Mental Math
Mental Computation
Grade 5

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Sharon Boudreau—Cape Breton-Victoria Regional School Board
Anne Boyd—Strait Regional School Board
Estella Clayton—Halifax Regional School Board (Retired)
Jane Chisholm—Tri-County Regional School Board
Paul Dennis—Chignecto-Central Regional School Board
Robin Harris—Halifax Regional School Board
Keith Jordan—Strait Regional School Board
Donna Karsten—Nova Scotia Department of Education
Ken MacInnis—Halifax Regional School Board (Retired)
Ron MacLean—Cape Breton-Victoria Regional School Board
Sharon McCready—Nova Scotia Department of Education
David McKillop—Chignecto-Central Regional School Board
Mary Osborne—Halifax Regional School Board (Retired)
Sherene Sharpe—South Shore Regional School Board
Martha Stewart—Annapolis Valley Regional School Board
Susan Wilkie—Halifax Regional School Board

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Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

While there are many aspects to mental math, this booklet, *Mental Computation*, deals with fact learning, mental calculations, and computational estimation — mental math found in General Curriculum Outcome (GCO) B. Therefore, teachers must also remember to incorporate mental math strategies from the six other GCOs into their yearly plans for Mental Math, for example, measurement estimation, quantity estimation, patterns and spatial sense. For more information on these and other strategies see *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

A. Addition — Fact Learning

Facts and the Fact Learning Strategies

At the beginning of grade 5, it is important to ensure that students review the addition facts to 18 and the fact learning strategies addressed in previous grades as listed below. Students will use these facts and strategies when doing mental math addition with numbers in the 10s, 100s, 1000s and 10 000s in grade 5. Further information about these strategies can be found in the grade 3 mental math booklet.

The strategies are:

1) Doubles Facts
2) Near- Doubles (1-Aparts) Facts
3) Doubles Plus 2 (2-Aparts Facts)
4) Plus 0-“No Change”
5) Plus 2s or “Next Even/Odd “Number

Examples

1) 400 + 400 =
2) 300 + 400 = (300 + 300) + 100 = 700
3) 300 + 500 = (300 + 300) + 200 =
4) 700 + 0 =
5) 700 + 200 = 900 or 400 + 200 = 600

Examples of Some Practice Items

Here are some practice items:

40 + 40 =	90 + 90 =	50 + 50 =
300 + 300 =	7000 + 7000 =	2000 + 2000 =
70 + 80 =	50 + 60 =	7000 + 8000 =
3000 + 2000 =	40 + 60 =	50 + 30 =
700 + 500 =	100 + 300 =	7000 + 9000 =
8000 + 6000 =	3000 + 5000 =	4000 + 2000 =
55 + 0 =	0 + 47 =	376 + 0 =
5678 + 0 =	0 + 9098 =	811 + 0 =
70 + 20 =	30 + 20 =	60 + 20 =
800 + 200 =	100 + 200 =	4000 + 2000 =

Some practice items for numbers in the 10 000 are:

20 000 + 30 000 =	60 000 + 30 000 =
10 000 + 80 000 =	40 000 + 40 000 =
70 000 + 70 000 =	50 000 + 60 000 =
80 000 + 90 000 =	50 000 + 70 000 =
90 000 + 70 000 =	30 000 + 50 000 =
60 000 + 0 =	0 + 40 000 =
40 000 + 20 000 =	60 000 + 20 000 =
70 000 + 20 000 =	20 000 + 50 000 =

B. Addition — Mental Calculations

Quick Addition — No Regrouping

This pencil-and-paper strategy is used when there are more than two combinations in the calculations, but no regrouping is needed and the calculations are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end. It is important to present these addition questions both horizontally and vertically.

Examples

1) For 56 + 23, simply record , starting at the front end, 79.
2) For 543 + 256, simply record, starting at the front end, 799.
3) For 2 341 + 3 4 00, simply record, starting at the front end, 5 741.
4) For 0.34 + 0.25, simply record, starting at the front end, 0.59 and remember, no regrouping.

Examples of Some Practice Items

1) Here are some practice items for numbers in the 10s are.

71 + 12 =	63 + 33 =
44 + 53 =	37 + 51 =
15 + 6 2 =	66 + 23 =
43 + 54 =	234 + 52 =

2) Some practice items for numbers in the 100s are.

291 + 703 =	144 + 333 =
507 + 201 =	623 + 234 =
770 + 129 =	534 + 435 =

3) Some practice items for numbers in the 1000s.

5200 + 3700 =	6200 + 1700 =
4423 + 1200 =	6334 + 2200 =
4067 + 4900 =	4300 + 2078 =
6621 + 2100 =	1452 + 8200 =

4) Some practice items for numbers in the 10ths and 100ths are.

0.3 + 0.6 =	0.5 + 0.1 =	0.6 + 0.2 =
0.7 + 0.1 =	0.2 + 0.5 =	0.23 + 0.43 =
0.71 + 0.16 =	0.45 + 0.33 =	0.93 + 0.04 =
0.07 + 0.52 =	2.34 + 0.54 =	7.36 + 0.51 =
2.45 + 3.33 =	7.15 + 2.84 =	6.04 + 2.82 =

Front End Addition

This strategy involves adding the highest place values and then adding the sums of the next place value(s).

Example

For 37 + 26, think: 30 and 20 is 50, 7 and 6 is 13 and 50 plus 13 is 63.

For 450 + 380 think: 400 + 300 is 700, and 50 and 80 is 130 and 700 plus 130 is 830.

Examples of Some Practice Items

Some practice items are:

45 + 38 =	34 + 18 =	53 + 29 =
15 + 66 =	74 + 19 =	190 + 430 =
340 + 220 =	470 + 360 =	607 + 304 =
3500 + 2300 =	5400 + 3 400 =	6800 + 2100 =
2900 + 6 000 =	3700 + 3200 =	7500 + 2400 =
8800 + 1100 =	2700 + 7200 =	6300 + 4400 =

Some practice items for numbers in the 10ths and 100ths are.

4.9 + 3.2 =	5.4 + 3.7 =	1.4 + 2.5 =
3.6 + 2.9 =	6.6 + 2.5 =	0.36 + 0.43 =
0.62 + 0.23 =	0.75 + 0.05 =	0.45 + 0.44 =

Finding Compatibles

This strategy for addition involves looking for pairs of numbers that add to powers of ten (10, 100, and 1000) to make the addition easier. Some examples of compatible numbers are: 1 and 9; 40 and 60; 300 and 700; and 75 and 25, a readily known compatible. Compatible numbers are also referred to as “friendly” numbers or “nice” numbers in some professional resources.

Example:

For 3 + 8 + 7 + 6 + 2, think- 3 + 7 is 10, 8 + 2 is 10, so 10 + 10 + 6 is 26.

For 25 + 47 + 75, think: 25 and 75 is 100, so 100 and 47 is 147.

For 400 + 720 + 600, think: 400 and 600 is 1000, so the sum is 1720.

For 3000 + 7000 + 2400, think: 3000 and 7000 is 10 000, so 10 000 and 2400 is 12 400.

Examples of Some Practice Items

Some practice items are:

11 + 59 =	33 + 27 =	60 + 30 + 40 =
75 + 95 + 25 =	80 + 20 + 79 =	40 + 72 + 60 =
90 + 86 + 10 =	125 + 25=	475 + 25 =
625 + 75 =	290 + 510 =	300 + 437 + 700 =
800 + 740 + 200 =		900 + 100 + 485 =
4400 + 1600 + 3000 =		9000 + 3300 + 1000 =
3250 + 3000 + 1750 =		2200 + 2800 + 600 =
3000 + 300 + 700 + 2000 =		3400 + 5600=

Some practice items for numbers in the 10ths and 100ths are.

0.6 + 0.9 + 0.4 + 0.1 =	0.2 + 0.4 + 0.8 + 0.6 =
0.7 + 0.1 + 0.9 + 0.3 =	0.2 + 0.4 + 0.3 + 0.8 + 0.6 =
0.4 + 0.5 + 0.6 + 0.2 + 0.5 =	0.25 + 0.50 + 0.75 =
0.80 + 0.26 =	0.45 + 0.63 =

Break Up and Bridge

This strategy for addition involves starting with the first number and adding the values in the place values, starting with the largest of the second number.

Example:

1) For 45 plus 36, think: 45 and 30(from the 36) is 75 and 75 plus 6(the rest of the 36) is 81.

   For 537 plus 208, think: 537 and 200 is 737 and 737 plus 8 is 745.

2) For example: For 5300 plus 2 400, think: 5300 and 2000 (from the 2400) is 7300 and 7 300 plus 400(from the rest of 2 400) is 7700.

3) For example: For 3.6 plus 5.3, think: 3.6 and 5(from the 5.3) is 8.6 and 8.6 plus 0.3(the rest of 5.3) is 8.9.

Examples of Some Practice Items

1) Some practice items are.

37 + 42 =	72 + 21 =	88 + 16 =
74 + 42 =	325 + 220 =	301 + 435 =
747 + 150 =	142 + 202 =	370 + 327 =

2) Some practice items for numbers in the 1000s are:

7700 + 1200 =	4100 + 3600 =	5700 + 2200 =
7300 + 1400 =	2800 + 6100 =	3300 + 3400 =
5090 + 2600 =	17 400 + 1300 =	

3) Some practice items for numbers in the 10ths and 100ths are:

4.2 + 3.5 =	6.3 + 1.6 =	4.2 + 3.7 =
6.1 + 2.8 =	0.32 + 0.56 =	2.08 + 3.2 =
4.15 + 3.22 =	5.43 + 2.26 =	6.03 + 2.45 =
15.46 + 1.23 =	43.30 + 8.49 =	70.32 + 9.12 =

Compensation

This strategy for addition involves changing one number to a ten or hundred; carrying out the addition and then adjusting the answer to compensate for the original change.

