnaires to obtain data from a sample of the population • sorting of data using technology	• conversion between metric units; for example, L to mL, and understanding of the significance of thousands and thousandths in the metric system • simulation of simple random events • calculation and analysis of the stability of a sequence of long run frequencies where the number of trials increases, say from 5 to 10 to 20 to 100 • interpretation of pie charts and histograms • identification of the median for a set of data
Structure	**Structure**	**Structure**
• conversion between venn diagrams and karnaugh maps as representations of relationships between two sets • recognition and completion of patterns formed	• incorporation of tables of information relating pairs of everyday variables • sorting of sequences into certain types (constant addition, constant multiplication,	• construction of diagrams illustrating the possible relationship between two sets and the truth of statements involving the words *all*, *some* or *none*

Progression Point 3.25

At 3.25, the work of a student progressing towards the standard at Level 4 demonstrates, for example:

by constant addition or subtraction

- use of *add* and *subtract* as inverse operations to solve simple word equations such as 'I am thinking of a number. If I add 6 I get 18, what number did I start with?'
- use of trial and error to find a missing number in a number sentence; for example, 4 × ? + 6 = 22
- use of language to describe change in everyday items or attributes whose value varies over time

Progression Point 3.5

At 3.5, the work of a student progressing towards the standard at Level 4 demonstrates, for example:

fibonacci, square, triangular)

- use of division and multiplication as inverses; for example, multiplication by 25 can be carried out as 'multiplication by 100 followed by division by 4'
- consistent and correct use of conventions for order of operations

Progression Point 3.75

At 3.75, the work of a student progressing towards the standard at Level 4 demonstrates, for example:

- construction of number patterns and tables of values from an equation or a recurrence relation
- recognition that a given number pattern can be represented by an apparently unrelated equation and recurrence relation; for example, 5, 9, 13 … represented by 'multiply position in the pattern (first, second, third ...) by 4 and add 1' and 'start with 5 then repeatedly add 4 to the previous term'
- understanding of zero and its characteristic of not having a multiplicative inverse, and the consequences of attempting division by zero

Working mathematically

- consideration of problems with a similar mathematical structure as a problem solving strategy
- use of familiar problems to focus on strategies to help in solving an unfamiliar problem
- search for counter-examples in an attempt to disprove a conjecture
- location of data sources, including use of the world wide web
- collection of mathematical data using

Working mathematically

- application of mathematics to model and solve simple practical problems; for example, the construction of a pair of stilts
- efficient communication when using mathematical language, symbols and representations
- appreciation of the history of mathematics in development of geometry and number concepts
- development and testing of conjectures with the aid of a calculator; for example, divisibility

Working mathematically

- knowledge of interpretation of maps, graphs and models
- understanding of patterns through the use of systematic strategies such as calculating first differences
- application of a set of questions linked to an area of investigation
- knowledge of appropriate historical information

Progression Point 3.25	Progression Point 3.5	Progression Point 3.75
At 3.25, the work of a student progressing towards the standard at Level 4 demonstrates, for example:	*At 3.5, the work of a student progressing towards the standard at Level 4 demonstrates, for example:*	*At 3.75, the work of a student progressing towards the standard at Level 4 demonstrates, for example:*
technology; for example, using data logging	tests • incorporation of text, data, images and graphs using technology, to report the results of an investigation	

Mathematics – Level 4

Number

At Level 4, students comprehend the size and order of small numbers (to thousandths) and large numbers (to millions). They model integers (positive and negative whole numbers and zero), common fractions and decimals. They place integers, decimals and common fractions on a number line. They create sets of number multiples to find the lowest common multiple of the numbers. They interpret numbers and their factors in terms of the area and dimensions of rectangular arrays (for example, the factors of 12 can be found by making rectangles of dimensions 1 × 12, 2 × 6, and 3 × 4).

Students identify square, prime and composite numbers. They create factor sets (for example, using factor trees) and identify the highest common factor of two or more numbers. They recognise and calculate simple powers of whole numbers (for example, $2^4 = 16$).

Students use decimals, ratios and percentages to find equivalent representations of common fractions (for example, $\frac{3}{4} = \frac{9}{12} = 0.75 = 75\% = 3 : 4 = 6 : 8$).

They explain and use mental and written algorithms for the addition, subtraction, multiplication and division of natural numbers (positive whole numbers). They add, subtract, and multiply fractions and decimals (to two decimal places) and apply these operations in practical contexts, including the use of money. They use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Space

At Level 4, students classify and sort shapes and solids (for example, prisms, pyramids, cylinders and cones) using the properties of lines (orientation and size), angles (less than, equal to, or greater than 90°), and surfaces. They create two-dimensional representations of three dimensional shapes and objects found in the surrounding environment. They develop and follow instructions to draw shapes and nets of solids using simple scale. They describe the features of

shapes and solids that remain the same (for example, angles) or change (for example, surface area) when a shape is enlarged or reduced. They apply a range of transformations to shapes and create tessellations using tools (for example, computer software).

Students use the ideas of size, scale, and direction to describe relative location and objects in maps. They use compass directions, coordinates, scale and distance, and conventional symbols to describe routes between places shown on maps. Students use network diagrams to show relationships and connectedness such as a family tree and the shortest path between towns on a map.

