Bridging the Gap

Reflections on First-Year University Calculus

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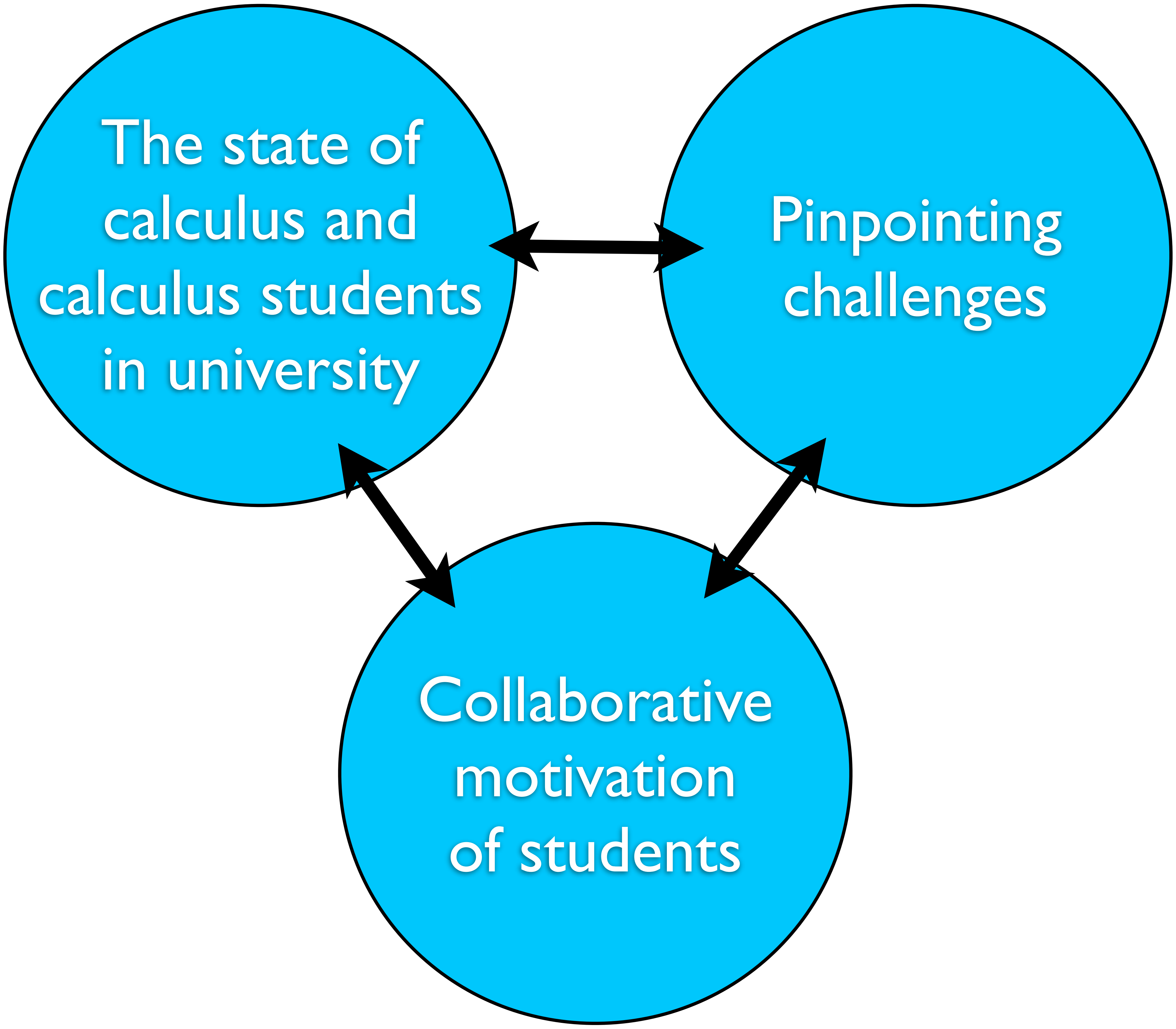
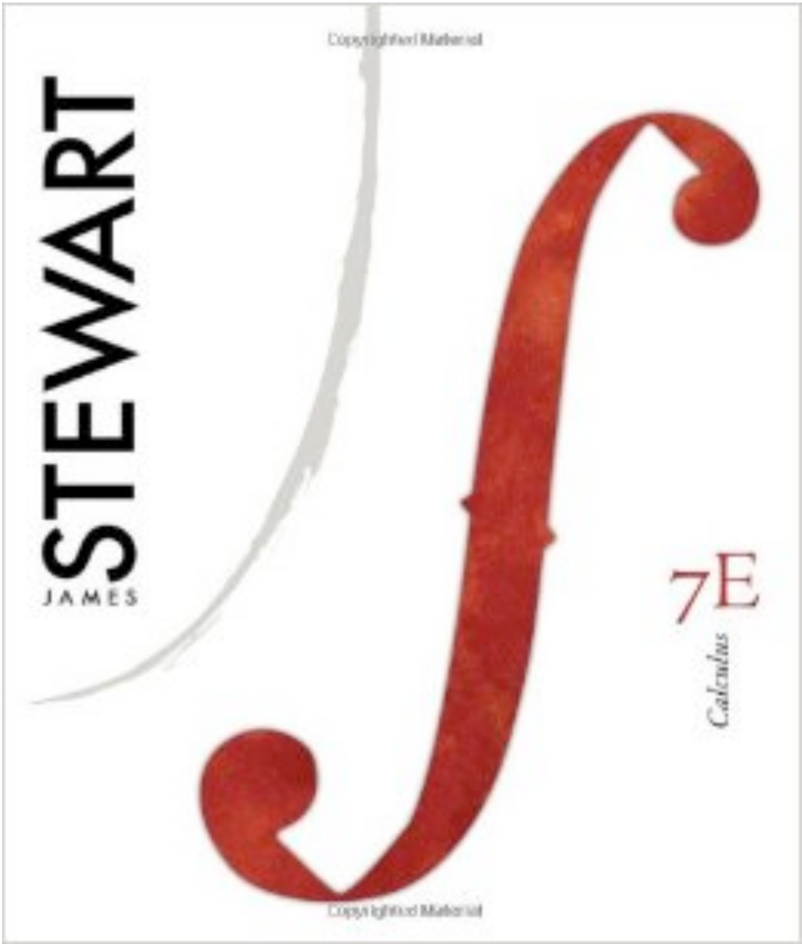
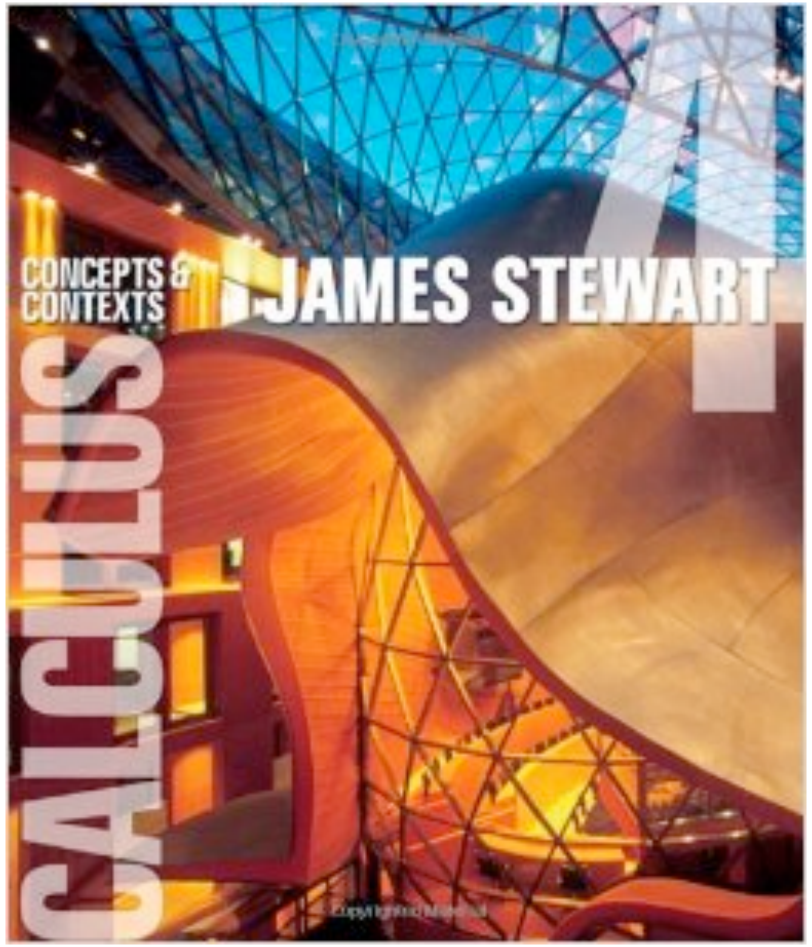
Introduction