Examples

1) For 52 plus 39, think, 52 plus 40 is 92, but I added 1 too many to take me to the next 10 (compensate - 39 to 40), so I subtract one from my answer, 92, to get 91.

   For example: For 345 + 198, think: 345 + 200 is 545, but I added 2 too many; so, I subtract 2 from 545 to compensate, to get 543.

2) For example: For 4500 plus 1900, think: 4500 + 2000 is 6500 but I added 100 too many; so, I subtract 100 from 6500 to get 6400.

3) For example: For 0.54 plus 0.29, think: 0.54 + 0.3 is 0.84 but I added 0.01 too many; so, I subtract 0.01 from 0.84 to compensate, to get 0.83.

Examples of Some Practice Items

1) Some practice items

56 + 8 =	72 + 9 =	44 + 27 =
14 + 58 =	21 + 48 =	255 + 49 =
371 + 18 =	125 + 49 =	504 + 199 =
354 + 597 =	826 + 99 =	676 + 197 =
304 + 399 =	526 + 799 =	

2) Some practice for numbers in the 1000s are:

1300 + 800 =	5400 + 2900 =	6421 + 1900 =
3450 + 4800 =	2330 + 5900 =	15 200 + 2900 =
4621 + 3800 =	2111 + 4900 =	2050 + 6800 =

3) Some practice items for numbers in the 10ths and 100ths are:

0.71 + 0.09 =	0.56 + 0.08 =	0.32 + 0.19 =
0.44 + 0.29 =	0.17 + 0.59 =	2.31 + 0.99 =
4.52 + 0.98 =	1.17 + 0.39 =	25.34 + 0.58 =

Make 10, 100 or 1000.

These facts all have one addend of 8 or 9. The strategy for these facts is to build onto the 8 or 9 up to 10 and then add on the rest.

Examples

For 9 + 6, think: 9 +1(from the 6) is 10, and 10 + 5 (the other part of the 6) is 15 or 9 + 6 = (9 + 1) + 5 = 10 + 5 = 15.

The "make 10" strategy can be extended to facts involving 7.

For 7 + 4, think: 7 and 3 (from the 4) is 10, and 10 + 1 (the other part of the 4) is 11or 7 + 4 = (7 + 3) + 1 = 10 + 1 = 11.

Examples of Some Practice Items

Some practice items are:

58 + 6 =	5 + 49 =	29 + 3 =
38 + 5 =	680 + 78 =	490 + 18 =
170 + 40 =	570 + 41 =	450 + 62 =
630 + 73 =	560 + 89 =	870 + 57 =
780 + 67 =	2800 + 460 =	5900 + 660 =
1700 + 870 =	8900 + 230 =	3500 + 590 =
2200 + 910 =	3600 + 522 =	4700 + 470 =

C. Subtraction — Fact Learning

Review Subtraction Facts to 18 and the Fact Learning Strategies

At the beginning of grade 4, it is important to ensure that students review the subtraction facts to 18 and the fact learning strategies addressed in previous grades as outlined below. It is necessary that subtraction strategies are always related to students' understanding of the basic addition facts and strategies. Students will use these facts and strategies when doing mental math subtraction in the 10s, 100s, 1000s, ten-thousands tenths and hundredths.

D. Subtraction — Mental Calculations

Using Subtraction Facts for 10s, 100s, 1000s, and 10 000s

This strategy applies to calculations involving the subtraction of two tens, hundreds or thousands with only one non-zero digit in each number. The strategy involves subtracting the single non-zero digits as if they were the single-digit subtraction facts. This strategy should be modeled with base ten materials.

Examples

For 80 – 30, think- 8 tens subtract 3 tens is 5 tens, or 50.

For 500 – 200 , think- 5 hundreds subtract 2 hundreds is 3 hundreds, or 300.

For 6000 subtract 2 000, think: 6 thousands subtract 2 thousands is 4 thousands, or 4 000.

Examples of Some Practice Items

Some practice items:

90 - 10 =	60 – 30 =	70 – 60 =
40 – 10 =	80 – 30 =	700 – 300 =
400 – 100 =	800 – 700 =	600 – 400 =
300 – 200 =	1400 – 100 =	1800 – 900 =

Here are some practice items for numbers in the ten thousands are:

70 000 – 30 000 =	80 000 – 50 000 =	90 000 – 80 000 =
60 000 – 30 000 =	60 000 – 40 000 =	20 000 – 7000 =
30 000 – 9000 =	80 000 – 4 000 =	100000 - 20 000 =

Quick Subtraction

This pencil-and-paper strategy is used when there are more than two combinations in the calculations, but no regrouping is needed and the calculations are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end. It is important to present these subtraction questions both horizontally and vertically.

Examples

1) For 86 - 23, simply record, starting at the front end, 63.

   For 568 - 135, simply record, starting at the front end, 433.

2) For 3 700 subtract 2 400, simply record , starting at the front end, 1 300.

3) For 0.65 subtract 0.23, simply record, starting at the front end, 0.42.

Examples of Some Practice Items

1) Some practice items:

38 – 25 =	27 – 15 =	97 – 35 =
82 – 11 =	67 – 43 =	745 – 23 =
947 – 35 =	357 - 135 =	845 – 542 =
704 – 502 =	809 – 408 =	2654 – 341 =

2) Some practice items for numbers in the 1000s and 10 000s are.

9800 – 7200 =	5600 – 4100 =	4850 – 2220 =
8520 – 7200 =	56 000 – 23 000 =	78 000 – 47 000 =
390 000 – 200 000 =	460 000 – 130 000 =	

3) Some practice items for numbers in the 10ths and 100ths are.

0.38 – 0.21 =	0.66 – 0.42 =	0.78 – 0.50 =
0.96 – 0.85 =	3.86 – 0.45 =	17.36 – 0.24 =

Back Through 10/100

This strategy involves subtracting a part of the subtrahend to get to the nearest one, ten, hundred or thousand and then subtracting the rest of the subtrahend. This strategy is most effective when the two numbers involved are quite far apart.

Example

1) For 15 – 8, think: 15 subtract 5 (one part of the 8) is ten and ten subtract 3 (the other part of the 8) is 7.

   For 530 - 70, think: 530 subtract 30 (one part of the 70) is 500 and 500 subtract 40(the other part of the 70) is 460.

2) For example: For 8 600- 700, think: 8 600 subtract 600(one part of the 700) is 8 000 and 8 000 subtract 100(the rest of the 700) is 7 900.

Examples of Some Practice Items

1) Some practice items:

74 – 7 =	97 – 8 =	53 – 5 =
420 – 60 =	340 -70 =	630 – 60 =
540 – 70 =	760 – 70 =	320 – 50 =

2) Some practice items for numbers in the 1000s are:

9200 – 500 =	4700 - 800 =	6100 – 300 =
7500 – 700 =	8 00 – 600 =	4200 – 800 =
9 500 – 600 =	3 400 – 700 =	2 300 – 600 =

Counting on to Subtract

This strategy involves counting the difference between the two numbers by starting with the smaller; keeping track of the *distance* to the nearest one, ten, hundred or thousand; and adding to this amount the rest of the *distance* to the greater number. *This strategy is most effective when the two numbers involved are quite close together*.

Example

1) For 613 – 594, think- It is 6 from 594 to 600 and 13 from 600 to 613; therefore, the difference is 6 plus 13, or 19.

   For example: For 84 – 77, think- It is 3 from 77 to 80 and 4 from 80 to 84; therefore, the difference is 3 plus 4, or 7.

2) For 2 310 – 1 800, think: It is 200 from 1 800 to 2 000 and 310 from 2 000 to 2 310; therefore, the difference is 200 plus 310, or 510.

3) For 12.4 – 11.8, think: It is 2 tenths from 11.8 to 12 and 4 tenths from 12 to 12.4; therefore, the difference is 2 tenths plus 4 tenths, or 0.6.

4) For 6.12 – 5.99, think: It is 1hundredth from 5.99 to 6.00 and 12 hundredths from 6.00 to 6.12 ; therefore, the difference is 1 hundredth plus 12 hundredths, or 0.13.

Examples of Some Practice Items

1) Add your own items:

11 -7 =	17 – 8 =	13 – 6 =
12 – 8 =	15 – 6 =	16 – 7 =
95 -86 =	67 -59 =	46 – 38 =
88 – 79 =	62 – 55 =	42 – 36 =
715 – 698 =	612 – 596 =	817 – 798 =
411 – 398 =	916 – 897 =	513 – 498 =
727 – 698 =	846 – 799 =	631 – 597 =

2) Some practice items for numbers in the 1000s are:

5170 – 4800 =	3210 – 2900 =	8220 – 7800 =
9130 – 8950 =	2400 – 1800 =	4195 – 3900 =
7050 – 6750 =	1280 – 900 =	8330 – 7700 =

3) Some practice items for numbers in the 10ths are:

15.3 – 14.9 =	27.2 – 26.8 =	19.1 – 18.8 =
45.6 – 44.9 =	23.5 – 22.8 =	50.1 – 49.8 =
34.4 – 33.9 =	52.8 – 51.8 =	70.3 – 69.7 =

4) Some practice items for numbers in the 100ths are:

3.25 – 2.99 =	5.12 – 4.99 =	4.05 – 3.98 =
3.24 – 2.99 =	8.04 – 7.98 =	6.53 – 5.97 =
24.12 – 23.99 =	36.11 – 35.98 =	100.72 – 99.98 =

Compensation

This strategy for subtraction involves changing one number to a ten, hundred or thousand; carrying out the subtraction and then adjusting the answer to compensate for the original change.