Measurement, **chance and data**

At Level 4, students use metric units to estimate and measure length, perimeter, area, surface area, mass, volume, capacity time and temperature. They measure angles in degrees. They measure as accurately as needed for the purpose of the activity. They convert between metric units of length, capacity and time (for example, L–mL, sec–min).

Students describe and calculate probabilities using words, and fractions and decimals between 0 and 1. They calculate probabilities for chance outcomes (for example, using spinners) and use the symmetry properties of equally likely outcomes. They simulate chance events (for example, the chance that a family has three girls in a row) and understand that experimental estimates of probabilities converge to the theoretical probability in the long run.

Students recognise and give consideration to different data types in forming questionnaires and sampling. They distinguish between categorical and numerical data and classify numerical data as discrete (from counting) or continuous (from measurement). They present data in appropriate displays (for example, a pie chart for eye colour data and a histogram for grouped data of student heights). They calculate and interpret measures of centrality (mean, median, and mode) and data spread (range).

Structure

At Level 4 students form and specify sets of numbers, shapes and objects according to given criteria and conditions (for example, 6, 12, 18, 24 are the even numbers less than 30 that are also multiples of three). They use venn diagrams and karnaugh maps to test the validity of statements using the words *none*, *some* or *all* (for example, test the statement '*all* the multiples of 3, less than 30, are even numbers').

Students construct and use rules for sequences based on the previous term, recursion (for example, the next term is three times the last term plus two), and by formula (for example, a term is three times its position in the sequence plus two).

Students establish equivalence relationships between mathematical expressions using properties such as the distributive property for multiplication over addition (for example, $3 \times 26 = 3 \times (20 + 6)$).

Students identify relationships between variables and describe them with language and words (for example, how hunger varies with time of the day).

Students recognise that addition and subtraction, and multiplication and division are inverse operations. They use words and symbols to form simple equations. They solve equations by trial and error.

Working mathematically

At Level 4, use students recognise and investigate the use of mathematics in real (for example, determination of test results as a percentage) and historical situations (for example, the emergence of negative numbers).

Students develop and test conjectures. They understand that a few successful examples are not sufficient proof and recognise that a single counter-example is sufficient to invalidate a conjecture. For example, in:

- number (all numbers can be shown as a rectangular array)
- computations (multiplication leads to a larger number)
- number patterns (the next number in the sequence 2, 4, 6 … must be 8)
- shape properties (all parallelograms are rectangles)
- chance (a six is harder to roll on die than a one).

Students use the mathematical structure of problems to choose strategies for solutions. They explain their reasoning and procedures and interpret solutions. They create new problems based on familiar problem structures.

Students engage in investigations involving mathematical modelling. They use calculators and computers to investigate and implement algorithms (for example, for finding the lowest common multiple of two numbers), explore number facts and puzzles, generate simulations (for example, the gender of children in a family of four children), and transform shapes and solids.

Mathematics – Progressing towards Level 5

Progression Point 4.25

At 4.25, the work of a student progressing towards the standard at Level 5 demonstrates, for example:

Number

- identification of square numbers up to, and including, 100
- knowledge of decimal and percentage equivalents for ½, ¼, ¾, ⅓, ⅔
- expression of single digit decimals as fractions in simplest form and conversion between ratio, fraction, decimal and percentage forms
- use of index notation to represent repeated multiplication
- division of fractions using multiplication by the inverse

Space

- construction of a plan, elevations and cross-sections for a three-dimensional object
- knowledge of how features (for example, an angle) change, or not, when a shape undergoes a transformation (for example, a rotation)
- classification of polygons with reference to a

Progression Point 4.5

At 4.5, the work of a student progressing towards the standard at Level 5 demonstrates, for example:

Number

- representation of collections of objects in base 2 notation
- location of the square roots from $\sqrt{(1)}$ to by $\sqrt{(100)}$ their approximate position on the real number line
- construction of factor trees for the expression of numbers in terms of powers of prime factors
- use of calculations involving operations with mixed numbers
- knowledge of the first several digits of decimal approximations to *pi*, π

Space

- identification of congruent shapes
- tessellation of suitable irregular shapes
- use of angle facts for a triangle
- use of conventional symbols and contours to describe a route marked on a map

Progression Point 4.75

At 4.75, the work of a student progressing towards the standard at Level 5 demonstrates, for example:

Number

- addition, multiplication and division of integers
- representation of subtraction of integers through the use of a physical model, and of integer subtraction as an equivalent integer addition, and as the difference between integers
- calculation of squares and cubes of rational numbers
- mental computation of square roots of rational numbers associated with known perfect squares; for example, $\sqrt{(0.64)}$ = 0.8 because $8^2 = 64$; $\sqrt{(6.4)}$ is not related to 8
- use of technology to confirm the results of operations with squares and square roots

Space

- knowledge of methods for creating the illusion of depth in a two-dimensional image, and description of the related process in geometrical terms
- production and analysis of images based on projection from a point (one point perspective)