- B.Sc., StFX 2003, M.Sc., Dal 2005, Ph.D UBC 2009, prof @ StFX 2009-present
- Taught calculus at both UBC and StFX
- Taught calculus every year (3 times with double sections) at StFX
- Dept. Coordinator for Calculus since 2010



The state of calculus and calculus students in university
Pinpointing challenges
Collaborative motivation of students

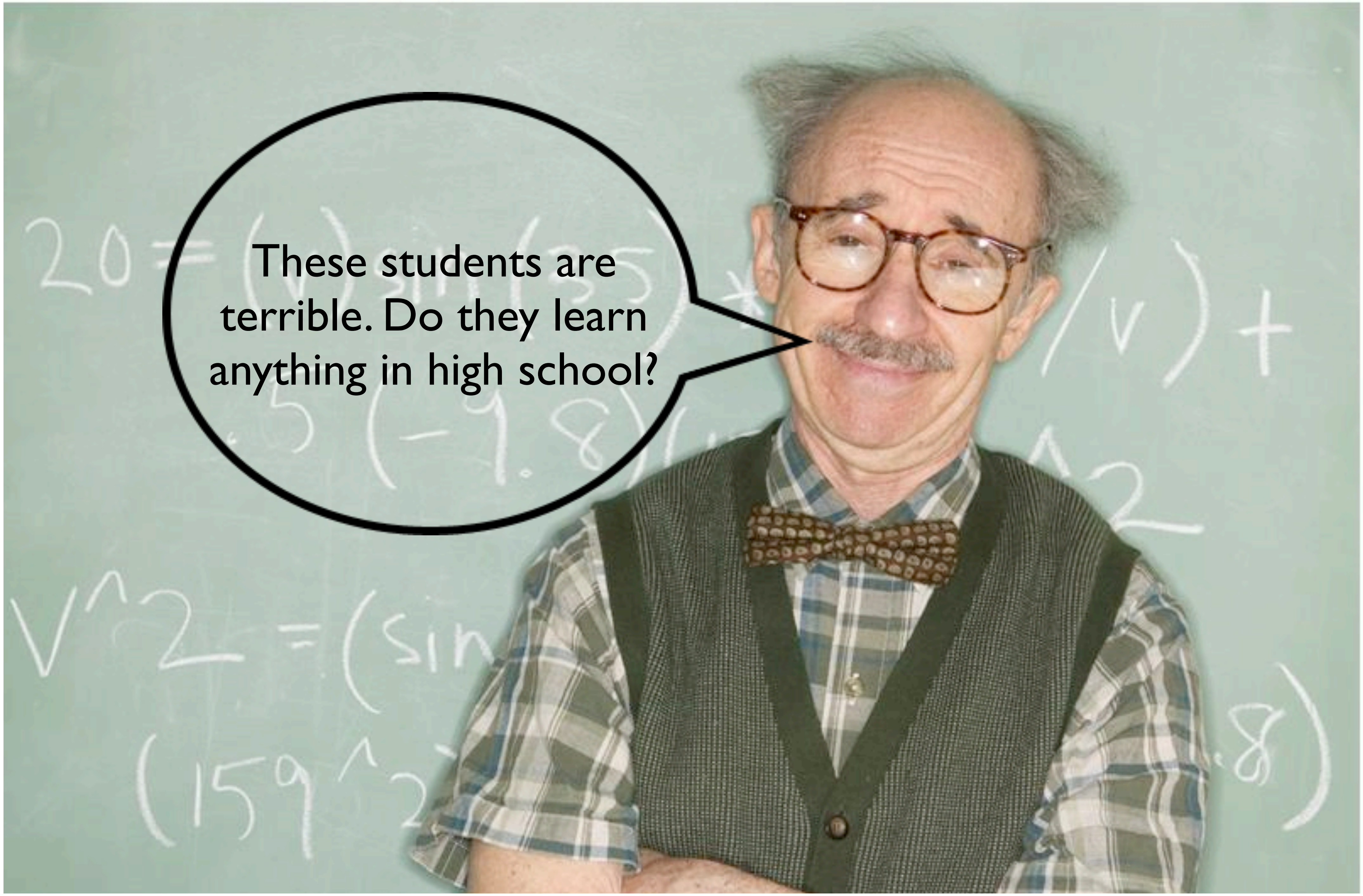


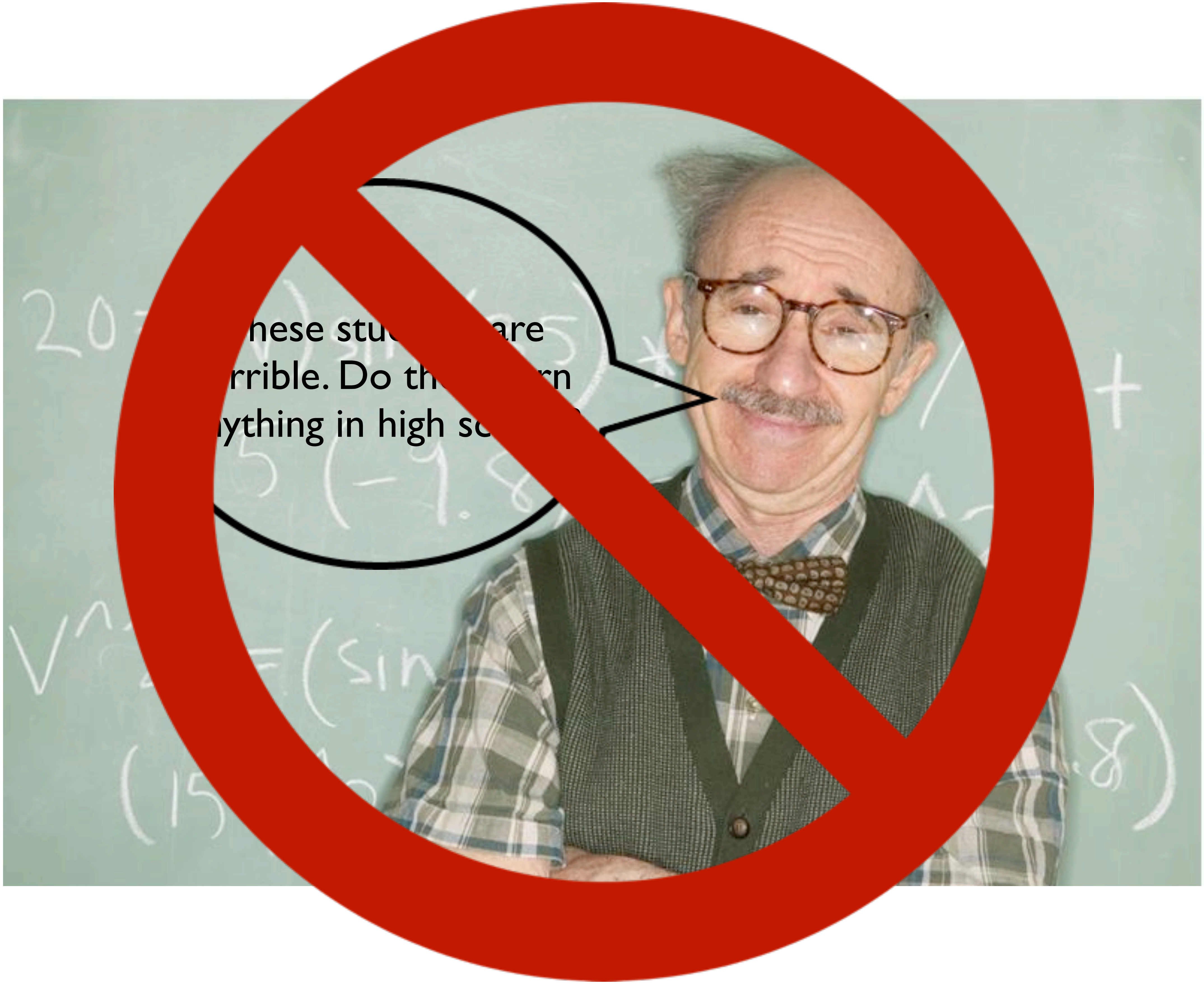


Stewart's Calculus Book is used by over 70% of the first-year market in North America

Typical Evaluation

- Weekly quiz based on homework (15%)
- One-hour per week lab component using Maple (10%)
- Midterms (30%)
- Final (45%)
- class sizes: 45-70 students at StFX, class averages: around 60%-70%