Examples

1) For 17 – 9, think: 17 – 10 = 7; but I subtracted one too many; so I add 1 to the answer to compensate to get 8.

   For 36 – 8, think: 36 – 10 = 26; but I subtracted 2 too many; so I add 2 to the answer to get 28 to compensate.

   For 85 – 9, think: 85 – 10 + 1 = 76.

2) For 145 – 99, think: 145 – 100 is 45; but I subtracted 1 too many; so, I add 1 to 45 to get 46.

   For 756 – 198, think: 756 – 200 + 2 = 558.

3) For 5 760 - 997, think: 5 760- 1000 is 4 760; but I subtracted 3 too many; so, I add 3 to 4 760 to compensate to get 4 763.

   For 3 660 - 996, think: 3 660 -1000 + 4 = 2 664.

Examples of Some Practice Items

1) Some practice items for numbers in the 10s are:

15 – 8 =	17 – 9 =	23 – 8 =
74 - 9 =	84 – 7 =	92 – 8 =
65 – 9 =	87 – 9 =	73 – 7 =

2) Some practice items for numbers in the 100s are:

673 – 99 =	854 – 399 =	953 - 499 =
775 – 198 =	534 – 398 =	647 – 198 =
641 – 197 =	802 – 397 =	444 – 97 =
765 – 99 =	721 – 497 =	513 – 298 =

3) Some practice items for numbers in the 1000s are:

8 620 – 998 =	4 100 – 994 =	5 700 – 397 =
9 850 – 498 =	3 720 – 996 =	2 900 – 595 =
4 222 – 998 =	7 310 – 194 =	75 316 – 9 900

Balancing For a Constant Difference

This strategy for subtraction *involves adding **or** subtracting* the same amount from both the subtrahend and the minuend to get a ten, hundred or thousand in order to make the subtraction easier. This works because the two numbers are still the same distance apart. *Many students may need to record at least the first changed number to keep track.*

Examples

1) For 87 -19, think: Add 1 to both numbers to get 88 – 20, so 68 is the answer.

   For 76 – 32, think: Subtract 2 from both numbers to get 74 -30, so the answer is 44.

2) For 345 – 198, think: Add 2 to both numbers to get 347 – 200; so the answer is 147.

   For 567 – 203, think: Subtract 3 from both numbers to get 564 -200; so the answer is 364.

3) For 8.5 – 1.8, think: Add 2 tenths to both numbers to get 8.5 – 2.0, so 6.5 is the answer.

   For 5.4 - 2.1, think: Subtract 1 tenth from both numbers to get 5.4 – 2.0, so the answer is 3.3.

4) For 6.45 – 1.98, think: Add 2 hundredths to both numbers to get 6.47 – 2.00, so 4.47 is the answer.

   For 5.67 – 2.03, think: Subtract 3 hundredths from both numbers to get 5.64 – 2.00, so the answer is 3.64.

Examples of Some Practice Items

1) Some practice items for numbers in the 10s are:

   In these items you add to balance.

85 – 18 =	42 - 17 =	36 – 19 =
78 – 19 =	67 -18 =	75 – 38 =
88 – 48 =	94 – 17 =	45 – 28 =

   In these items you subtract to balance:

83 – 21 =	75 – 12 =	68 – 33 =
95 – 42 =	72 - 11 =	67 – 51 =
67 – 32 =	88 – 43 =	177 – 52 =

2) Some practice items for numbers in the 100s are:

   In these items you add to balance.

649 - 299 =	563 – 397 =	823 – 298 =
912 – 797 =	737 - 398 =	456 – 198=
631 – 499 =	811 – 597 =	628 – 298 =
971 – 696 =		

In these items you subtract to balance:

486 – 201 =	829 – 503 =	659 – 204 =
382 – 202 =	293 - 102 =	736 – 402 =
564 – 303 =	577 – 102 =	948 – 301 =
437 – 103 =	819 – 504 =	

3) Some practice items for numbers in the 10ths where you add *or* subtract to balance are:

6.4 – 3.9 =	7.6 – 4.2 =	8.7 – 5.8 =
4.3 – 1.2 =	9.1 – 6.7 =	5.0 – 3.8 =
6.3 – 2.2 =	4.7 – 1.9 =	12.5 – 4.3 =
15.3 – 5.7 =		

4) Here are some practice items for numbers in the 100ths where you add *or* subtract to balance:

In these items you add to balance.

8.36 – 2.99 =	7.45 – 1.98 =	5.40 – 3.97 =
6.92 – 4.98 =	27.84 – 6.99 =	

In these items you subtract to balance:

7.58 – 3.02 =	8.49 – 4.03 =	6.25 – 2.01 =
8.53 – 6.02 =	38.66 – 5.03 =	

Break Up and Bridge

This strategy for subtraction involves starting with the first number and subtracting the values in the place values, starting with the highest, of the second number.

Examples

1) For 92 – 26, think: 92 subtract 20(from the 26) is 72 and 72 subtract 6 is 66.
2) For 745 – 203, think: 745 subtract 200 (from the 203) is 545 and 545 minus 3 is 542.
3) For 8 369 - 204, think: 8 369 subtract 200 (from the 204) is 8 169 and 8 16 minus 4(the rest of the 204) is 8 165 .

Examples of Some Practice Items

1) Some practice examples for numbers in the 10s are:
Add your own examples:

79 – 37 =	93 – 72 =	98 – 22 =
79 – 41=	74 – 15 =	77 – 15 =
95 – 27 =	85 – 46 =	67 - 42 =
56 – 31 =	86 – 54 =	156 – 47 =

2) Some practice examples for numbers in the 100s are:
Add your own examples:

736 – 301 =	848 – 207 =	927 – 605 =

632 – 208 =	741 – 306 =	758 – 205 =
928 – 210 =	847 – 412 =	746 – 304 =
647 – 102 =	3586 – 302 =	9564 – 303 =

3) Some practice examples for numbers in the 1000s are:

9275 -8100 =	6350 – 4200 =	8461 - 4050 =
10 270 – 8100 =	15 100 – 3003 =	4129 – 2005 =
3477 – 1060 =	38 500 -10 400 =	137 400 – 6100 =

E. Multiplication & Division — Fact Learning

Multiplication and Division Fact Learning Strategies

In grade 4 students learned the multiplication facts to a 3 second response after careful instruction of the strategies. Review of the multiplication facts and the related fact learning strategies should be done at the beginning of grade 5. See grade 4 booklet for multiplication facts. Students will then apply these strategies to the related division facts and work toward a 3 second response.

1. The Doubles Facts

Multiplication: This strategy involves using knowledge of the addition doubles to learn the related multiplication facts. The multiplication facts that have 2 as a factor are known to students as "addition doubles". It is important to make sure students are aware that 2 × 7 is a double as is 7 × 2 (think: 2 × 7 is 7 + 7 the double). Flash cards displaying the 2s facts and the times 2 function on the calculator are effective tools to used when learning the multiplication doubles.

Examples of Some Practice Items

Here are the items for multiplication doubles facts:

2 × 2 =	3 × 2 =	4 × 2 =
5 × 2 =	6 × 2 =	1 × 2 =
7 × 2 =	2 × 1 =	8 × 2

Division: After the students have mastered their doubles multiplication facts, it is appropriate to have them learn the corresponding division facts. The strategy for learning the division facts is "think-multiplication".

Examples

For 16 ÷ 2, think: 2 times what equals 16; it is 8, so 16 ÷ 2 = 8.

Examples of Some Practice Items

4 ÷ 2=	10 ÷ 2 =	14 ÷ 2 =
8 ÷ 2 =	16 ÷2 =	6 ÷ 2 =
12 ÷ 2 =	18 ÷ 2 =	2 ÷ 2 =

2. The Nifty Nines

Multiplication: There are two strategies for learning the nines facts.

1) Using the example of 9 × 6, the tens digit in the product(9 × 6 = 54) is one less than the variable factor (6- the factor other than 9) and the sum of the digits in the product (5 + 4) equals 9. Therefore, for 6 × 9, 1 less than 6 is 5, and 5 + ___ = 9 and it is 4, so 6 × 9 = 54. This strategy can be used to learn the nines facts to 9 × 9.

2) This strategy involves thinking of 10 instead of 9 and subtracting the number you are multiplying nine by.

Example

9 × 3, think: 10 × 3= 30, subtract 3 (variable factor or the other factor), so 30 – 3 = 27.

Examples of Some Practice Items

Here are the items for multiplication of nifty nines:

9 × 9=	9 × 5 =	9 × 7 =
9 × 4 =	9 × 6 =	9 × 8 =
9 × 0 =	9 × 2 =	9 × 3 =
9 × 1 =	5 × 9 =	0 × 9 =
8 × 9 =	1 × 9 =	7 × 9 =
2 × 9 =	4 × 9 =	6 × 9 =

Division: After the students have mastered the nifty nines multiplication, it is appropriate to learn the corresponding division facts.