Progression Point 4.25	Progression Point 4.5	Progression Point 4.75
At 4.25, the work of a student progressing towards the standard at Level 5 demonstrates, for example:	*At 4.5, the work of a student progressing towards the standard at Level 5 demonstrates, for example:*	*At 4.75, the work of a student progressing towards the standard at Level 5 demonstrates, for example:*
definition or a key property • construction of parallel and perpendicular lines • use of a map reference to locate a point or region on a map • use of networks to display relationships between people and pathways between objects	• representation of pathways between objects as part of a network	and a line • calculation of size of objects using a numerical map scale • use of bearings and distances to plot a route on a map • equivalence of components of a three-dimensional object and its net; for example, vertices and nodes, arcs and edges, faces and regions
Measurement, chance and data	**Measurement, chance and data**	**Measurement, chance and data**
• development and use of formulas for the area and perimeter of triangles and parallelograms • determination of the internal and external angle sums for a polygon and confirmation by measurement • estimation of the likely maximum and minimum error associated with a measurement • appropriate use of zero to indicate accuracy of measurement; for example, a piece of timber 2.100m long is accurate to the nearest mm • recognition of the mean value of a set of measurements as the best estimate, and that the range could represent the associated error	• use of appropriate units and measurement of length, perimeter, area, surface area, mass, volume, capacity, angle, time and temperature, in context • calculation of total surface area of prisms, including cylinders, by considering their nets • contrast between the stability of long run relative frequency and the variation of observations based on small samples • construction of dot plots, and stem and leaf plots to represent data sets	• understanding of the distinction between error and percentage error • use of random numbers to assist in probability simulations and the arithmetic manipulation of random numbers to achieve the desired set of outcomes • calculation of theoretical probability using ratio of number of 'successful' outcomes to total number of outcomes • use of tree diagrams to explore the outcomes from multiple event trials • display and interpretation of dot plots, and stem and leaf plots, including reference to mean, median and mode as measures of centre

Progression Point 4.25

At 4.25, the work of a student progressing towards the standard at Level 5 demonstrates, for example:

Structure

- use of inverse and identity when subtracting and dividing rational numbers

- identification of domain and range; independent and dependent variable and their role in graphing
- representation of data by plotting points in the first quadrant and explanation of key features
- collection and classification of sets of data as either linear or non-linear depending on whether the slope is constant
- interpretation of a letter as a symbol for any one of a set of numbers and use in symbolic description of relationships

Working mathematically

- consideration of evidence to support theorems; for example, in geometry
- exploration of the appropriateness of linear models for data
- translation between verbal descriptions and

Progression Point 4.5

At 4.5, the work of a student progressing towards the standard at Level 5 demonstrates, for example:

Structure

- use of *inequality*, *equality*, *approximately equal* and *not equal*, including in symbolic expressions

- translation from verbal description to algebraic representation, and of the structure of algebraic expressions; for example, if $500 is shared between n people, each receives $500/n$
- solution of simple linear equations using tables, graphs and inverse operations (backtracking)
- representation of inequalities as parts of the number line; for example, $x < -5$
- translation between symbolic rules, patterns and tables for linear functions

Working mathematically

- application of logic to the creation and use of a database
- identification of the mathematical information needed to solve a problem or carry out an investigation
- development of deductive proof to reach new

Progression Point 4.75

At 4.75, the work of a student progressing towards the standard at Level 5 demonstrates, for example:

Structure

- lists of sets in the power set of a given set and knowledge that the total number of set equals 2^n for n elements in the given set

- solution of equations such as $x^2 = 17$ as required in measurement situations; for example, using pythagoras theorem
- graphical representation of simple inequalities such as $y \leq 2x + 4$
- selection of a type of function (linear, exponential, quadratic) to match a set of data
- translation between forms (table, graph, rule, recurrence relation) of representation of a function

Working mathematically

- communication of the results of a mathematical investigation in an appropriate form
- creation and manipulation of tables and graphs using technology
- numerical and graphical solution of algebraic

Progression Point 4.25	Progression Point 4.5	Progression Point 4.75
At 4.25, the work of a student progressing towards the standard at Level 5 demonstrates, for example:	*At 4.5, the work of a student progressing towards the standard at Level 5 demonstrates, for example:*	*At 4.75, the work of a student progressing towards the standard at Level 5 demonstrates, for example:*
algebraic rules • use of technology to extend their own ability to make and test conjectures • use of spreadsheets to manipulate data and generate graphs	conclusions • use of interpolation to make predictions • development of simple geometric and algebraic models for real situations; for example, representation of an animal as a cylinder	problems using technology • exploration of geometrical propositions using technology

Mathematics – Level 5

Number

At Level 5, students identify complete factor sets for natural numbers and express these natural numbers as products of powers of primes (for example, $36\,000 = 2^5 \times 3^2 \times 5^3$).

They write equivalent fractions for a fraction given in simplest form (for example, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \ldots$). They know the decimal equivalents for the unit fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{9}$ and find equivalent representations of fractions as decimals, ratios and percentages (for example, a subset: set ratio of 4:9 can be expressed equivalently as $\frac{4}{9} = 0.\overline{4} \approx 44.44\%$). They write the reciprocal of any fraction and calculate the decimal equivalent to a given degree of accuracy.

Students use knowledge of perfect squares when calculating and estimating squares and square roots of numbers (for example, $20^2 = 400$ and $30^2 = 900$ so $\sqrt{700}$ is between 20 and 30). They evaluate natural numbers and simple fractions given in base-exponent form (for example, $5^4 = 625$ and $(\frac{2}{3})^2 = \frac{4}{9}$). They know simple powers of 2, 3, and 5 (for example, $2^6 = 64$, $3^4 = 81$, $5^3 = 125$). They calculate squares and square roots of rational numbers that are perfect squares (for example, $\sqrt{0.81} = 0.9$ and $\sqrt{\frac{9}{16}} = \frac{3}{4}$). They calculate cubes and cube roots of perfect cubes (for example, $\sqrt[3]{64} = 4$). Using technology they find square and cube roots of rational numbers to a specified degree of accuracy (for example, $\sqrt[3]{200} = 5.848$ to three decimal places).

