

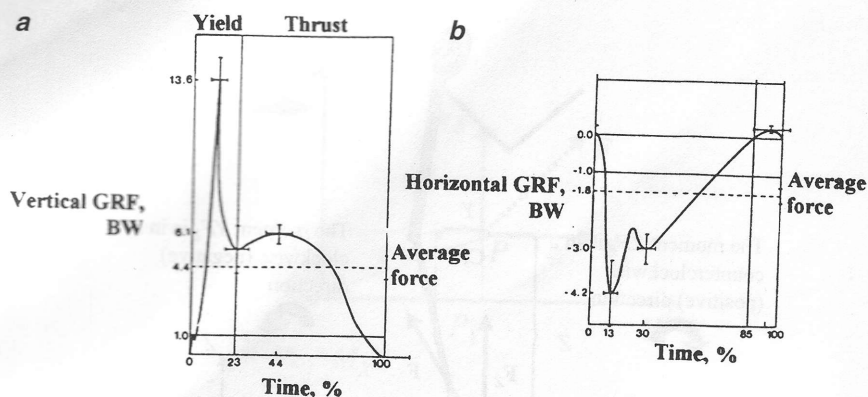


Figure 1.9 (a) Vertical and (b) horizontal components of the ground reaction force (GRF) exerted on the athlete's body in the running forward somersault, based on average data from eight experienced gymnasts. The force values are normalized in body weights (BW); the time is expressed as a percentage of total support duration.

Reprinted, by permission, from D.I. Miller and M.A. Nissinen, 1987, "Critical examination of ground reaction forces in the triple jump," *International Journal of Sport Biomechanics* 3:189–206.

■ ■ ■ FROM THE LITERATURE ■ ■ ■

Takeoff Forces in the Running Forward Somersault

Source: Miller, D.I., and M.A. Nissinen. 1987. Critical examination of ground reaction force in the running forward somersault. *Int. J. Sport Biomech.* 3: 189–206.

The authors studied the ground reaction forces elicited by male gymnasts during a running forward somersault. Very large braking forces in the anteroposterior direction exceeding four body weights (BW) were registered (figure 1.9). In the vertical direction, an initial impact force of 13.6 BW was recorded, followed by a second peak of 6.1 BW. The average duration of the support was 135 ms.

1.1.2 Couples

A *force couple*, or simply a *couple*, consists of two equal, opposite, and parallel forces, $\mathbf{F}$ and $-\mathbf{F}$, that are acting concurrently at a distance d apart (figure 1.10). For instance, two equal, parallel, and opposite forces applied by the

■ ■ ■ FROM THE LITERATURE ■ ■ ■

Moment Arm of a Muscle Force

Source: Pandy, M.G. 1999. Moment arm of a muscle force. *Exerc. Sport Sci. Rev.* 27: 79–119.

The moment arm of a muscle about an axis $O-O$ was defined as the moment applied by a muscle force of magnitude 1, $\hat{\mathbf{F}}$.

$$\mathbf{r}^M = \frac{\mathbf{M}_{oo}}{F} = (\mathbf{U}_{oo} \cdot \mathbf{r} \times \hat{\mathbf{F}}) \mathbf{U}_{oo} \quad (1.28)$$

Equation 1.28 can easily be derived from equations 1.23 and 1.27. The author performed a detailed mechanical analysis of the muscle moment arms in the joints with one and multiple degrees of freedom. The moment arms were determined about the instantaneous screw axes (ISA) for joints with intersecting and nonintersecting joint axes. When movement of the bones is restricted to a single plane and the muscle acts in the plane of bone movement, the magnitude of the moment arm is equal to the perpendicular distance from the instantaneous center of rotation of the joint to the line of action of the muscle force. In all other cases, the magnitude of the moment arm equals the perpendicular distance between the ISA and the line of action of the muscle multiplied by the sine of the angle between these two lines.

hands to a steering wheel during driving or by the thumb and index finger when turning a nut on a bolt form a couple. The plane in which the forces lie is called the plane of the couple. Because the vector sum of the forces constituting a couple is zero in every direction, the couple does not have a tendency to translate the body on which the forces act. The couple makes the body rotate. The measure of this tendency is called the *moment of a couple* or *torque*. A couple C that produces moment of couple $\mathbf{M}_C$ is customarily called the “couple $\mathbf{M}_C$ ” for brevity. This is similar to calling a person by his or her function, for instance, “teacher” or “plumber.”