These students are terrible. Do they learn anything in high school?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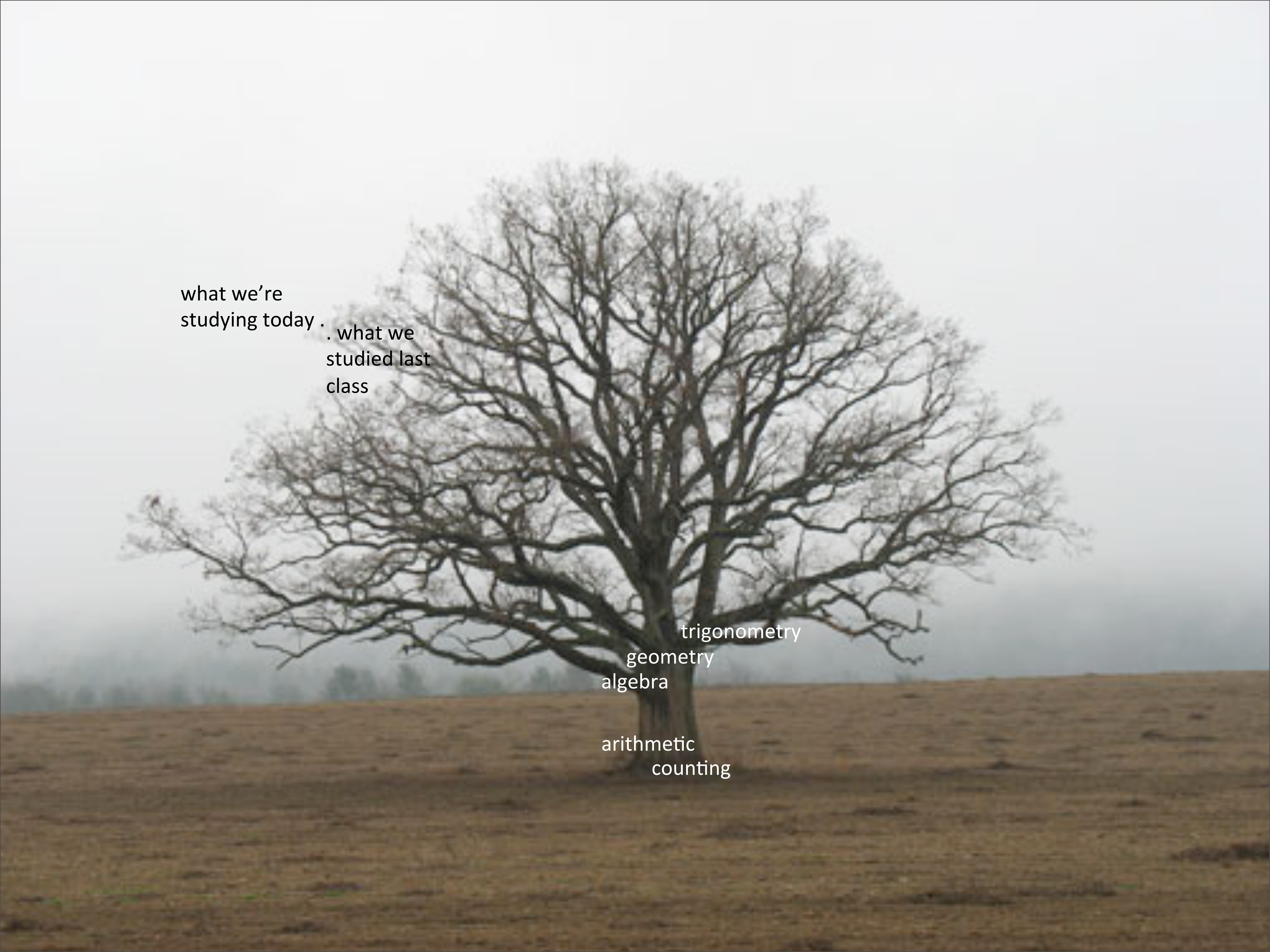
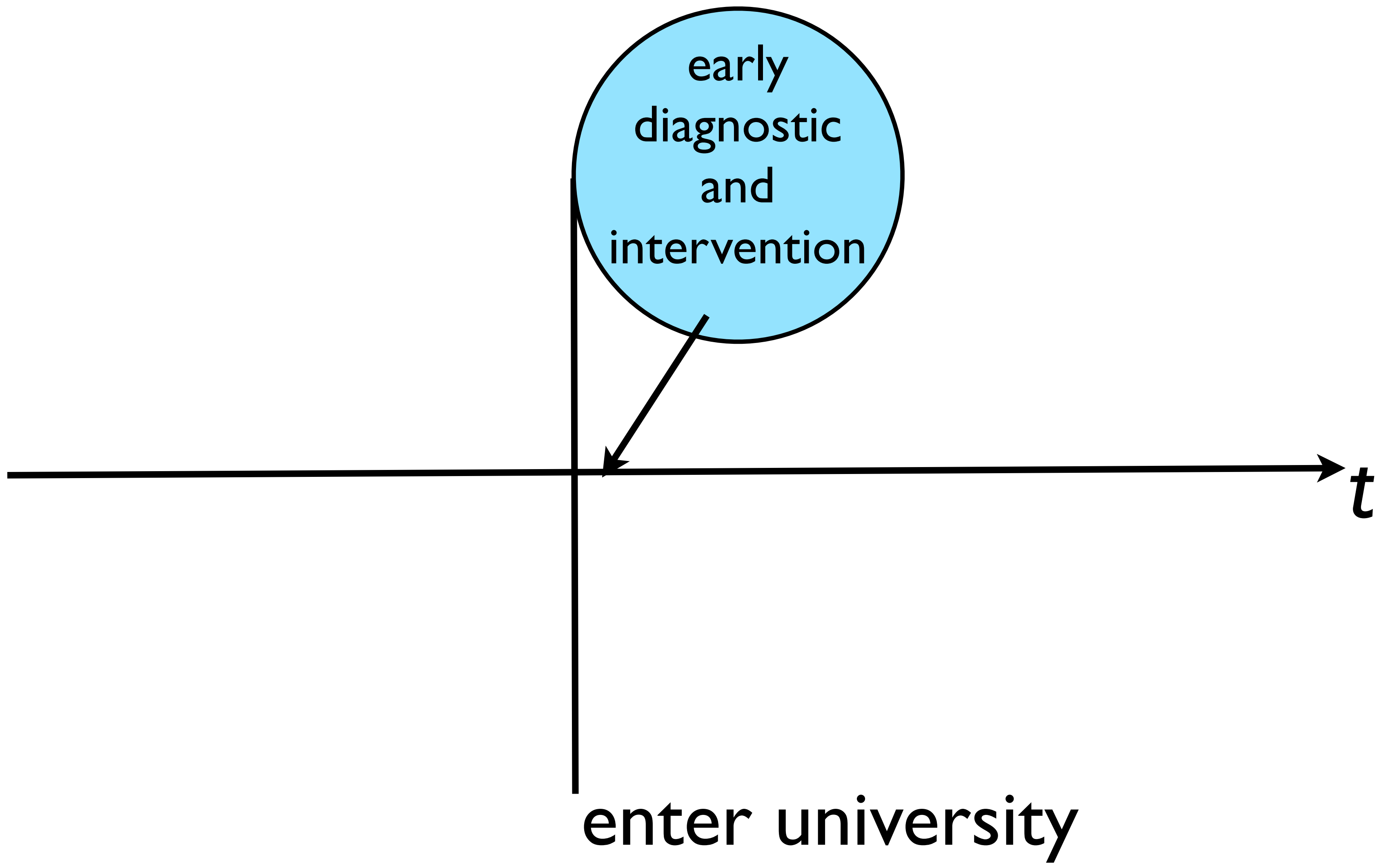

what we're
studying today .
. what we
studied last
class
trigonometry
geometry
algebra
arithmetic
counting

Identifying a problem at X

- 2010-2013, increasing problem with student retention in first-term calculus (peaking in 2012)
- 2012-2013: initiated review of calculus course
- Changed text, syllabus, introduced diagnostic component (pre-calc), removed 3 class review of precalculus, extra midterm

		OCT. DROP	REMAINED	% DROP
2012	**M111:11**	17	37	
	M111:13	11	50	**24.3**
2013	**M111:11**	12	49	
	M111:13	5	57	**13.8**
2014	**M111:11**	10	59	**14.5**


early
diagnostic
and
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enter university

Digging Deeper

- What did you take in High School?