Students express natural numbers base 10 in binary form, (for example, $42_{10} = 101010_2$), and add and multiply natural numbers in binary form (for example, $101_2 + 11_2 = 1000_2$ and $101_2 \times 11_2 = 1111_2$).

Students understand ratio as both set: set comparison (for example, number of boys : number of girls) and subset: set comparison (for example, number of girls : number of students), and find integer proportions of these, including percentages (for example, the ratio number of girls: the number of boys is 2 : 3 = 4 : 6 = 40% : 60%).

Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding (for example, $925 \div 34 \approx 900 \div 30 = 30$).

Students use efficient mental and/or written methods for arithmetic computation involving rational numbers, including division of integers by two-digit divisors. They use approximations to π in related measurement calculations (for example, $\pi \times 5^2 = 25\pi = 78.54$ correct to two decimal places).

They use technology for arithmetic computations involving several operations on rational numbers of any size.

Space

At Level 5, students construct two-dimensional and simple three-dimensional shapes according to specifications of length, angle and adjacency. They use the properties of parallel lines and transversals of these lines to calculate angles that are supplementary, corresponding, allied (co-interior) and alternate. They describe and apply the angle properties of regular and irregular polygons, in particular, triangles and quadrilaterals. They use two-dimensional nets to construct a simple three-dimensional object such as a prism or a platonic solid. They recognise congruence of shapes and solids. They relate similarity to enlargement from a common fixed point. They use single-point perspective to make a two-dimensional representation of a simple three-dimensional object. They make tessellations from simple shapes.

Students use coordinates to identify position in the plane. They use lines, grids, contours, isobars, scales and bearings to specify location and direction on plans and maps. They use network diagrams to specify relationships. They consider the connectedness of a network, such as the ability to travel through a set of roads between towns.

Measurement, chance and data

At Level 5, students measure length, perimeter, area, surface area, mass, volume, capacity, angle, time and temperature using suitable units for these measurements in context. They interpret and use measurement formulas for the area and perimeter of circles, triangles and parallelograms and simple composite shapes. They calculate the surface area and volume of prisms and cylinders.

Students estimate the accuracy of measurements and give suitable lower and upper bounds for measurement values. They calculate absolute percentage error of estimated values.

Students use appropriate technology to generate random numbers in the conduct of simple simulations.

Students identify empirical probability as long-run relative frequency. They calculate theoretical probabilities by dividing the number of possible successful outcomes by the total number of possible outcomes. They use tree diagrams to investigate the probability of outcomes in simple multiple event trials.

Students organise, tabulate and display discrete and continuous data (grouped and ungrouped) using technology for larger data sets. They represent univariate data in appropriate graphical forms including dot plots, stem and leaf plots, column graphs, bar charts and histograms. They calculate summary statistics for measures of centre (mean, median, mode) and spread (range, and mean absolute difference), and make simple inferences based on this data.

Structure

At Level 5 students identify collections of numbers as subsets of natural numbers, integers, rational numbers and real numbers. They use venn diagrams and tree diagrams to show the relationships of intersection, union, inclusion (subset) and complement between the sets. They list the elements of the set of all subsets (power set) of a given finite set and comprehend the partial-order relationship between these subsets with respect to inclusion (for example, given the set {*a*, *b*, *c*} the corresponding power set is {Ø, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*b*, *c*}, {*a*, *c*}, {*a*, *b*, *c*}}.)

They test the validity of statements formed by the use of the connectives *and, or, not,* and the quantifiers *none*, *some* and *all*, (for example, '*some* natural numbers can be expressed as the sum of two squares'). They apply these to the specification of sets defined in terms of one or two attributes, and to searches in data-bases.

Students apply the commutative, associative, and distributive properties in mental and written computation (for example, 24 × 60 can be calculated as 20 × 60 + 4 × 60 or as 12 × 12 × 10). They use exponent laws for multiplication and division of power terms (for example $2^3 \times 2^5 = 2^8$, $2^0 = 1$, $2^3 \div 2^5 = 2^{-2}$, $(5^2)^3 = 5^6$ and $(3 \times 4)^2 = 3^2 \times 4^2$).

Students generalise from perfect square and difference of two square number patterns (for example, $25^2 = (20 + 5)^2 = 400 + 2 \times (100) + 25 = 625$.
And $35 \times 25 = (30 + 5)(30 - 5) = 900 - 25 = 875$)

Students recognise and apply simple geometric transformations of the plane such as translation, reflection, rotation and dilation and combinations of the above, including their inverses.

They identify the identity element and inverse of rational numbers for the operations of addition and multiplication (for example, $\frac{1}{2} + -\frac{1}{2} = 0$ and $\frac{2}{3} \times \frac{3}{2} = 1$).

Students use inverses to rearrange simple mensuration formulas, and to find equivalent algebraic expressions (for example, if $P = 2L + 2W$, then $W = P/2 - L$. If $A = \pi r^2$ then $r = \sqrt{A}/\pi$ for $r > 0$).