The strategy for learning the division facts is by think-multiplication.

Example

For 36 ÷ 9 , think: 9 times what equals 36; it is 4, so 36 ÷ 9 = 4

Examples of Some Practice Items

18 ÷ 9 =	9 ÷ 9 =	27 ÷ 9 =
63 ÷ 9 =	81 ÷ 9 =	36 ÷ 9 =
72 ÷ 9 =	45 ÷ 9 =	54 ÷ 9 =

3. The Five Facts

Multiplication: There are several ways to teach fives facts.

1) Have students skip count 5s to at least 45 and relate it to rows of 5, for example:

   5 (one row of 5 is 5)

   10 two rows of 5 is 10)

   15 …

2) Products of an even number and 5 always end in zero, for example: 2 × 5 = 10; 6 × 5 = 30 etc.
3) Products of an odd number and 5 always end in five, for example: 3 × 5 = 15; 9 x5 = 45 etc.
4) Use referents when teaching the fives facts; for example use the 5s on an analog clock. Teach students to identify the related facts.

Examples of Some Practice Items

Here are the items for multiplication of the "fives facts".

5 × 1 =	5 × 3 =	5 × 9 =
5 × 8 =	5 × 2 =	5 × 7 =
5 × 6 =	5 × 4 =	5 × 5 =

Note: use nickels as referents, for example; 5 nickels, how much money?

10 nickels = $0.___ 7 nickels = $0.___

4 nickels = ______ cents 9 nickels = $_____

Division: After the students have mastered their nifty nines multiplication facts, it is appropriate to have them learn the corresponding division facts.

The strategy for learning the division facts is by "think-multiplication".

Example

For 35 ÷ 5, think: 5 times what equals 35; it is 7, so 35 ÷ 5 = 7.

Examples of Some Practice Items

45 ÷ 5 =	5 ÷ 5 =	40 ÷ 5 =
10 ÷ 5 =	20 ÷ 5 =	35 ÷ 5 =
15 ÷ 5 =	25 ÷ 5 =	30 ÷ 5 =

Nickel referents:

25 cents = ____ nickels	45 cents = _____ nickels	1 loonie = ______ nickels
$\frac{1}{2}$ dollar = _____ nickels	$\frac{1}{4}$ dollar = _____ nickels	$\frac{3}{4}$ dollar = ______nickels
35 cents = ______ nickels	_____ nickels = a toonie	$5 = ______ nickels

4. The Ones Facts

Multiplication: To understand the ones facts, knowing what is happening when we multiply by one is most important. For example 6 × 1 means six groups of 1 or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means one group of 6. It is important to avoid teaching arbitrary rules such as "any number multiplied by one is that number". Students will come to this rule on their own given opportunities to develop understanding. When teaching the ones facts, make sure to present questions correctly, both visually and orally; for example, 4 groups of 1 are (4 × 1 = 4) or 1 group of 4 is (1 × 4 = 4). The ones facts are the "no change" facts.

Examples of Some Practice Items

Here are the items for the ones facts:

1 × 5 =	1 × 2 =	1 × 9 =
1 × 1 =	1 × 8 =	1 × 0 =
1 × 4 =	1 × 3 =	1 × 6 =
1 × 7 =	9 × 1 =	5 × 1 =
2 × 1 =	8 × 1 =	0 × 1 =

Division: After the students have mastered the ones multiplication facts, it is appropriate to have them learn the corresponding division facts.

The strategy for learning the division facts is by "think-multiplication".

Example

For $6 \div 1$, think: 1 times what equals 6; it is 6, so $6 \div 1 = 6$

Examples of Some Practice Items

$5 \div 1 =$	$2 \div 1 =$	$0 \div 1 =$
$8 \div 1 =$	$3 \div 1 =$	$1 \div 1 =$
$6 \div 1 =$	$1 \div 1 =$	$4 \div 1 =$
$9 \div 1 =$	$7 \div 1 =$	

5. The Tricky Zeros Facts

Multiplication & **Division**: To understand the tricky zeros facts, knowing what is happening when we multiply by zero is important. When teach the tricky zeros facts, connecting to meaning is necessary, for example; Think: 6×0 means "six 0s or "six sets of nothing" or $0 + 0 + 0 + 0 + 0 + 0$. Zero times six (0×6) is much more difficult to conceptualize, but if students are asked draw two sets of 6, then one set of 6, and finally zero sets of 6 where they don't draw anything, they will realize why zero is the result. Similar to the previous strategy for teaching the facts, it is important not to teach a rule like "any number multiplied by zero is zero". Students will come to this rule on their own given opportunities to develop understanding.

Examples of Some Practice Items

Here are the items for multiplication and division of the tricky zeros facts:

$4 \times 0 =$	$0 \times 4 =$	$7 \times 0 =$
$0 \times 7 =$	$0 \times 0 =$	$1 \times 0 =$
$0 \times 1 =$	$4 \times 0 =$	$2 \times 0 =$
$0 \times 2 =$	$5 \times 0 =$	$0 \times 5 =$

6. The Threes Facts

Multiplication: The way to teach the threes facts is to use a "double plus one more set" strategy.

Example

Think of 3×7 is thought of as 2 sets of 7(double) plus one set of 7 or $7 \times 2) + 7 = 14 + 7 = 21$. This strategy will require review of the doubles facts and then discussion of quick addition strategies for the third set.

Examples of Some Practice Items

Here are the items for the threes facts.

$4 \times 3 =$	$2 \times 3 =$	$9 \times 3 =$
$7 \times 3 =$	$3 \times 9 =$	$5 \times 3 =$
$3 \times 5 =$	$3 \times 7 =$	$3 \times 2 =$

Division: After the students have mastered the threes multiplication facts, it is appropriate to have them learn the corresponding division facts.

The strategy for learning the division facts is by “think-multiplication”.

Example

For 18 ÷ 3, think: 3 times what equals 18; it is 6, so 18 ÷ 3 = 6.

Examples of Some Practice Items

9 ÷ 3 =	3 ÷ 3 =	27 ÷ 3 =
12 ÷ 3 =	24 ÷ 3 =	6 ÷ 3 =
21 ÷ 3 =	18 ÷ 3 =	15 ÷ 3 =

7. The Fours Facts

Multiplication: The fours facts are taught using a “double-double” strategy.

Example

Think: 4 × 7 is thought of as 2 × 7(double) to get 14 and then 2 × 14 to get 28. Discussion of quick mental strategies for the doubles of 12, 14, 16 and 18 will be required for students to master their fours facts. For 4 × 6, 2 × 6 equals 12 (think one dozen) then two 12s are 24, or 2 × 10= 20 plus 2 × 2 = 4 which equals 24. This strategy could be used with all of these doubles.

Examples of Some Practice Items

Here are the items for the fours facts.

2 × 4 =	8 × 4 =	4 × 4 =
7 × 4 =	1 × 4 =	6 × 4 =
3 × 4 =	5 × 4 =	4 × 9 =

Use money referents, a quarter, 4 quarters = one dollar.

0ne loonie = _____quarters	3 loonies = ______quarters	one toonie = _____quarters
$5 = ______ quarters	$10 = _____ quarters	$7 = ______ quarters

Division: After the students have mastered the fours multiplication facts it is appropriate to learn the corresponding division facts.

The strategy for learning the division facts is by “think-multiplication”.

Example

For 32 ÷ 4, think: 4 times what equals 32; it is 8, so 32 ÷ 4 = 8.

Examples of Some Practice Items

36 ÷ 4 =	8 ÷ 4 =	28 ÷ 4 =
4 ÷ 4 =	24 ÷ 4 =	32 ÷ 4 =
20 ÷ 4 =	12 ÷ 4 =	16 ÷ 4 =

The Last Nine Facts (6 × 6; 6 × 7; 6 × 8; 7 × 7; 7 × 8; 8 × 8; 7 × 6; 8 × 7; and 8 × 6)

The last nine facts may be learned by using decomposition and helping facts.

1) For 6 × 6, think: 5 sets of 6 plus 1 set of 6; therefore, (6 × 5) =30 plus 6 which equals 36.
2) For 6 × 7(7 × 6), think: 5 sets of 6 plus 2 sets of 6; therefore, (6 × 5) 30 plus (2 × 6) 12 equals 30 plus 12 which equals 42.
3) For 6 × 8(8 × 6), think: 5 sets of 8 plus 1 set of 8; therefore, (5 × 8) = 40 plus 8 equals 48.

Another strategy is to think of 3 sets of 8 doubled, therefore; (3 × 8) 24 doubled which equals 48.

4) For 7 × 7, think: 5 sets of 7 plus 2 sets of 7; therefore, (5 × 7) 35 plus (2 × 7) 14 equals 49. Another strategy for 7 × 7 is double and one more; think: (3 × 7) 21 double 21 which is 42 and one more group of 7 equals 49.
5) For 7 × 8, think: 5 sets of 8 plus 2 sets of 8; therefore, (5 × 8) 40 plus (2 × 8) 16 equals 56.
6) For 8 × 8, think: 4 sets of 8 doubled; therefore, (4 × 8) 32 doubled which equals 64.