2014	HS CALC	HS PRECAL	NEITHER
number	35	22	7
avg. grade	76.2%	59.3%	35.6%
failures	0	6	4
drops	2	3	2
fail/drop %	5.7%	40.9%	85.7%

Observations

- By and large, students with HS calculus adequately prepared for Calculus I.
- problems at university level stem from:
  - complacency, delay in identifying gaps in knowledge
  - the background they enter with
  - time away from math

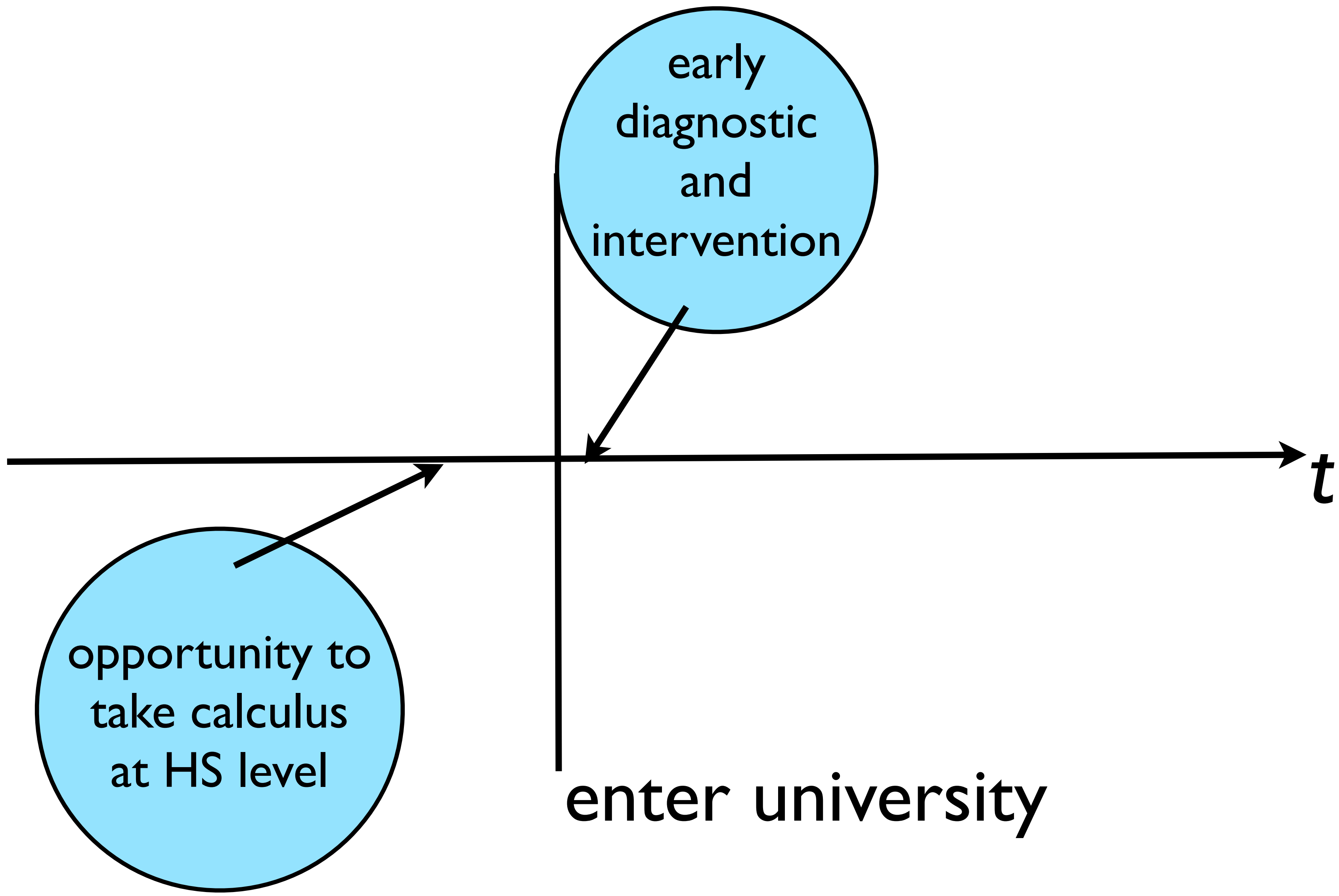
Male vs. Female Performance

2013	MALE	FEMALE	
number	37	69	65% female
avg. grade	59.4	66.4	
2014			
number	21	44	68% female
avg. grade	64.8	69.6	

Female students on average consistently outperform male students by 5-7% in first-term calculus

Possible gender factors

- Talented male students disproportionately enter engineering versus females
- Delayed versus instant gratification
- Higher prevalence in males of over-confidence due to taking high school calculus


early
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opportunity to
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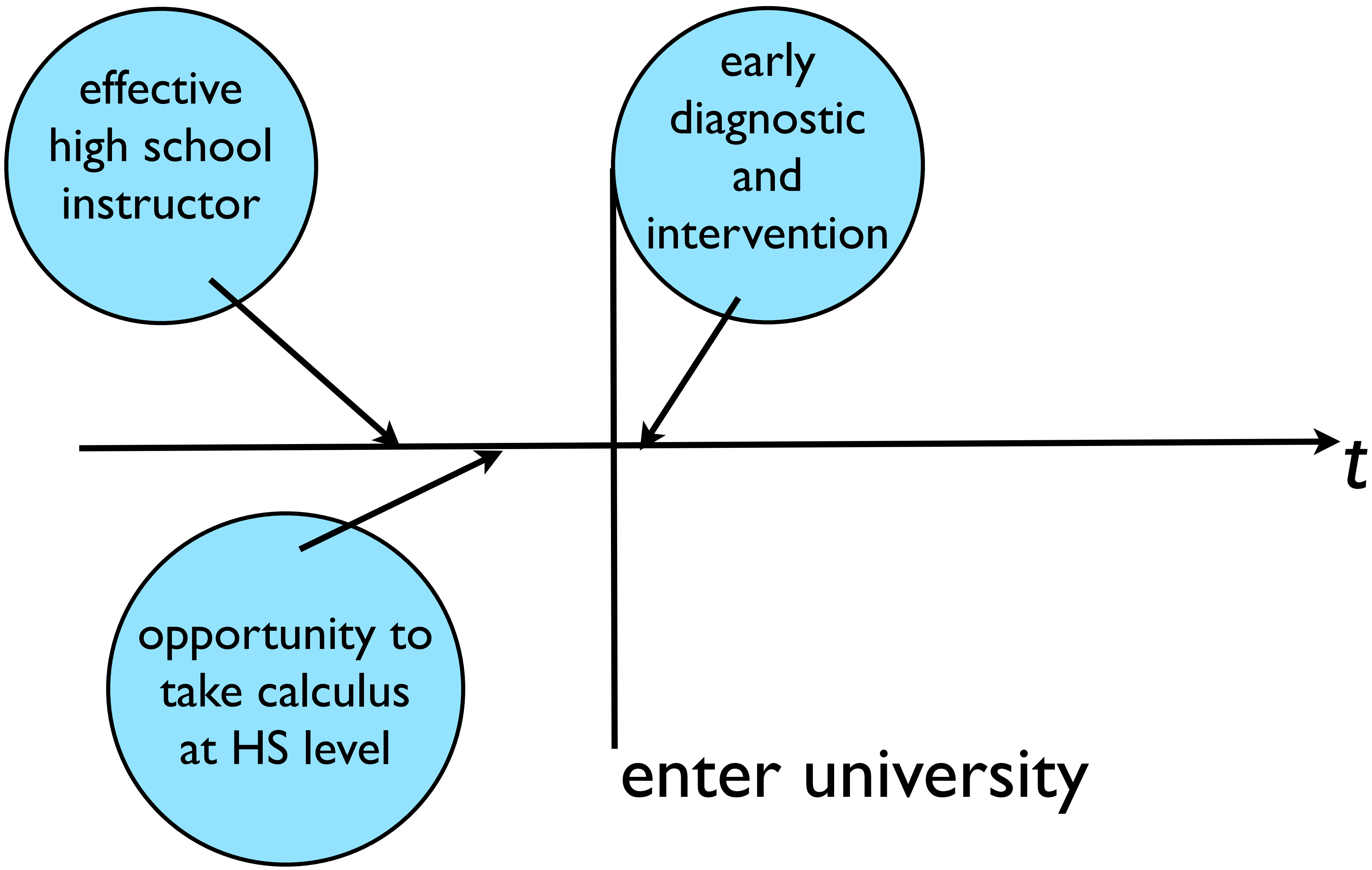
enter university

The ‘Good High School Teacher’ Effect: An Example

- In 2011, I noticed that students from Dr. J. H. Gillis Regional (local school) were performing very well.

2011	JHGILLIS	OTHER
number	12	48
top 6	5	1
avg. grade	86.7%	65.1%


effective high school instructor
early diagnostic and intervention
t
opportunity to take calculus at HS level
enter university

The state of the university calculus class: Questions?