They solve simple equations (for example, $5x + 7 = 23$, $1.4x - 1.6 = 8.3$, and $4x^2 - 3 = 13$) using tables, graphs and inverse operations. They recognise and use inequality symbols. They solve simple inequalities such as $y \leq 2x + 4$ and decide whether inequalities such as $x^2 > 2y$ are satisfied or not for specific values of x and y.

Students identify a function as a one-to-one correspondence or a many-to-one correspondence between two sets. They represent a function by a table of values, a graph, and by a rule. They describe and specify the independent variable of a function and its domain, and the dependent variable and its range. They construct tables of values and graphs for linear functions. They use linear and other functions such as $f(x) = 2x - 4$, $xy = 24$, $y = 2^x$ and $y = x^2 - 3$ to model various situations.

Working mathematically

At Level 5, students formulate conjectures and follow simple mathematical deductions (for example, if the side length of a cube is doubled, then the surface area increases by a factor of four, and the volume increases by a factor of eight).

Students use variables in general mathematical statements. They substitute numbers for variables (for example, in equations, inequalities, identities and formulas).

Students explain geometric propositions (for example, by varying the location of key points and/or lines in a construction).

Students develop simple mathematical models for real situations (for example, using constant rates of change for linear models). They develop generalisations by abstracting the features from situations and expressing these in words and symbols. They predict using interpolation (working with what is already known) and extrapolation (working beyond what is already known). They analyse the reasonableness of points of view, procedures and results, according to given criteria, and identify limitations and/or constraints in context.

Students use technology such as graphic calculators, spreadsheets, dynamic geometry software and computer algebra systems for a range of mathematical purposes including numerical computation, graphing, investigation of patterns and relations for algebraic expressions, and the production of geometric drawings.

Mathematics – Progressing towards Level 6

Progression Point 5.25

At 5.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example

Number

- relationships between real, rational, irrational, integer and natural numbers on a venn diagram
- determination of lowest common multiple through investigation of prime factors
- solution of problems involving ratio and proportion
- representation and recognition of large and small numbers in scientific notation
- calculation and use of percentage change in practical situations; for example, discounts

Space

- use of two-dimensional nets and line-segment models to investigate regular, semi-regular and irregular solids
- application of the angle properties of parallel lines and transversals to other geometrical

Progression Point 5.5

At 5.5, the work of a student progressing towards the standard at Level 6 demonstrates, for example:

Number

- simplification of surds; for example, $\sqrt{(12)} = 2\sqrt{(3)}$
- calculation of the whole given the size of a percentage; for example, if a 20% discount is $7, what was the original value?
- solution of proportion problems using real numbers
- calculation of approximate values for φ, the golden ratio, using measurement, definition, and successive ratios of fibonacci sequence
- computation involving natural numbers, integers, finite decimals and surds without the aid of technology, giving exact answers as applicable
- calculation of the remainder after division by using multiplication (Euclid's method)

Space

- recognition of the features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and the associated angle relationships
- investigation of angle properties of circles and

Progression Point 5.75

At 5.75, the work of a student progressing towards the standard at Level 6 demonstrates, for example:

Number

- division and multiplication of numbers in index form, including application to scientific notation
- knowledge of the equivalence of $(\frac{1}{10})^3$ and 10^{-3}
- application of scientific notation and recalled approximations to squares and square roots to approximate values for expressions
- rationalisation of expressions where division by a square root is involved; for example, $\frac{\sqrt{(5)}}{\sqrt{(3)}} = \frac{\sqrt{(15)}}{3}$

Space

- location of the great circle pathway between two points on a sphere
- application of geometrical transformations to graphs

Progression Point 5.25 *At 5.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example*	Progression Point 5.5 *At 5.5, the work of a student progressing towards the standard at Level 6 demonstrates, for example:*	Progression Point 5.75 *At 5.75, the work of a student progressing towards the standard at Level 6 demonstrates, for example:*
problems • knowledge of sets of conditions for pairs of triangles to be congruent • use of Euler's formula for polyhedra and their nets	tangents • representation of a point on the Earth's surface in terms of its latitude and longitude • identification of paths and circuits in network diagrams that illustrate connections between objects, locations and events	• knowledge of latitude and longitude in geometrical terms
Measurement, chance and data • conversion between units and between derived units • use of pythagoras theorem to calculate the length of a hypotenuse • use of symmetry and scale to calculate side lengths in triangles • representation of compound events involving two categories and the logical connectives *and*, *or* and *not* using lists, grids (lattice diagrams), tree diagrams, venn diagrams and karnaugh maps (two-way tables) and the calculation of associated probabilities • representation of statistical data using technology	**Measurement, chance and data** • calculation and application of ratio, proportion and rate of change such as concentration, density and the rate of filling a container • use of pythagoras theorem to calculate the length of a side other than a hypotenuse • use of trigonometric ratios to calculate unknown sides in a right-angled triangle • display of data as a box plot including calculation of quartiles and inter-quartile range and the identification of outliers • qualitative judgment of positive or negative correlation and strength of relationship and, if appropriate, application of gradient to find a line of good fit by eye	**Measurement, chance and data** • conversion between degrees and radians, and use of radians when calculating arc length and area of sectors • use of pythagoras theorem in three-dimensional applications • calculation of unknown angle in a right-angled triangle using trigonometric ratios • use of surveys as a means of obtaining information about a population, including awareness that sample results will not always provide a reasonable estimate of population parameters • placement of a line of best fit on a scatter plot using technology and, where appropriate, use of a line of best fit to make predictions
Structure	**Structure**	**Structure**