Some practice examples

Here are the examples for the last nine facts:

6 × 6 =	6 × 7 =	6 × 8 =
7 × 7 =	7 × 6 =	8 × 6 =
7 × 8 =	8 × 7 =	8 × 8 =

Division: After the students have mastered the last nine multiplication facts, it is appropriate to learn the corresponding division facts.

The strategy for learning the division facts is by “think-multiplication”.

Example

For 56 ÷ 7, think: 7 times what equals 56; it is 8, so 56 ÷ 7 = 8

Examples of Some Practice Items

36 ÷ 6 =	42 ÷ 7 =	42 ÷ 6 =
48 ÷ 6 =	49 ÷ 7 =	48 ÷ 8 =
56 ÷ 8 =	64 ÷ 8 =	56 ÷ 7 =

Multiplication and Division — Mental Calculations

Quick Multiplication – No Regrouping

Note: This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end, recording the sub-steps.

Examples

For 52 × 3, simply record, starting at the front end, 150 + 6 = 156.

For 423 x 2, simply record, starting at the front end, 800 + 40 + 6 = 846.

Examples of Some Practice Items

Some practice items are:

43 x 2 =	72 × 3=	84 × 2 =
142 × 2 =	803 × 3 =	342 × 2 =
12.3 × 3 =	143 × 2 =	63 000 × 2 =
1220 × 3 =	42 000 × 4 =	43.4 × 2 =

Division Using the Think Multiplication Strategy

This is a convenient strategy to use when dividing mentally. For example, when dividing 60 by 12, think: “What times 12 is 60?” This could be used in combination with other strategies.

Example

For 920 ÷ 40, think: “20 groups of 40 would be 800, leaving 120, which is 3 more groups of 40 for a total of 23 groups.

Examples of Some Practice Items

Some practice items are:

240 ÷ 12 =	3600 ÷ 12 =	660 ÷ 30 =
880 ÷ 40 =	1260 ÷ 60 =	690 ÷ 30 =
1470 ÷ 70 =	6000 ÷ 12 =	650 ÷ 50 =

Using Multiplication Facts for Tens, Hundreds and Thousands

1) This strategy applies to tens, hundreds and thousands (with one non-zero digit in the number) multiplied by a 1-digit number.

Examples

For 3 × 70,think: 3 times 7 tens is 21 tens, or 210.

For 6 × 900, think: 6 times 9 hundreds is 54 hundreds, or 5400.

For 4 × 6000, think: 4 times 6 thousand is 24 thousand, or 24 000.

Examples of Some Practice Items

Some practice items are:

10s:	4 × 30 =	8 × 40 =	9 × 30 =
	6 × 50 =	70 × 7 =	90 × 40 =
100s:	6 × 200 =	8 × 600 =	9 × 800 =
	300 × 4 =	800 × 7 =	5 × 900 =
1000s:	3 × 2000 =	4 × 5000 =	8 × 3000 =
	6 × 6000 =	4000 × 7 =	8000 × 9 =

2) Another strategy also involves combining the two non-zero digits as if they were single-digit multiplication facts, and then attaching the appropriate place value name to the result.

Examples

For 30 × 80, think: tens by tens is hundreds, so 3 tens by 8 tens is 24 hundreds, or 2 400.

For 20 × 300, think, tens by hundreds is thousands, so 2 tens by 3 hundreds is 6 thousands, or 6 000.

Examples of Some Practice Items

40 × 80 =	60 × 20 =	30 × 50 =
90 × 60 =	40 × 40 =	70 × 90 =
10 × 400 =	30 × 600 =	80 × 200 =
100 × 50 =	700 × 30 =	900 × 50 =

Division Where the Divisor is a Multiple of 10 and the Dividend is a Multiple of the Divisor

Division by a power of ten should be understood to result in a uniform "shrinking" of hundreds, tens and units which could be demonstrated and visualized with base -10 blocks.

Example

For 400 ÷ 20, think: 400 shrinks to 40 and 40 divided by 2 is 20.

Examples of Some Practice Items

500 ÷ 10=	700 ÷ 10 =	900 ÷ 10 =
900 ÷ 30 =	600 ÷ 20 =	4000 ÷ 10 =
8000 ÷ 40 =	120 ÷ 10 =	240 ÷ 40 =
12 000 ÷ 20 =	2000 ÷ 50 =	18 000 ÷ 600 =

Multiplying by 10, 100, and 1000

Multiplication: This strategy involves keeping track of how the place values have changed.

Multiplying by 10 increases all the place values of a number by one place. For 10 × 67, think: the 6 tens will increase to 6 hundreds and the 7 ones will increase to 7 tens; therefore, the answer is 670.

Multiplying by 100 increases all the place values of a number by two places. For 100 × 86, think: the 8 tens will increase to 8 thousands and the 6 ones will increase to 6 hundreds; therefore, the answer is 8 600. It is necessary that students use the correct language when orally answering questions where they multiply by 100. For example the answer to 100 × 86 should be read as 86 hundred and not 8 thousand 6 hundred.

Multiplying by 1000 increases all the place values of a number by three places. For 1000 × 45, think: the 4 tens will increase to 40 thousands and the 5 ones will increase to 5 thousands; therefore, the answer is 45 000. It is necessary that students use the correct language when orally answering questions where they multiply by 1000. For example, the answer to 1000 × 45 should be read as 45 thousand and not 4 ten thousands and 5 thousand.

Examples of Some Practice Items

10 × 53 =	10 × 34 =	87 × 10 =
10 × 20 =	47 × 10 =	78 × 10 =
92 × 10 =	10 × 66 =	40 × 10 =
100 × 7 =	100 × 2 =	100 × 15 =
100 × 74 =	100 × 39 =	37 × 100 =
10 × 10 =	55 × 100 =	100 × 83 =
100 × 70 =	40 × 100 =	
1000 × 6 =	1000 × 14 =	83 × 1000 =
$73 × 1000 =	$20 × 1000 =	16 × $1000 =
5m = ___ cm	8m = ___cm	3m =___cm
$3 × 10 =	$7 × 10 =	$50 × 10 =
3 m = _____mm	7m = ____mm	4.2m = ___mm
6.2m = ____mm	6cm = ___mm	9km = ____m
7.7km = ____m	3dm = ____mm	3dm = ____ cm
10 × 3.3 =	4.5 × 10 =	0.7 × 10 =
8.3 × 10 =	7.2 × 10 =	10 × 4.9 =
100 × 2.2 =	100 × 8.3 =	100 × 9.9 =
7.54 × 10 =	8.36 x10 =	10 × 0.3 =
100 × 0.12 =	100 × 0.41 =	100 × 0.07 =
3.78 × 100 =	1000 × 2.2 =	1000 × 43.8 =
1000 × 5.66 =	8.02 × 1000 =	0.04 × 1000 =

Dividing by 0.1, 0.01, and 0.001

When students fully understand decimal tenths and hundredths they will be able to use this knowledge in understanding multiplication and division by tenths, hundredths and thousandths in mental math situations.

Multiplying by 10s, 100s and 1000s is similar to dividing by tenths, hundredths and thousandths. Division – tenths (0.1), hundredths (0.01) and thousandths (0.001).

Dividing by tenths increases all the place values of a number by one place.

Dividing by hundredths increases all the place values of a number by two places .

Dividing by thousandths increases all the place values of a number by three places.

Examples

1) Tenths:

For 3 × 0.1, think: the 3 ones will increase to 3 tens, therefore the answer is 30.

For 0.4 ÷ 0.1, think: the 4 tenths will increase to 4 ones, therefore the answer is 4.

2) Hundredths:

For 3 × 0.01, think: the 3 ones will increase to 3 hundreds, therefore the answer is 300

For 0.4 ÷ 0.01, think: the 4 tenths will increase to 4 tens, therefore the answer is 40.

For 3.7 ÷ 0.01, think: the 3 ones will increase to 3 hundreds and the 7 tenths will increase to 7 tens, therefore the answer is 37.

3) Thousandths:

For 3 ÷ 0.001, think: the 3 ones will increase to 3 thousands, therefore the answer is 3000. For 0.4 ÷ 0.001, think: the 4 tenths will increase to 4 hundreds, therefore the answer is 400.

For 3.7 ÷ 0.001, think: the 3 ones will increase to 3 thousands and the 7 tenths will increase to 7 hundreds, therefore the answer is 3700.

Examples of Some Practice Items:

1) Tenths:

5 ÷ 0.1 =	7 ÷ 0.1 =	23 ÷ 0.1 =
46 ÷ 0.1 =	0.1 ÷ 0.1 =	2.2 ÷ 0.1 =
0.5 ÷ 0.1 =	1.8 ÷ 0.1 =	425 ÷ 0.1 =
0.02 ÷ 0.1 =	0.06 ÷ 0.1 =	0.15 ÷ 0.1 =
14.5 ÷ 0.1 =	1.09 ÷ 0.1 =	253.1 ÷ 0.1 =

2) Hundredths:

4 ÷ 0.01 =	7 ÷ 0.01 =	4 ÷ 0.01 =
1 ÷ 0.01 =	9 ÷ 0.01 =	0.5 ÷ 0.01 =
0.2 ÷ 0.01 =	0.3 ÷ 0.01 =	0.1 ÷ 0.01 =
0.8 ÷ 0.01 =	5.2 ÷ 0.01 =	6.5 ÷ 0.01 =
8.2 ÷ 0.01 =	9.7 ÷ 0.01 =	17.5 ÷ 0.01 =

3) Thousandths:

5 ÷ 0.001 =	7 ÷ 0.001 =	1 ÷ 0.001 =
0.2 ÷ 0.001 =	3.4 ÷ 0.001 =	0.1 ÷ 0.001 =

Multiplying by 0.1, 0.01, and 0.001

Multiplying by tenths, hundredths and thousandths is similar to dividing by tens, hundreds and thousands.