Specific challenges

- Overwhelmingly, students struggle with mechanics of performing calculus, not concepts of calculus.
  - Algebra, algebra, algebra.

The nefarious trio

$$\sqrt{x+7} = \sqrt{x} + \sqrt{7}$$

$$(x+7)^2 = x^2 + 7^2$$

$$\frac{x}{x^2+7} = \frac{x}{x^2} + \frac{x}{7}$$

Canceling everything in sight

$$\frac{3x^2+2x-1}{2x-x^2} = \frac{3\cancel{x^2}+2x-1}{2x-\cancel{x^2}} = \frac{3+2x-1}{2x-1} = \frac{2+\cancel{2x}}{\cancel{2x}-1} = \frac{2}{-1} = -2.$$

Being cheap with parentheses

$$x \cdot (x+7)^2 = x \cdot x^2 + 14x + 49 = x^3 + 14x + 49$$

And a host of others...

Top Algebra Errors Made by Calculus Students

by **Thomas L. Scofield**
Assistant Professor of Mathematics
Calvin College

Sept. 5, 2003

http://www.calvin.edu/~scofield/courses/materials/tae/

Getting the student ready

- Over a decade ago, regional mathematics professors convened to create a calculus readiness document for entering students.

Preparing for University Calculus

Prepared by the APICS Committee
on Mathematics and Statistics

Edited by Robert Dawson

May 9, 2007

Disclaimer: This booklet is intended to give prospective students an idea of what a typical introductory calculus course at a university in the Atlantic region is like. Readers should note, however, that there are differences between calculus courses at different universities, and even within universities.

http://cs.smu.ca/apics/calculus/CalcPrep1.pdf

3 The Math You'll Need

In this section, we present over 175 questions that cover the absolute minimum of mathematical skills that you will need for university calculus. We give worked answers for a few; the answers to the rest are in the back of the booklet.

There are important topics in high school mathematics that are not covered here, and there may even be some things you'll need for calculus that we have missed. A good student should be able to answer significantly more challenging problems than these. However, if you understand all of this material, you will be reasonably well prepared for calculus.

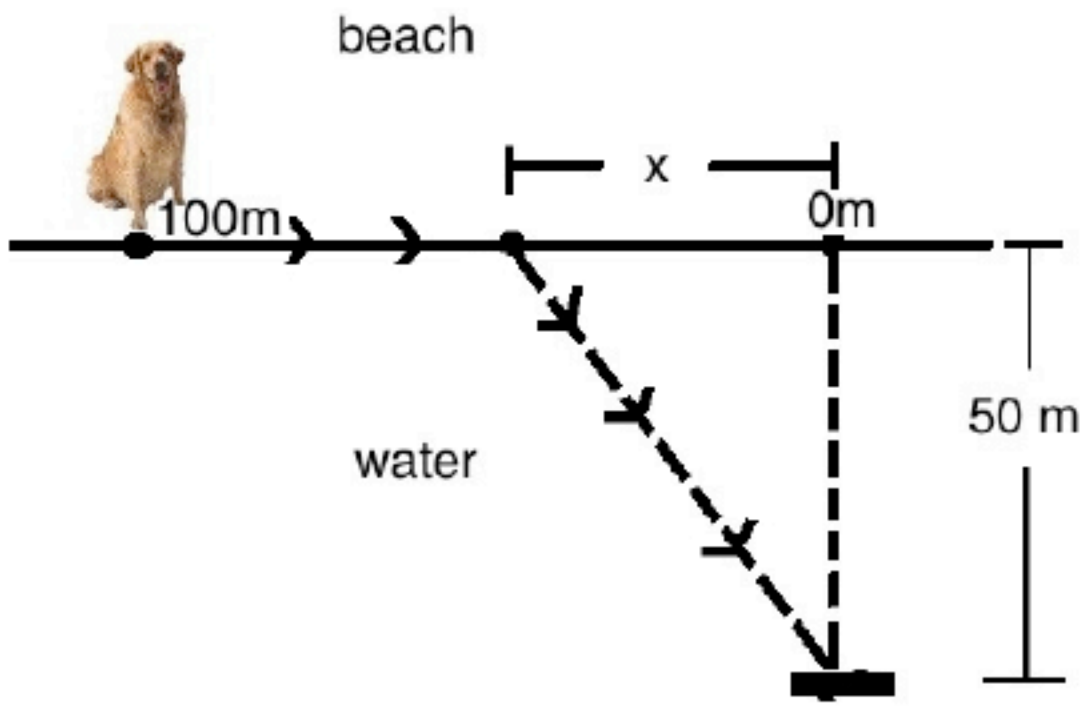
3.1 Arithmetic

You should be able to do basic arithmetic without a calculator, including operations on fractions, negative numbers, and decimals. You should be able to compute simple powers and roots. This material, which is from the elementary and junior

$$\vdots$$

An example

(10 pts) A dog wants to retrieve a stick thrown into the ocean. The stick is sitting in the water, 50 m out from shore and 100 m along the shore from the dog's position. The dog can run at 4 m/s and swim at 2 m/s. The dog retrieves the stick by running along the shore, then entering the water at some point, running and swimming in straight lines (see diagram).



Find the location the dog should enter the water to **minimize** the time to get the stick.