Progression Point 5.25

At 5.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example

- relationships between two sets using a venn diagram, tree diagram and karnaugh map
- factorisation of algebraic expressions by extracting a common factor
- solution of equations by graphical methods
- identification of linear, quadratic and exponential functions by table, rule and graph in the first quadrant
- knowledge of the quantities represented by the constants *m* and *c* in the equation $y = mx + c$

Progression Point 5.5

At 5.5, the work of a student progressing towards the standard at Level 6 demonstrates, for example:

- expression of the relationship between sets using membership, $\in$, complement, $'$, intersection, $\cap$, union, $\cup$, and subset, $\subset$, for up to two sets
- representation of numbers in a geometric sequence (constant multiple, constant percentage change) as an exponential function
- knowledge of the relationship between geometrical and algebraic forms for transformations
- expansion of products of algebraic factors, for example, $(2x + 1)(x - 5) = 2x^2 - 9x - 5$
- equivalence between algebraic forms; for example, polynomial, factorised and turning point form of quadratics
- use of inverse operations to rearrange formulas to change the subject of a formula

Progression Point 5.75

At 5.75, the work of a student progressing towards the standard at Level 6 demonstrates, for example:

- expression of irrational numbers in both exact and approximate form
- factorisation of simple quadratic expressions and use of the null factor law for solution of equations
- testing of sequences by calculating first difference, second difference or ratio between consecutive terms to determine existence of linear, quadratic and exponential functions
- formulation of pairs of simultaneous equations and their graphical solution
- representation of algebraic models for sets of data using technology

Working mathematically

- development of alternative algebraic models for a set of data and evaluation of their relative merits
- presentation of algebraic arguments using appropriate mathematical symbols and conventions

Working mathematically

- generation of reports from a database by using *and*, *or* and *not* as search tools
- justification or proof of generalisations made from specific cases
- selection and use of technology to explore geometrical and algebraic relationships and

Working mathematically

- use of an 'equations editor' to insert mathematical material in a text document
- simulation of events using technology
- representation and manipulation of symbolic expressions using technology
- recognition of functionality of technology and

Progression Point 5.25	Progression Point 5.5	Progression Point 5.75
At 5.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example	*At 5.5, the work of a student progressing towards the standard at Level 6 demonstrates, for example:*	*At 5.75, the work of a student progressing towards the standard at Level 6 demonstrates, for example:*
• evaluation of the appropriateness of the results of their own calculations	data trends	its limitations, such as image resolution, discontinuities in graphs and systematic error in computation through rounding

Mathematics – Level 6

Number

At Level 6, students comprehend the set of real numbers containing natural, integer, rational and irrational numbers. They represent rational numbers in both fractional and decimal (terminating and infinite recurring) forms (for example, $1\frac{4}{25} = 1.16$, $0.\overline{47} = \frac{47}{99}$). They comprehend that irrational numbers have an infinite non-terminating decimal form. They specify decimal rational approximations for square roots of primes, rational numbers that are not perfect squares, the golden ratio φ, and simple fractions of π correct to a required decimal place accuracy.

Students use the Euclidean division algorithm to find the greatest common divisor (highest common factor) of two natural numbers 9 (for example, the greatest common divisor of 1071 and 1029 is 21 since 1071 = 1029 × 1 + 42, 1029 = 42 × 24 + 21 and 42 = 21 × 2 + 0).

Students carry out arithmetic computations involving natural numbers, integers and finite decimals using mental and/or written algorithms (one- or two-digit divisors in the case of division). They perform computations involving very large or very small numbers in scientific notation (for example, $0.0045 \times 0.000028 = 4.5 \times 10^{-3} \times 2.8 \times 10^{-5} = 1.26 \times 10^{-7}$).

They carry out exact arithmetic computations involving fractions and irrational numbers such as square roots (for example, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{(\frac{3}{2})} = \frac{\sqrt{6}}{2}$) and multiples and fractions of π (for example $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$). They use appropriate estimates to evaluate the reasonableness of the results of calculations involving rational and irrational numbers, and the decimal approximations for them. They carry out computations to a required accuracy in terms of decimal places and/or significant figures.

Space

At Level 6, students represent two- and three-dimensional shapes using lines, curves, polygons and circles. They make representations using perspective, isometric drawings, nets and computer-generated images. They recognise and describe boundaries, surfaces and interiors of common plane and three-dimensional shapes, including cylinders, spheres, cones, prisms and polyhedra. They recognise the features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use associated angle properties.

Students explore the properties of spheres.

Students use the conditions for shapes to be congruent or similar. They apply isometric and similarity transformations of geometric shapes in the plane. They identify points that are invariant under a given transformation (for example, the point (2, 0) is invariant under reflection in the *x*-axis, so the *x* axis intercept of the graph of $y = 2x - 4$ is also invariant under this transformation). They determine the effect of changing the scale of one characteristic of two- and three-dimensional shapes (for example, side length, area, volume and angle measure) on related characteristics.

They use latitude and longitude to locate places on the Earth's surface and measure distances between places using great circles.

Students describe and use the connections between objects/location/events according to defined relationships (networks).