This strategy involves keeping track of how the place values have changed.

Multiplying by 0.1 decreases all the place values of a number by one place.

Multiplying by 0.01 decreases all the place values of a number by two places.

Multiplying by 0.001decreases all the place values of a number by three places.

Examples

1) Tenths:

   For 5 × 0.1, think: the 5 ones will decrease to 5 tenths, therefore the answer is 0.5.

   For, 0.4 × 0.1, think: the 4 tenths will decrease to 4 hundredths, therefore the answer is 0.04.

2) Hundredths:

   For 5 × 0.01, think: the 5 ones will decrease to 5 hundredths, therefore the answer is 0.05.

   For, 0.4 × 0.01, think: the 4 tenths will decrease to 4 thousandths, therefore the answer is 0.004.

3) Thousandths:

   For 5 × 0.001, think: the 5 ones will decrease to 5 thousandths; therefore, the answer is 0.005.

Examples of Some Practice Items

1) Tenths:

6 × 0.1 =	8 × 0.1 =	3 × 0.1 =
9 × 0.1 =	1 × 0.1 =	12 × 0.1 =
72 × 0.1 =	136 × 0.1 =	406 × 0.1 =
0.7 × 0.1 =	0.5 × 0.1 =	0.1 × 10 =
1.6 × 0.1 =	0.1 × 84 =	0.1 × 3.2 =

2) Hundredths:

6 × 0.01 =	8 × 0.01 =	1.2 × 0.01 =
0.5 × 0.01 =	0.4 × 0.01 =	0.7 × 0.01 =
2.3 × 0.01 =	3.9 × 0.01 =	10 × 0.01 =
100 × 0.01 =	330 × 0.01 =	46 × 0.01 =

3) Thousandths:

3 × 0.001 =	7 × 0.001 =	80 × 0.001 =
21 × 0.001 =	45 × 0.001 =	12 × 0.001 =
62 × 0.001 =	9 × 0.001 =	75 × 0.001 =
4mm =_____m	9mm = _____m	6m = ______km

Dividing by 10, 100, and 1000

Dividing by 10 decreases all the place values of a number by one place.

Dividing by 100 decreases all the place values of a number by two places.

Dividing by 1000 decreases all the place values of a number by three places.

1) Tens:

   For,60 ÷ 10, think: the 6 tens will decrease to 6 ones; therefore, the answer is 6.

   For, 500 ÷ 10; think: the 5 hundreds will decrease to 5 tens; therefore, the answer is 50. This is an opportunity to show the relationship between multiplying by one tenth and dividing by 10.

2) Hundreds:

   For, 7 500 ÷ 100; think: the 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75. This is an opportunity to show the relationship between multiplying by one hundredth and dividing by 100.

3) Thousands:

   For, 75 000 ÷ 1000; think: the 7 ten thousands will decrease to 7 tens and the 5 thousands will decrease to 5 ones; therefore, the answer is 75. This is an opportunity to show the relationship between multiplying by one thousandth and dividing by 1000.

Examples of Some Practice Items

1) Tens:

70 ÷ 10 =	90 ÷ 10 =	40 ÷ 10 =
200 ÷ 10 =	800 ÷ 10 =	100 ÷ 10 =

2) Hundreds:

400 ÷ 100 =	900 ÷ 100 =	6000 ÷ 100 =
4200 ÷ 100 =	7600 ÷ 100 =	8500 ÷ 100 =
9700 ÷ 100 =	4400 ÷ 100 =	10 000 ÷ 100 =
600 pennies = $_____	1 800 pennies =$_____	56 000 pennies = $____

3) Thousands:

82 000 ÷ 1000 =	98 000 ÷ 1000 =	12 000 ÷ 1000 =
66 000 ÷ 1000 =	70 000 ÷ 1000 =	100 000 ÷ 1000 =
430 000 ÷ 1000 =	104 000 ÷ 1000 =	4 500 ÷ 1000 =
77 000m = _____km	84 000m = ____km	7 700m = _____km

Front End Multiplication or the Distributive Principle

This strategy involves finding the product of the single-digit factor and the digit in the highest place value of the second factor, and adding to this product a second sub-product.

Examples

1) For 3 × 62, think: 3 times 6 is 18 tens, or 180, and 3 times 2 is 6; so, 180 plus 6 is 186.
2) For 2 × 706, think: 2 times 7 hundreds is 14 hundreds, or 1400; and 2 times 6 is 12; so 1 400 plus 12 is 1 412.

3) For 5 × 6100, think: 5 times 6 thousand is 30 thousands, or 30 000; and 5 times 100 is 500; so 30 000 plus 500 is 30 500.

4) For 3.2 × 6, think: 3 times 6 is 18 and 6 times 0.2 is 12 tenths or 1 and 2 tenths; so 18 plus 1.2 is 19.2.

   For 62 × 0.2, think: 2 tenths times 60 is 120 tenths or 12; and 2 tenths times 2 is 4 tenths or 0.4; so 12 plus 0.4 is 12.4.

   For 47 × 0.3, think: 3 tenths times 40 is 120 tenths or 12; and 3tenths times 7 is 21 tenths or 2.1; so 12 plus 2.1 is 14.1.

Examples of Some Practice Items

1) Some practice items for numbers in the 10s are:

53 × 3 =	32 × 4 =	41 × 6 =
29 × 2 =	83 × 3 =	75 × 3 =
62 × 4 =	92 × 5 =	35 × 4 =

2) Some practice items in the 100s are:

3 × 503 =	209 × 9 =	703 × 8 =
606 × 6 =	503 × 2 =	8 04 × 6 =
309 × 7 =	122 × 4 =	320 × 3 =
410 × 5 =		

3) Some practice items in the 1000s are:

3 × 4200 =	4 × 2100 =	6 × 3100 =
5 × 5100 =	2 × 4300 =	3 × 3200 =
2 × 4300 =	7 × 2100 =	4 × 4200 =

4) Some practice items in the tenths are:

4. 6 × 2 =	36 × 0.2 =
8.3 × 5 =	43 × 0.5 =
7.9 × 6 =	96 × 0.3 =
3.7 × 4 =	52 × 0.4 =
8.9 × 5 =	75 × 0.8 =
3.3 × 7 =	83 × 0.9 =

Compensation

This strategy for multiplication involves changing one of the factors to a ten, hundred or thousand; carrying out the multiplication; and then adjusting the answer to compensate for the change that was made. This strategy could be used when one of the factors is near ten, hundred or thousand.

Examples

1) For 6 × 39, think: 6 times 40 is 240, but this is six more than it should be because 1 more was put into each of the six groups; therefore, 240 subtract 6 is 234.

   For 7 × 198, think: 7 times 200 is 1400, but this is 14 more than it should be because there were 2 extra in each of the 7 groups; therefore, 1400 subtract 14 is 1 368.

2) For 19 × 60, think: 20 times 60 is 1200, but this is 60 more than it should be because 1 200 includes one more set of 60 than it should; therefore 1200 – 60 is 1 140.

Examples of Some Practice Items

1) Some practice items are:

Tens:

6 × 39 =	8 × 29 =	5 × 49 =
2 × 79 =	6 × 89 =	7 × 59 =
4 × 49 =	9 × 69 =	

Hundreds:

5 × 399 =	3 × 199 =	4 × 198 =
9 × 198 =	8 × 698 =	7 × 598 =

2) Some practice items are:

29 × 50 =	39 × 40 =	89 × 20 =
49 × 90 =	79 × 30 =	59 × 60 =

Finding Compatible Factors

This strategy for multiplication involves looking for pairs of factors whose product is a power of ten and re-associating the factors to make the overall calculation easier. This is possible because of the associative property of multiplication.

Examples

For 25 × 63 × 4, think: 4 times 25 is 100, and 100 times 63 is 6 300.

For 2 × 78 × 500, think: 2 times 500 is 1000, and 1000 times 78 is 78 000.

For 5 × 450 × 2, think: 2 times 5 is 10, and 10 times 450 is 4500.

Examples of Some Practice Items

5 × 19 × 2 =	2 × 43 × 50 =	4 × 38 × 25 =
500 × 86 × 2 =	250 × 56 × 4 =	40 × 25 × 33 =

Open Frames in Addition, Subtraction, Multiplication and Division

Open frames are introduced in grade 4 and will be a mental math activity in grade 5. Sometimes there will be a missing digit and sometimes a missing number. Following are a variety of examples using the four operations.