- difficulty I: making functions work for you

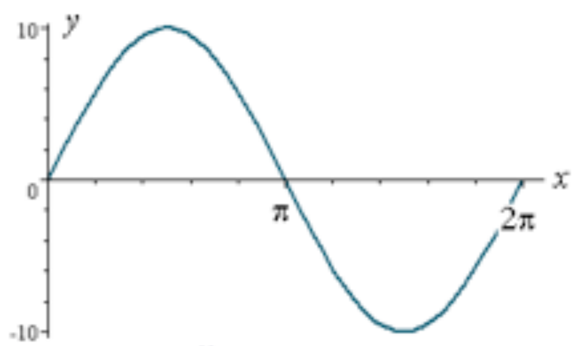
$$y = \sin(x)$$

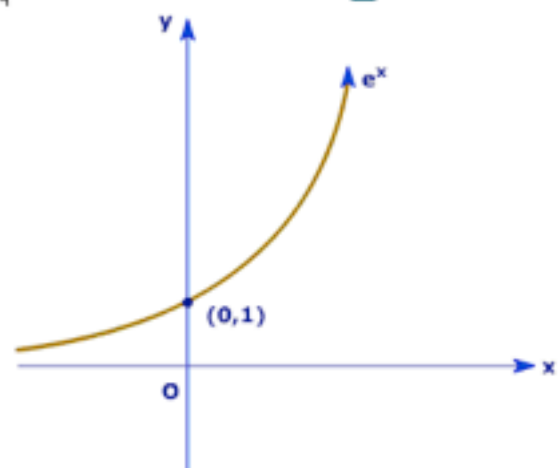
$$y = \cos(x)$$

$$y = e^{2x}$$

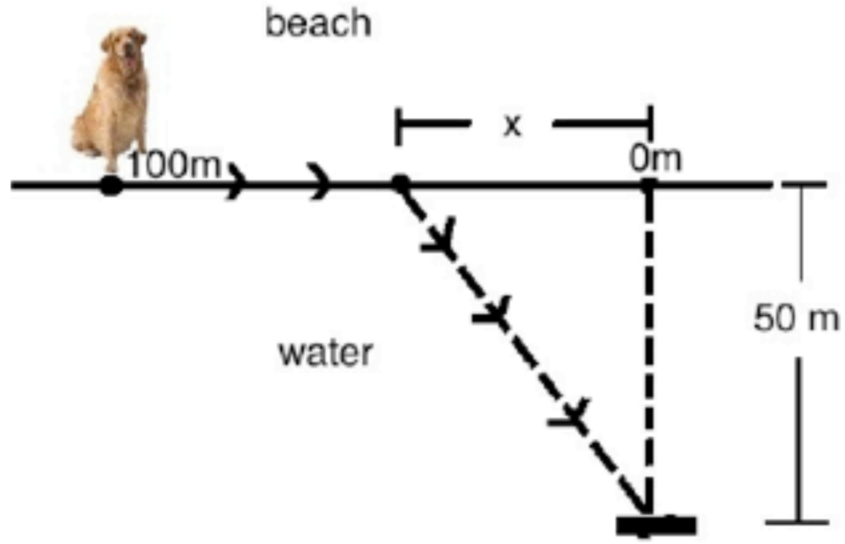
$$y = x^2 - 6x + 4$$

$$y = \arctan(1/x)$$

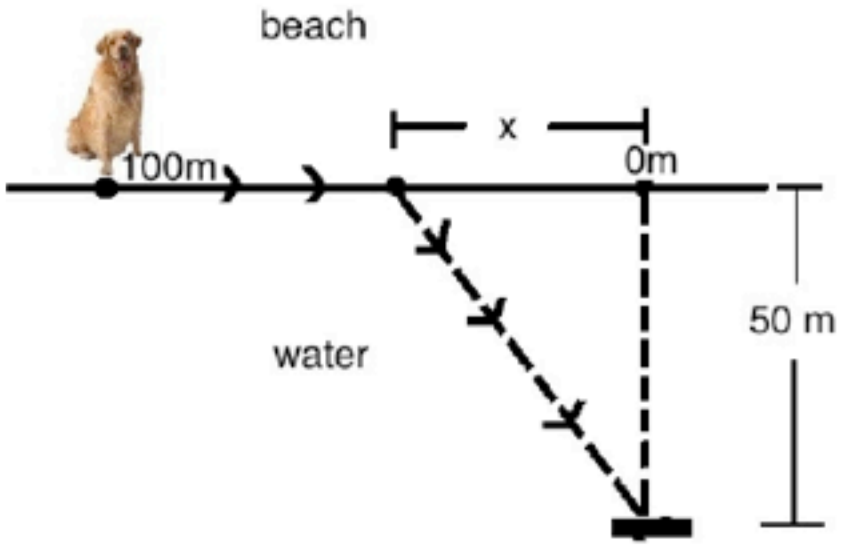




versus...



$T(x)$, time to get stick based on various entry points x



(pythagorean theorem)

$$T = \frac{100 - x}{4} + \frac{\sqrt{x^2 + 50^2}}{2}$$

(run) (swim)

$$T'(x) = \frac{-1}{4} + \frac{1}{4}\left(x^2 + 50^2\right)^{-1/2} \cdot 2x$$

$$T'(x) = \frac{-1}{4} + \frac{x}{2\sqrt{x^2 + 50^2}}$$

-comfort converting radical and exponent form

-need to know there's a friendlier form for the algebraic solution down the road

$$T'(x) = 0 \to \frac{-1}{4} + \frac{x}{2\sqrt{x^2 + 50^2}} = 0$$

$$\frac{1}{4} = \frac{x}{2\sqrt{x^2 + 50^2}}$$

$$2\sqrt{x^2 + 50^2} = 4x$$

-need to have a toolbox of available manipulations

-cross-multiplying percolates up as reasonable way to rewrite

$$\sqrt{x^2 + 50^2} = 2x$$

-(avoid trap of x + 50 = 2x)
-consider squaring both sides as the logical next step

$$x^2 + 50^2 = 4x^2$$

$$\frac{2500}{3} = x^2$$

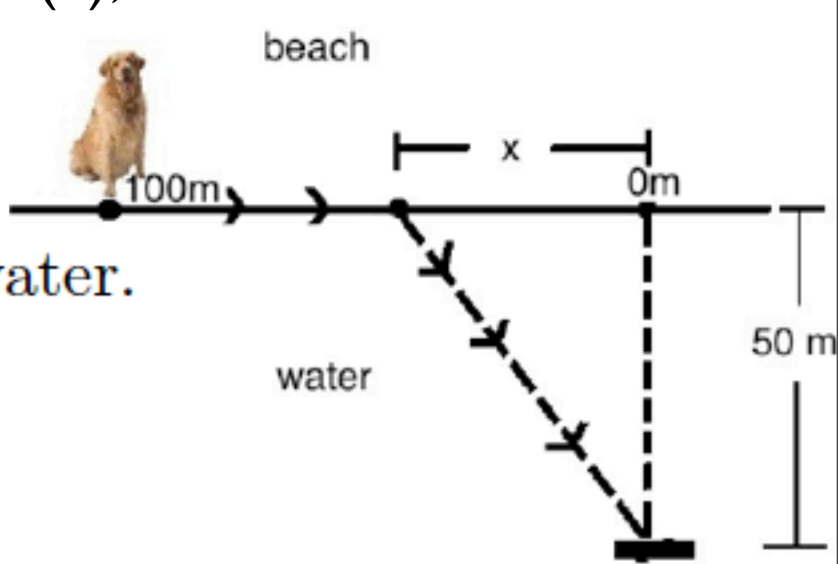
straightforward algebra to see the problem out

$$x = \pm\sqrt{\frac{2500}{3}} \approx 28.86$$

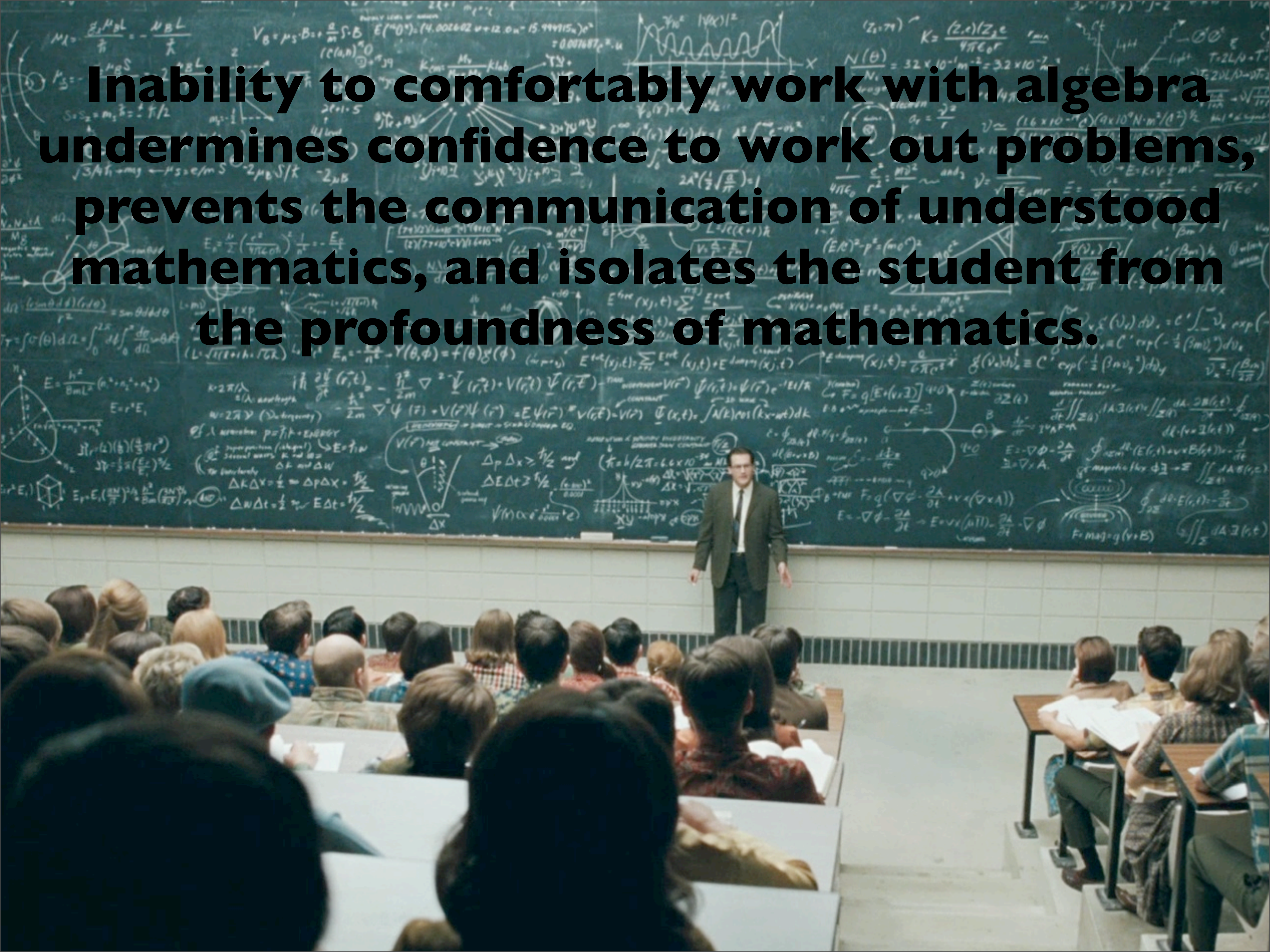
(discard the nonsensical negative root)

After testing endpoints [0,100] via global optimization in function T(x), we find...