Measurement, chance and data

At Level 6, students estimate and measure length, area, surface area, mass, volume, capacity and angle. They select and use appropriate units, converting between units as required. They calculate constant rates such as the density of substances (that is, mass in relation to volume), concentration of fluids, average speed and pollution levels in the atmosphere. Students decide on acceptable or tolerable levels of error in a given situation. They interpret and use mensuration formulas for calculating the perimeter, surface area and volume of familiar two- and three-dimensional shapes and simple composites of these shapes. Students use pythagoras theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles.

They use degrees and radians as units of measurement for angles and convert between units of measurement as appropriate.

Students estimate probabilities based on data (experiments, surveys, samples, simulations) and assign and justify subjective probabilities in familiar situations. They list event spaces (for combinations of up to three events) by lists, grids, tree diagrams, venn diagrams and karnaugh maps (two-way tables). They calculate probabilities for complementary, mutually exclusive, and compound events (defined using *and*, *or* and *not*). They classify events as dependent or independent.

Students comprehend the difference between a population and a sample. They generate data using surveys, experiments and sampling procedures. They calculate summary statistics for centrality (mode, median and mean), spread (box plot, inter-quartile range, outliers) and association (by-eye estimation of the line of best fit from a scatter plot). They distinguish informally between association and causal relationship in bi-variate data, and make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables.

Structure

At Level 6, students classify and describe the properties of the real number system and the subsets of rational and irrational numbers. They identify subsets of these as discrete or continuous, finite or infinite and provide examples of their elements and apply these to functions and relations and the solution of related equations.

Student express relations between sets using membership, $\in$, complement, $'$, intersection, $\cap$, union, $\cup$, and subset, $\subseteq$, for up to three sets. They represent a universal set as the disjoint union of intersections of up to three sets and their complements, and illustrate this using a tree diagram, venn diagram or karnaugh map.

Students form and test mathematical conjectures; for example, 'What relationship holds between the lengths of the three sides of a triangle?'

They use irrational numbers such as, π, φ and common surds in calculations in both exact and approximate form.

Students apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computation with number, to rearrange formulas, rearrange and simplify algebraic expressions involving real variables. They verify the equivalence or otherwise of algebraic expressions (linear, square, cube, exponent, and reciprocal, (for example, $4x - 8 = 2(2x - 4) = 4(x - 2)$; $(2a - 3)^2 = 4a^2 - 12a + 9$; $(3w)^3 = 27w^3$; $\frac{(x^3y)}{xy^2} = x^2y^{-1}$; $\frac{4}{xy} = \frac{2}{x} \times \frac{2}{y}$).

Students identify and represent linear, quadratic and exponential functions by table, rule and graph (all four quadrants of the cartesian coordinate system) with consideration of independent and dependent variables, domain and range. They distinguish between these types of functions by testing for constant first difference, constant second difference or constant ratio between consecutive terms (for example, to distinguish between the functions described by the sets of ordered pairs {(1, 2), (2, 4), (3, 6), (4, 8) …}; {(1, 2), (2, 4), (3, 8), (4, 14) …}; and {(1, 2), (2, 4), (3, 8), (4, 16) …}). They use and interpret the functions in modelling a range of contexts.

They recognise and explain the roles of the relevant constants in the relationships $f(x) = ax + c$, with reference to gradient and y axis intercept, $f(x) = a(x + b)^2 + c$ and $f(x) = ca^x$.

They solve equations of the form $f(x) = k$, where k is a real constant (for example, $x(x + 5) = 100$) and simultaneous linear equations in two variables (for example, $\{2x - 3y = -4$ and $5x + 6y = 27\}$ using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods.

Working mathematically

At Level 6, students formulate and test conjectures, generalisations and arguments in natural language and symbolic form (for example, 'if m^2 is even then m is even, and if m^2 is odd then m is odd'). They follow formal mathematical arguments for the truth of propositions (for example, 'the sum of three consecutive natural numbers is divisible by 3').

Students choose, use and develop mathematical models and procedures to investigate and solve problems set in a wide range of practical, theoretical and historical contexts (for example, exact and approximate measurement formulas for the volumes of various three dimensional objects such as truncated pyramids). They generalise from one situation to another, and investigate it further by changing the initial constraints or other boundary conditions. They judge the reasonableness of their results based on the context under consideration.

They select and use technology in various combinations to assist in mathematical inquiry, to manipulate and represent data, to analyse functions and carry out symbolic manipulation. They use geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

Mathematics – Progressing beyond Level 6

Progression Point 6.25

At 6.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:

Number

- representation of various rational and irrational real numbers by their infinite decimal expansion, as a limiting value of a sequence of rational numbers, or by location on a geometric number-line model; for example, the compass and straight edge constructible numbers
- knowledge of the relation ≤ as a total (linear) order on the set of real numbers (the continuum) and use of $|x| = \sqrt{(x^2)}$ to specify the magnitude of a real number
- efficient and reliable use of written algorithms for all four arithmetic operations with integers, rational numbers (decimal and fractional) and exact form irrational numbers

Space

- proof of properties of shapes in plane (Euclidean) geometry, for example, circle and tangent properties
- practical applications of geometry on a sphere, such as methods for determining latitude and longitude, and navigation on the Earth's surface

Progression Point 6.5

At 6.5, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:

Number

- recognition that rational numbers are a dense subset of real numbers and that real numbers are complete
- use of closed and open intervals to specify an interval (or union of intervals) over which a given inequality is true
- use of real number properties to reformulate computations involving several operations so that they can be carried out more efficiently using a suitable combination of mental, written or technology-assisted methods

Space

- locus definitions of paths, and their corresponding forms, in various coordinate systems; for example, cartesian, polar, parametric
- application of properties of non-Euclidean geometry; for example, projective geometry and the problems of representation in maps,

Progression Point 6.75

At 6.75, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:

Number

- use of concepts and operations of other number systems, such as modulo (clock) arithmetic, matrices or Gaussian integers
- knowledge of the properties that formally define the set of real numbers with the operations of + and × and the relation < as an ordered number field, and their application to proving simple number properties or order results about real numbers; for example, $-a \times -b = ab$; if $0 < a < b$ then $a^2 < b^2$
- efficient and reliable use of number facts and techniques to carry out mental computation where a written algorithm or technology might otherwise be used; for example, $49^2 = 2500 - 100 + 1$

Space

- identification and application of self-similarity in spatial constructions; for example, fractal patterns in nature and art
- solution of mathematical puzzles involving topological properties of connectedness; for example, the geometry of knots, puzzles where apparently 'linked' three-dimensional

Progression Point 6.25 *At 6.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:*	Progression Point 6.5 *At 6.5, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:*	Progression Point 6.75 *At 6.75, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:*
	art and engineering, affine transformations and digital images	shapes can be separated
• use of networks and properties to solve practical problems involving paths and circuits, length and coverage	• understanding of dual relationships between faces and vertices involving polyhedra	• invariance of some geometric properties under certain transformations
Measurement, chance and data • derivation of measurement formulas for composite shapes and objects; for example, the surface area of a closed cone • use of measurement formulas, including cases where more than one type of unit, and/or conversion of units, is involved • use of cumulative frequency distribution to represent and interpret uni-variate data	**Measurement, chance and data** • recognition of the effect of rounding and measurement error in numerical computations, for example, where a formula is used • use of tree diagrams to determine the probability of outcomes for sampling with or without replacement • identification of random variation and possible hidden variables in analysing association and possible causal relationship in bi-variate data	**Measurement, chance and data** • informal use of limiting values to approximate instantaneous rate of change, arc length, area and surface area and volume measures of regular and irregular curves, shapes and objects • use of conditional probability to distinguish between dependent and independent events • awareness of sampling errors and possible sources of bias
Structure • description of linear, reciprocal, quadratic, exponential and logarithmic functions by recursion or other functional relation, for example, if $f(x) = \log_a(x)$, then $f(xy) = f(x) + f(y)$ • knowledge of analytic solution of general equations of the form $f(x) = k$ and corresponding numerical solution of particular equations of this form by	**Structure** • exploration of periodic functions where $f(x + k) = f(x)$ for some non-zero real k, for example, $\sin(x + 2\pi) = \sin(x)$ • solution of simultaneous equations of linear-linear, linear-quadratic, linear-hyperbola types by analytic, numerical and graphical methods • use of Boolean connectives to conduct searches in relational databases, for example	**Structure** • knowledge of the properties that characterise and distinguish mathematical systems; for example, number systems (such as integers and rational numbers), matrices, sets and logic, coordinates and vectors • knowledge of how mathematical properties in a given system relate to computation and proof in that system; for example, condition for the existence and nature of solutions to an

Progression Point 6.25

At 6.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:

algorithm, using technology as applicable

- selection and use of venn diagrams, karnaugh maps or tree diagrams to solve logic and/or combinatoric problems

Progression Point 6.5

At 6.5, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:

the Internet

Progression Point 6.75

At 6.75, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:

equation, or a system of simultaneous equations

- equivalence of Boolean relations using venn diagrams, karnaugh maps or truth tables; for example, the de Morgan laws, $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Working mathematically

- identification of assumptions used to develop a model for a practical situation, and consideration of related constraints and limitations
- efficient and effective use of mathematical concepts, skills and processes, including the effective use of technology, to solve a broad range of problems in familiar situations
- consistent, accurate and appropriate use of mathematical notation, symbols, diagrams and graphs in solving problems and in the presentation of mathematical arguments

Working mathematically

- consideration of alternative models for given situations and the application of criteria to select an appropriate model from these alternatives; for example, the best model to use for illustrating the time taken for an aspirin tablet to dissolve with respect to water temperature
- selection and use of mathematical concepts, skills and processes, including the appropriate selection and effective use of technology, to solve challenging problems, and consideration of alternative approaches in familiar and some unfamiliar situations
- use of informal mathematical reasoning to establish general results; for example, the formula for the area of a circle by triangle

Working mathematically

- development or generalisation of an existing model to enhance its applicability in other contexts; for example, bounce of a ball over a larger domain where a maximal bounce height occurs
- development of new approaches or methods, including those that may require the use of technology for their solution, to formulate and solve challenging problems in unfamiliar and new situations
- use of deductive proof techniques; for example, Euler's proof that there is an infinite number of primes, including indirect proof; for example, the irrationality of $\log_2(5)$, and mathematical induction; for example, the Tower of Hanoi relationship, to establish

Progression Point 6.25	Progression Point 6.5	Progression Point 6.75
At 6.25, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:	*At 6.5, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:*	*At 6.75, the work of a student progressing beyond the standard at Level 6 demonstrates, for example:*
	approximation of sectors	general results