Some practice examples are

Think Subtraction

$25 + \square = 85$	$163 + \square = 363$	$426 + 2\square 2 = 668$
$0.4 + \square = 0.9$	$0.32 + 0.\square 6 = 0.88$	$3.555 + \square = 4.000$
$29\ 000 + \square = 30\ 000$	$5\square 000 + 30\ 000 = 87\ 000$	$\$67 + \square = \$100$

Think Addition

36 - □ = 29	46 – 2□ = 23	□7 – 35 = 22
487 - □35 = 252	70cm - □ = 22cm	2.24 - □ = 2.00
3 567 – □222 = 1345	0.35mm - □ mm = 0.15mm	\$3.64 – \$1.□ = \$2.24

Think Division

25 × □ = 50	9 × □ = 81	3□ × 3 = 99
30 × □ = 60	40 × □0 = 2 000	52 × □ = 5.2
5 □ × 3 = 168	\$50 × □ = \$150	365mm = □ m
750cm = □ m	2□0 × 3 = 750	7 000m = □ km

Think Multiplication

10 ÷ □ = 5	□ ÷ 3 = 8	□ ÷ 7 = 7
120 ÷ □ = 12	3□ ÷ 5 = 6	5 000mm = □ m
34cm = □ mm	\$240 ÷ □ = \$40	\$240 ÷ □ = \$4
6.3 ÷ □ = 63	0.05 ÷ □ = 5	0.019 ÷ □ = 1.9

Addition, Subtraction, Multiplication and Division — Computational Estimation

It is essential that estimation strategies are used by students before attempting pencil/paper or calculator computation to help them find "ballpark" or reasonable answers.

When teaching estimation strategies, it is necessary to use the language of estimation with your students. Some of the common words and phrases are: about, just about, between, a little more than, a little less than, close, close to, and near.

Rounding

Note: This strategy involves rounding each number to the highest, or the highest two, place values and adding the rounded numbers. . Rounding to the highest place value would enable most students to keep track of the rounded numbers and do the calculation in their heads; however, rounding to two highest place values, as presented in rounding with 5s in the next section (B), would probably require most students to record the rounded numbers before performing the calculation mentally.

Examples

1) To estimate 348 + 230, think: 348 rounds to 300 and 230 rounds to 200, so 300 plus 200 is 500.

   To estimate 4276 + 3937, think: 4276 rounds to 4000 and 3937 rounds to 4000, so 4000 plus 4000 is 8000.

2) To estimate 594 - 203, think: 594 rounds to 600 and 203 rounds to 200, so 600 subtract 200 is 400.

   To estimate 6237 – 2945, think: 6 237 rounds to 6000 and 2945 rounds to 3000, so 6000 subtract 3000 is 3000.

3) Some examples of rounding *multiplication* questions with a double or triple digit factor by a single digit factor.

   To round 7 × 64, think: 64 rounds to 60 and 7 times 60 is 420.

   To round 8 × 693, think: 693 rounds to 700 and 8 times 700 is 560.

4) Some examples of rounding multiplication questions when the two factors are 2-digit with the ones digits 5 or more, consider rounding the smaller factor up and the larger factor down.

   To round 76 × 36, round 76 (the larger number down to 70) and round 36 (the smaller number up to 40) which equals 70 × 40 = 2800. This produces a closer estimate than rounding to 80 × 40 or 80 × 30.

5) Some practice examples of rounding *division* questions to estimate the quotient where it is necessary to look for compatible numbers.

   To round 471 ÷ 6, think: 480 ÷ 6, which is 80 because 6 × 8 = 48, therefore, 60 × 8 = 480.

   To round 822 ÷ 9, think: 810 ÷ 9, which is 90.

Examples of Some Practice Items

1) Some practice items for rounding *addition* of numbers in the 10s, 100s and 1000s.

28 + 57 =	41 + 34 =	123 + 62 =
303 + 49 =	137 + 641 =	223 + 583 =
490 + 770 =	684 + 824 =	530 + 660 =
8879 + 4238 =	6853 + 1280 =	3144 + 4870 =
6110 + 3950 =	4460 + 7745 =	1370 + 6410 =

2) Some practice items for rounding *subtraction* of numbers in the 10s, 100s and 1000s are.

36 – 22 =	43 – 8 =	54 – 18 =
68 – 34 =	99 – 47 =	93 – 12 =
427 – 198 =	984 – 430 =	872 – 399 =
594 – 301 =	266 – 98 =	843 – 715 =
834 – 587 =	947 – 642 =	780 - 270 =
4768 – 3068 =	6892 - 1812 =	7368 – 4817 =
4807 – 1203 =	7856 – 1250 =	5029 – 4020 =
8876 – 3640 =	9989 – 4140 =	1754 – 999 =

3) Here are some practice items:

4 × 59 =	7 × 22 =	8 × 61 =
9 × 43 =	295 × 6 =	7 × 402 =
889 × 3 =	5 × 503 =	2 × 888 =
7 × 821 =	1 × 795 =	712 × 4 =

4) Some practice items are:

57 × 29 =	49 × 28 =	38 × 27 =
66 × 57 =	87 × 19 =	36 × 58 =
27 × 68 =	87 × 37 =	96 × 78 =

5) Some practice items are:

370 ÷ 9 =	732 ÷ 8 =	243 ÷ 6 =
458 ÷ 5 =	331 ÷ 4 =	191 ÷ 2 =
638 ÷ 7 =	731 ÷ 9 =	268 ÷ 3 =
409 ÷ 6 =	341 ÷ 5 =	630 ÷ 8 =

Rounding with "Fives"

When the digit 5 is involved in the rounding procedure for numbers in the 10s, 100s and 1000s, the number should be rounded **up** or **down** depending upon the effect the rounding will have in the overall calculation. When both numbers are about 5, 50, 500, or 5 000 rounding one number up and one number down will minimize the effect the rounding will have in the estimation.

Example

To estimate 45 + 65, it would be best to round to 40 + 70; to estimate 452 + 329, it would be best to round to 500 + 300; and to estimate 4520 + 4610, it would be best to round to 4000 + 5000.

Examples of Some Practice Items

Some practice items using the rounding procedure for adding numbers with fives are:

35 + 55 =	45 + 31 =	26 + 35 =
250 + 650 =	653 + 128 =	179 + 254 =
2500 + 4500 =	4550 + 4220 =	6810 + 1550 =

Front End

This strategy involves combining only the values in the highest place value to get a “ball- park”. Such estimates are adequate in many circumstances.

Examples

1) To estimate 4276 + 3237, think: 4000 plus 3000 is 7000
2) To estimate 37 260 + 28 142, think: 30 000 plus 20 000 is 50 000.
3) To estimate 5475 – 3128, think: 5000 subtract 3000 is 2000.
4) To estimate 58 123 – 22 144, think: 50 000 subtract 20 000 is 30 000.
5) To estimate 3.8 + 2.1, think, 3 + 2 = 6.5 (nearest whole number).

   To estimate 8.95 + 3.21, think, 8 + 3 = 11 (nearest whole number).

   To estimate 0.41 + 0.27, think, 0.4 + 0.2 = 0.6. (nearest tenth).
6) For example: To estimate 4.7 – 3.1, think, 4 – 3 = 1. (to nearest whole number)

   To estimate 8.88 – 4.21, think, 8 – 4 = 4. (to nearest whole number)

   To estimate 0.92 – 0.61, think, 0.9 – 0.6 = 0.3. (to nearest tenth)
7) For example: To estimate 93 × 4, think: 90 × 4 is 4 groups of 9 tens, or 360.

   To estimate 231 × 5, think: 200 × 5 is 200 groups of 5 hundreds, or 1000.
8) To estimate 8 × 823.24, think: 8 × 800 or 6400.
9) To estimate 59 ÷ 3, think: 59 is about 60 and 6 ÷ 3 is 2, so 60 ÷ 3 is 20.

   To estimate 241 ÷ 4, think: 241 is about 240 and 24 ÷ 4 is 6, so 240 ÷ 4 is 60.

Examples of Some Practice Items

1) Some practice items for estimating *addition* of numbers are:

54 + 33 =	12 + 51 =	71 + 14 =
24 + 73 =	341 + 610 =	647 + 312 =
632 + 207 =	703 + 241 =	423 + 443 =
816 + 111 =	2467 + 5106 =	4275 + 2105 =
6125 + 2412 =	3321 + 6410 =	1296 + 6388 =

2) Some practice items for estimating *addition* of numbers in the 10 000s are.

23 404 + 19 123 =	47 409 + 22 222 =
71 246 + 23 511 =	39 332 + 41 100 =
79 012 + 9 123 =	53 199 + 31 001 =
18 945 + 23 135 =	68 987 + 12 321 =
78 043 + 49 075 =	92 463 + 68 987 =

3) Some practice items for estimating *subtraction* are:

72 – 33 =	84 – 61 =
54 – 21 =	73 – 44 =
639 – 426 =	718 – 338 =
248 – 109 =	823 – 240 =
431 - 206 =	743 – 519 =
639 – 426 =	718 – 338 =
248 – 109 =	823 – 240 =
431 - 206 =	743 – 519 =

4) Some practice items for estimating *subtraction* of numbers in the 10 000s are:

68 403 – 39 005 =	43 115 – 19 432 =
77 859 – 32 443 =	98 846 – 59 319 =
19 432 – 11 115 =	71 812 – 9 856 =
51 273 – 39 180 =	99 437 – 19 841 =
37 673 – 8 950 =	82 402 – 71 673 =

5) Some practice items for estimating *addition* of numbers in the tenths and hundredths are:

Estimate to nearest whole number.

5.2 + 3.8 =	6.7 + 1.2 =	4.8 + 4.1 =
6.2 + 0.85 =	0.2 + 4.9 =	0.1 + 0.2 =
3.01 + 2.86 =	5.32 + 0.97 =	13.40 + 3.89 =
26.67 + 3.21 =	0.86 + 0.93 =	1.01 + 2.13 =

Estimate to nearest tenth.