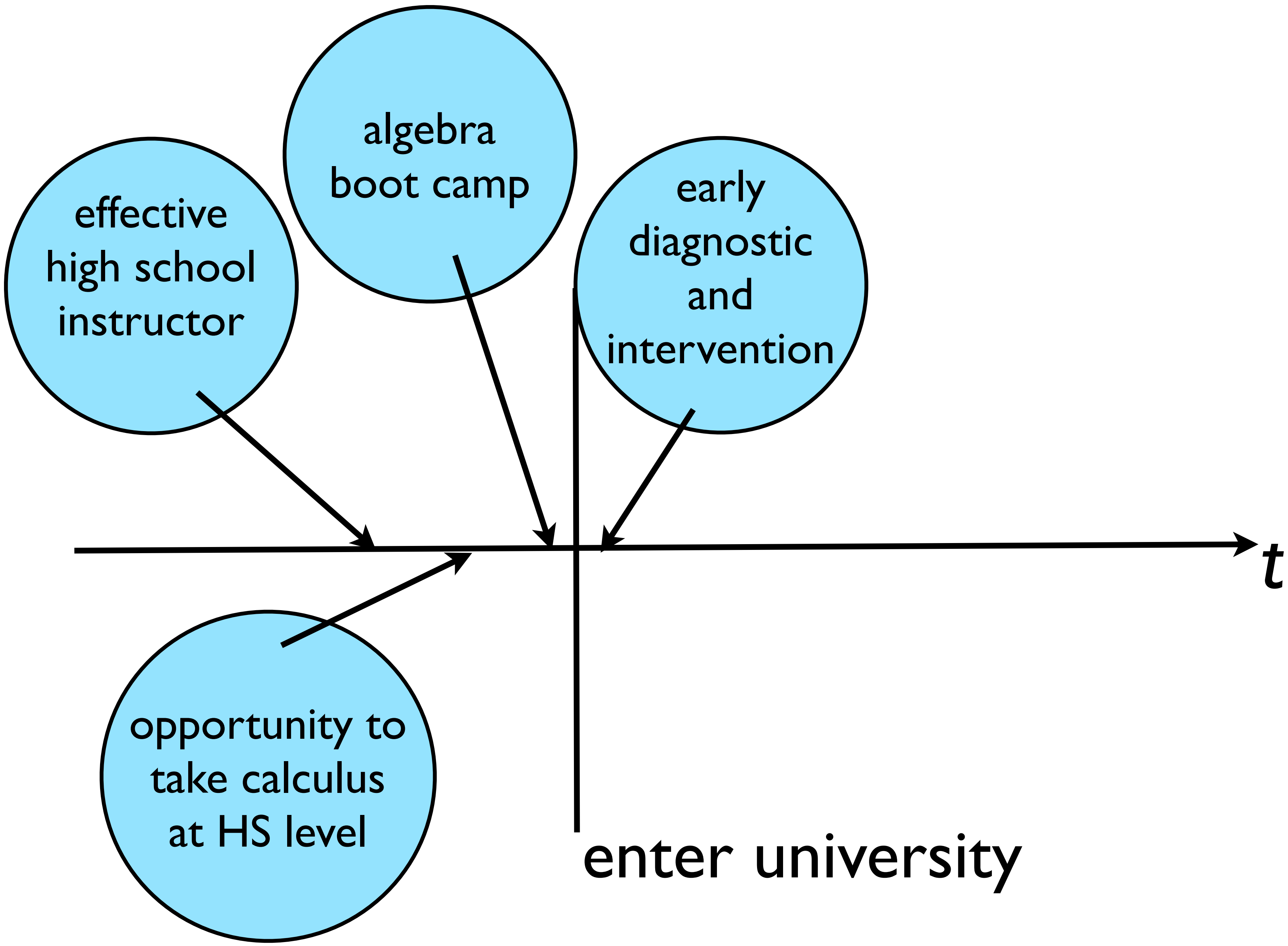
The dog should run down the shore 71.14 m, then enter the water.



(it will take about *T*(28.86) = 46.65 sec)



Inability to comfortably work with algebra undermines confidence to work out problems, prevents the communication of understood mathematics, and isolates the student from the profoundness of mathematics.


effective
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algebra
boot camp
early
diagnostic
and
intervention
opportunity to
take calculus
at HS level
t
enter university

How do we define success for students in first-year calculus?

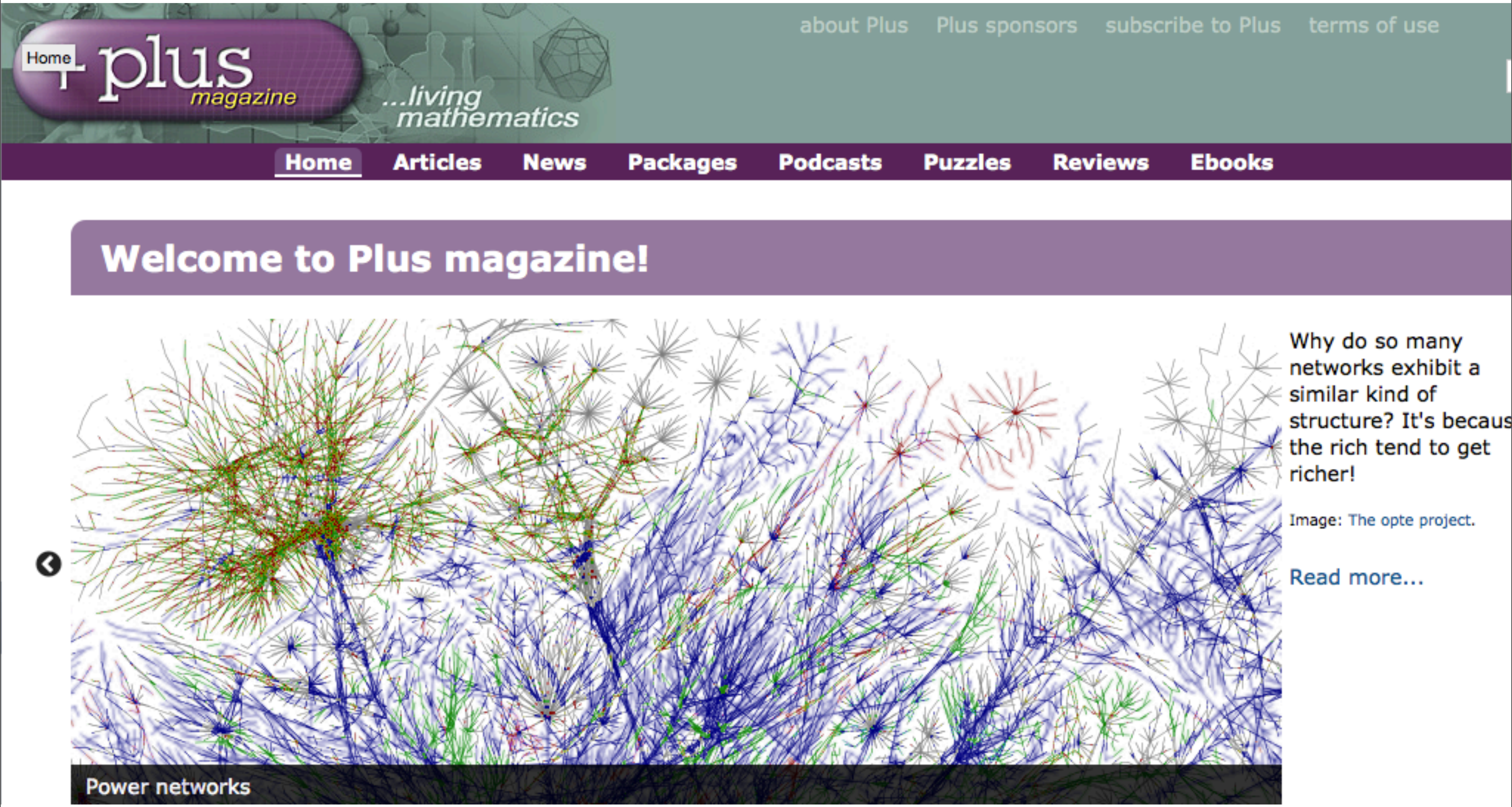
Motivating the Math

- Apart from ability, *interest* in mathematics is very rare for entering calculus students.
    - 2/124 listed it as their likely major (September, 2014)

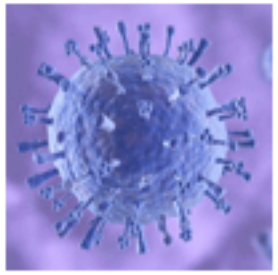
Motivating the Math

- Expose students as early and often as possible to
  - relatable, ‘uncooked’ applications
  - the world of research mathematics
    - not nearly as esoteric as it’s made out to be

Method 1: Online Resources



https://plus.maths.org/content/



Classroom activity: Build your own disease — Explore how to model the spread of an infectious disease.

Maths and climate change: the melting Arctic — The Arctic ice cap is melting fast and the consequences are grim. Mathematical modelling is key to predicting how much longer the ice will be around and assessing the impact of an ice free Arctic on the rest of the planet.

Sex, evolution and parasitic wasps — Some things are so familiar to us that they are simply expected, and we may forget to wonder why they should be that way in the first place. Sex ratios are a good example of this: the number of men and women in the world is roughly equal, but why should this be the case? A mathematical model provides an answer.

Pools of blood — A biologist has developed a blood test for detecting a certain minor abnormality in infants. Obviously if you have blood samples from 100 children, you could find out which children are affected by running 100 separate tests. But mathematicians are never satisfied by the obvious answer. Keith Ball uses information theory to explain how to cut down the number of tests significantly, by pooling samples of blood.

Matrix: Simulating the world, Part I — If you've ever watched a flock of birds flying at dusk, or a school of fish reacting to a predator, you'll have been amazed by their perfectly choreographed moves. Yet, complex as this behaviour may seem, it's not all that hard to model it on a computer. Lewis Dartnell presents a hands-on guide for creating your own simulations — no previous experience necessary.

Modelling, step by step — Why can't human beings walk as fast as they run? And why do we prefer to break into a run rather than walk above a certain speed? Using mathematical modelling, R. McNeill Alexander finds some answers.

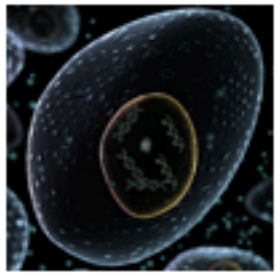
Guilt counts — Guilt, so some people have suggested, is what makes us nice. When we do someone a favour or choose not to exploit someone vulnerable, we do it because we fear the guilt we'd feel otherwise. A team of scientists have recently month produced some new results in this area, using a model from psychological game theory.

- details often not needed - just want to inspire curiosity, purpose, relevance

Counting calories — Struggling with that new year's resolution to lose a few pounds? Weight not dropping off as fast as you'd expected? A new mathematical model has some good news and some bad news for you. Which would you like to hear first?

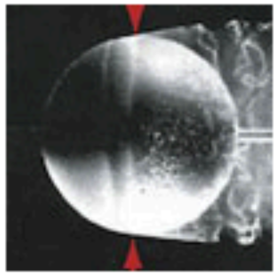
What can birds tell us about flying through ash clouds? — Why does a financial mathematician think about birds when trying to understand the grounding of aeroplanes after the Icelandic volcano eruption?

Baby robots feel the love — Researchers have unveiled the first prototypes of robots that can develop emotions and express them too. But how do you get emotions into machines that only understand the language of maths?

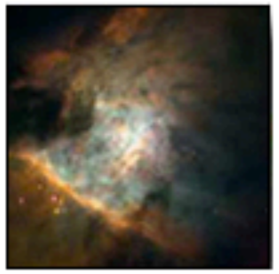


Modelling cell suicide — This article sheds light on suicidal cells and a mathematical model that could help fight cancer.

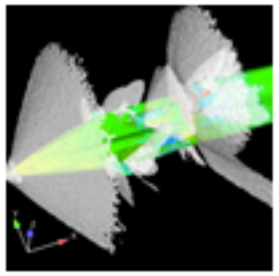
Uncoiling the Spiral: Maths and Hallucinations — Think drug-induced hallucinations, and the whirly, spirally, tunnel-vision-like patterns of psychedelic imagery immediately spring to mind. So what can these patterns tell us about the structure of our brains?



A fly walks around a football — What makes a perfect football? We find out that it's all down to the ball's surface.



Understanding turbulence — The study of turbulence is used to understand a range of phenomena from the simple squirting of a jet of water to the activity of the sun.



Supersonic Bloodhound — In 1997 Andy Green was the first to break the sound barrier in his car Thrust SSC. Now he and his team want to push things even further with a car called Bloodhound, designed to reach the dizzy heights of 1,000mph, about 1.3 times the speed of sound. This article explains how maths is used to build this car.

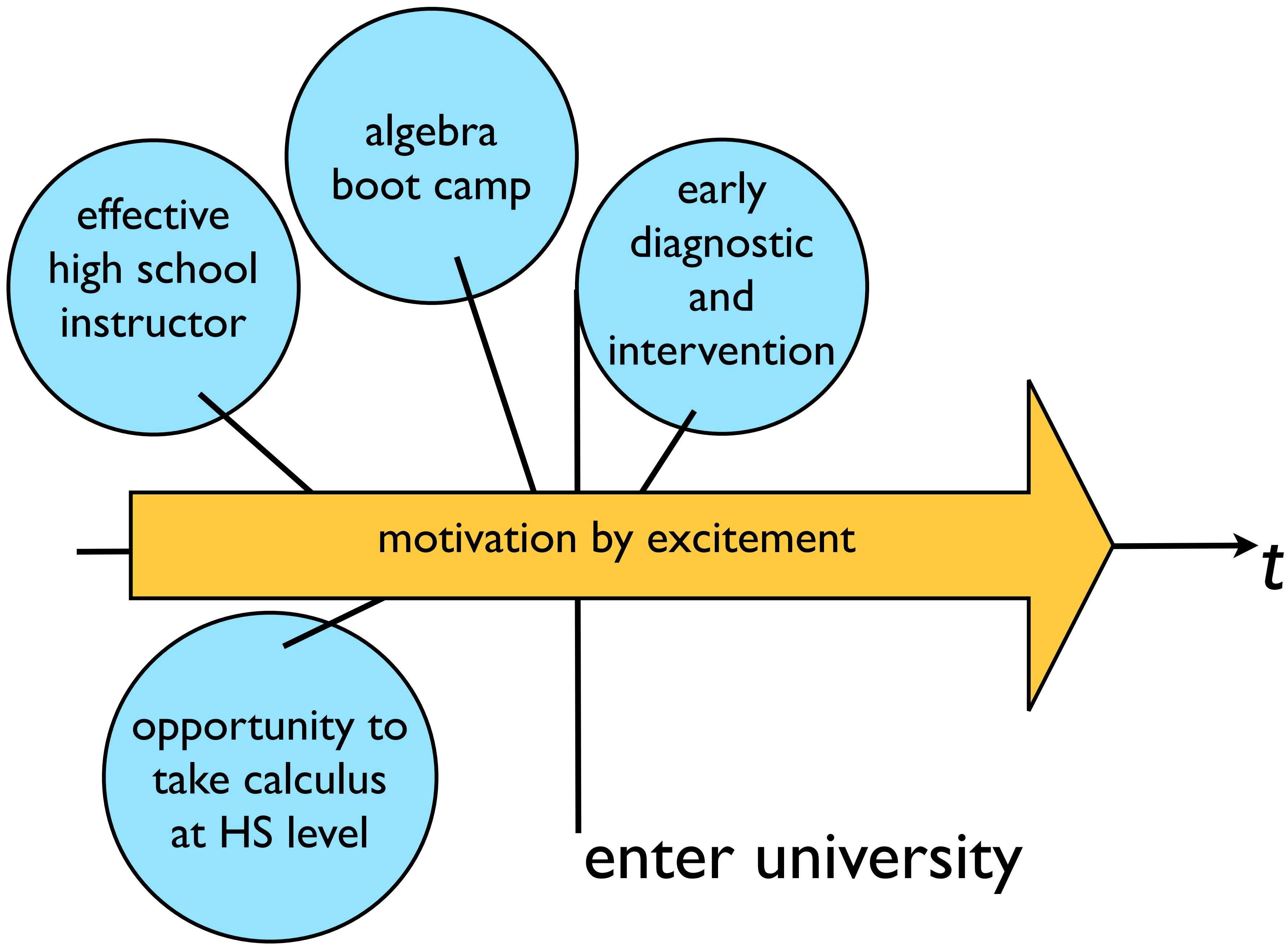
How maths can make you rich and famous — Understanding the behaviour of fluids and gases is one of the biggest open problems in maths.

Career interview: Aerodynamicist — *Plus* talks to Christine Hogan who studied aerodynamics planning to become a member of a Formula One team.

Method II: Connecting to Math Researchers

- Most university math departments in the region care deeply about outreach to high schools
- Encourage students to consider mathematics as a potential major
- Traditionally difficult access: schools avoiding university sales-pitches

1400
1200
1000
800
600
400
200
0
0
500
1000
1500
Relative Position of Neighbours
distance, in BL
distance, in BL
6
4
2
0
-2
-4
-6
-5
0
5
1
0.8
0.6
0.4
0.2
0
average neighbour direction
Att
Al
R
repulsion vector


algebra
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Thank you for your time.
Questions?

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