0.23 + 0.38 =	0.81 + 0.09 =	0.72 + 0.21 =
0.13 + 0.61 =	0.01 + 0.81 =	3.22 + 2.31 =
2.49 + 0.08 =	10.09 + 0.09 =	0.02 + 0.01 =

6) Some practice items for estimating *subtraction* of numbers in the tenths & hundredths are.

Estimate to nearest whole number.

6.1 – 2.2 =	5.9 – 3.1 =	8.3 – 5.8 =
4.1 - 0.9 =	12.3 – 10.1 =	6.2 – 0.8 =
1.9 – 0.2 =	4.3 – 0.8 =	15.1 – 14.9 =

Estimate to nearest tenth.

0.98 – 0.11 =	0.79 – 0.01 =	0.09 – 0.01 =
0.31 – 0.08 =	0.71 – 0.29 =	0.53 – 0.27 =
0.88 – 0.09 =	3.41 – 0.28 =	5.69 – 2.31 =

7) Some practice examples for estimating *multiplication* of numbers in the 10s and 100s are:

78 × 2 =	57 × 6 =	63 × 8 =
92 × 2 =	73 × 8 =	44 × 7 =
176 × 3 =	809 × 5 =	613 × 6 =
287 × 8 =	467 × 4 =	481 × 9 =

8) Some practice items for estimating *multiplication* of numbers in the tenths and hundredths by a single digit whole number are:

3 × 78.8 =	6 × 20.14 =	5 × 29.84 =
9 × 19.04 =	7 × 486.35 =	2 × 490.86 =
4 × 490.86 =	8 × 572.33 =	6 × 97.53 =

9) Some practice items for estimating *division* of numbers in the 10s and 100s are:

92 ÷ 3 =	83 ÷ 2 =	102 ÷ 5 =
119 ÷ 3 =	121 ÷ 6 =	357 ÷ 5 =
407 ÷ 8 =	141 ÷ 7 =	75 ÷ 3 =

Adjusted Front End

This strategy begins by getting a Front End estimate and then adjusting that estimate to get a better, or closer, estimate by either (a) considering the second highest place values or (b) by clustering all the values in the other place values to "eyeball" whether there would be enough together to account for an adjustment.

Examples

1) To estimate 437 + 541, think: 400 plus 500 is 900: but 37 and 41 would account for about another 100; therefore, the adjusted estimate is 900 + 100 or 1000.

   To estimate 6237 – 2954, think: 6000 subtract 2000 is 4000, and 954 would account for about another 1000; therefore, the adjusted estimate is 6000 - 2000 – 1000 or 3000.

2) To estimate 819 - 399, think: 800 subtract 300 is 500; but 99 would account for about another 100; therefore, the adjusted estimate is 800 – 300 -100 = 400.

   To estimate 6237 – 2954, think: 6000 subtract 2000 is 4000, and 954 would account for about another 1000; therefore, the adjusted estimate is 6000 - 2000 – 1000 or 3000.

3) Sometimes the numbers in the ones do not account for another ten and therefore, do not affect the estimation.

   To estimate 31 + 22, think: 30 plus 20 is 50, and 1plus 2 *would not* account for another 10, so the estimate is 30 + 20 or 50, with no adjustment.

   51 + 33 = 50 + 30 + (4 which would not account for another 10) = 80

4) Sometimes the number in the ones in the subtrahend does not account for another ten and therefore, does not affect the estimation.

   To estimate 89 – 31, think: 80 subtract 30 is 50; and 1 would not account for another 10, so the estimate is 50 with no adjustment.

   76 – 53 = 70 – 50 –(3 which would not account for another 10) =20

5) Sometimes the numbers in the tens and ones do not account for another hundred and therefore, do not affect the estimation.

   To estimate 512 + 207, think: 500 plus 200 is 700, and 12 and 7 *would not* account for another 100, so the estimate is 500 + 200 or 700, with no adjustment.

   305 + 614 = 300 + 600 + (19 which would not account for another 100) = 900.

6) Sometimes the numbers in the ones and tens in the subtrahend do not account for another hundred and therefore, do not affect the estimation.

   To estimate 819 – 310, think: 800 subtract 300 is 500, but 10 would *not* account for another 100, so the estimate is 800 – 300 = 500, with no adjustment.

   625 – 410 = 600 – 400 – (10 which would not account for another 100) = 200

7) Sometimes the numbers in the hundreds, tens and ones do not account for another thousand and therefore, do not affect the estimation.

   To estimate 4105 + 1045, think: 4000 plus 1000 is 5000, and 105 and 45 *would not* account for another 1000, so the estimate is 4000 + 1000 or 5000, with no adjustment.

   6040 + 2110 = 6000 + 2000 + (40 + 110) which would not account for another 1000) = 8 000.

8) Sometimes the numbers in the ones, tens and hundreds in the subtrahend do not account for another thousand and therefore, do not affect the estimation.

   To estimate 5055 – 2010, think: 5000 subtract 2000 is 3000, but 10 *would not* account for another 1000, so the estimate is 5000 – 2000 = 3000, with no adjustment.

   8040 – 3121 = 8000 – 3000 –(121 which would not account for another 1000) = 5 000

9) To estimate 8.64 + 5.28, think: 8 plus 5 is 13, and 0.64 plus 0.28 would account for another 1 whole; therefore, the adjusted estimate is 8 + 5 + 1= 14.

10) To estimate 7.12 – 3.89, think: 7 – 3, but 0.89 would account for about another 1 whole; therefore the adjusted estimate is 7 – 3 – 1 = 3.

11) To estimate 8 × 823.24, think: 8 × 800(6400) + 8 × 20(160) = 6400 + 160 = 6560.
    or 8 × 800(6400) + 8 × 25(200) = 6400 + 200 = 6600.

Examples of Some Practice Items

1) Some practice items for estimating *addition* are:

251 + 445 =	589 + 210 =	320 + 275 =
642 + 264 =	519 + 180 =	148 + 450 =
5695 + 2450 =	4190 + 1850 =	4550 + 3445 =
5240 + 3790 =	1910 + 5125 =	

2) Some practice items for estimating *subtraction* are:

645 – 290 =	720 – 593 =	834 – 299 =
935 – 494 =	6210 – 2987 =	8040 – 5899 =
9145 – 4968 =	7120 – 4975 =	6148 – 3920 =

3) Some other practice items for estimating *addition* of numbers in the 10s demonstrating this are:

42 + 22 =

4) Some other practice items for estimating *subtraction* of numbers in the 10s demonstrating this are:

45 -12 =

5) Some other practice items for estimating *addition* of numbers in the 100s demonstrating this are:

407 + 303 =

214 + 320 =

6) Some other practice items for estimating *subtraction* of numbers in the 100s demonstrating this are:

468 – 215 =

937 – 612 =

7) Some other practice items for estimating *addition* of numbers in the 1000s demonstrating this are:

8105 + 1210 =	1120 + 7140 =	4087 + 2120 =
3034 + 4230 =	1250 + 3100 =	6060 + 3140 =
5125 + 3085 =	2220 + 5120 =	4140 + 5050 =

8) Some other practice items for estimating *subtraction* of numbers in the 1000s demonstrating this are:

5025 – 2 050 =	7100 – 4040 =	9150 – 5045 =
6140 – 3 110 =	8202 – 5222 =	4004 – 1259 =
7334 – 6 009 =	3 067 – 1367 =	9254 – 8321 =

9) Here are some practice items for estimating *addition* of numbers in the hundredths.

7.45 + 1.56 =	5.89 + 2.10 =	3.20 + 2.75 =
6.43 + 2.67 =	3.19 + 2.81 =	3.48 + 4.50 =
$2.45 + $4.60 =	$3.95 + $4.07 =	$2.73 - $2.22 =

10) Some practice items for estimating *subtraction* of numbers in the hundredths are:

7.43 – 4.95 =	5.29 – 2.99 =	6.18 – 1.97 =
8.05 – 4.92 =	8.11 – 4.98 =	9.21 – 5.99 =
$6.45 - $5.98 =	$7.20 - $5.97 =	$8.34 - $2.99 =

11) Some practice items for estimating *multiplication* of numbers in the tenths and hundredths by a single digit whole number are:

7 × 341.25 =	2 × 722.56 =	5 × 331.43 =
3 × 943.19 =	8 × 776.43 =	4 × 609.98 =
6 × 280.53 =	9 × 371.05 =	568.99 × 7 =

Clustering of Near Compatibles for Addition and Mixed Computation

When adding a list of numbers it is sometimes useful to look for two or three numbers that can be grouped to make 10, 100s and 1000s (compatible numbers). If there are numbers that can be adjusted slightly (*near compatibles*) to produce these groups, it will make finding an estimate easier.

Examples

For 44 + 62 + 33 + 71, think: 44 + 62 is almost 100 and 33 + 71 is almost 100; the estimate would be 100 + 100 = 200.

For 692 – 204 + 298, think: 692 + 298 is almost 700 + 300 = 1000, and 1000 subtract 200 is about 800.

Examples of Some Practice Items

412 + 32 + 611 =	252 + 131 + 757 =
499 + 76 + 503 =	301 + 251 + 710 + 749 =
998 + 495 + 509 =	399 + 203 + 608 =
3100 + 2100 + 7050 =	1334 + 2501 + 7510 =
710- 62 – 41 + 320 =	921 – 38 – 63 + 121 =
7200 – 403 – 599 + 3200 =	8950 – 211 – 798 + 985 =