Consider two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ applied to a rigid body at the corresponding points A and B (figure 1.10). Let $\mathbf{r}_A$ and $\mathbf{r}_B$ be the position vectors of the points A and B , respectively, and $\mathbf{r}$ is the position vector of A with respect to B ($\mathbf{r} = \mathbf{r}_A - \mathbf{r}_B$). The vector $\mathbf{r}$ is in the plane of the couple but need not be perpendicular to the forces $\mathbf{F}$ and $-\mathbf{F}$. The combined moment of the two forces about O is

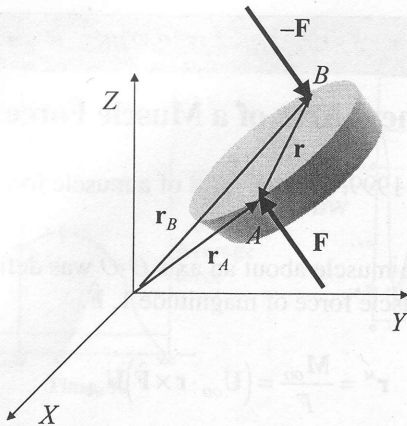


Figure 1.10 Two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ a distance d apart constitute a couple. The magnitude of their combined moment does not depend on the distance to any point, and hence the couple can be translated to any location in a parallel plane or in the same plane.

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \mathbf{r} \times \mathbf{F} \quad (1.29)$$

The product $\mathbf{r} \times \mathbf{F}$ is independent of the vectors $\mathbf{r}_A$ and $\mathbf{r}_B$; that is, it is independent of the choice of the origin O of the coordinate reference. Hence, the moment of couple $\mathbf{M}_C = \mathbf{r} \times \mathbf{F}$ does not depend on the position of O and has the same magnitude for all moment centers. The magnitude of the moment of couple is $M_C = Fd$, where d is the moment arm of the couple—the shortest distance between the lines of action of the two involved parallel forces. The moment arm d is perpendicular to the lines of force action. The magnitude of the moment of couple does not depend on the direction of the applied forces. The force magnitude F and the moment arm d determine the couple.

Because the moment of a couple $\mathbf{M}_C$ is the same for all moment centers and remains unchanged under parallel displacements, it is also called the *free moment*. Two couples having the same moment are equivalent: they produce the same effect on the rigid body on which they act. For instance, two couples $M_{C1} = Fd$ and $M_{C2} = 2F \cdot 0.5d$, if acting in the same or parallel planes, are equivalent, although the acting forces are different.

Couples and moments that they generate can be represented by vectors. A *couple vector* is normal to the plane of the couple. The sense of the couple vector is determined by the right-hand rule. By convention, counterclockwise couples are considered positive, and clockwise couples negative. Couple vectors obey the ordinary rules of vector algebra. For instance, the sum of two couples of moments $\mathbf{M}_1$ and $\mathbf{M}_2$ is a couple of moment $\mathbf{M}_C$ ($\mathbf{M}_C = \mathbf{M}_1 + \mathbf{M}_2$).

Couple vectors can be resolved into the component vectors $\mathbf{M}_x$, $\mathbf{M}_y$, and $\mathbf{M}_z$ along the axes of the coordinates. The component vectors represent the couples acting in the planes that are perpendicular to the corresponding coordinate axes. For instance, the vector $\mathbf{M}_x$ represents a couple acting in the YZ plane. Couple vectors are *free vectors*; they can be freely translated in space provided that their orientation remains constant. If so desired, the origin of the reference frame can be selected as the point of application of a couple vector.

In summary, the turning effect of a force depends on the point of force application, while the turning effect of a couple does not depend on the place where the couple is exerted.

1.1.3 Transformation of Forces and Couples

Because forces and couples are vectors, they can be transformed from one Cartesian coordinate system to another in the same manner as the coordinates themselves. In the biomechanics of human motion, a transformation from the global reference system $O-XYZ$ (e.g., fixed with a force plate) into a local system $O-xyz$ (fixed with a body or a body part) is quite common.

Let $\mathbf{F}_G$ be a column vector whose elements are the three components of force measured in a global system of coordinates $O-XYZ$ fixed, for instance, with a force platform. Let $\mathbf{F}_L$ be the same vector expressed in the local reference system $O-xyz$ fixed, for instance, with a body part. The global and local systems are related by rotation that is described by an orthogonal *rotation matrix* $[R]$. The coordinates of point P in the two reference systems are related by the equations

The rest of this section will likely only result in confusion

$$\mathbf{P}_G = [R]\mathbf{P}_L \quad (1.30a)$$

and

$$\mathbf{P}_L = [R]^T\mathbf{P}_G \quad (1.30b)$$

where $\mathbf{P}_G$ and $\mathbf{P}_L$ are the coordinates of point P in the global and local systems, respectively. These equations are equations 1.11 and 1.12 from *Kinematics of Human Motion*. The force vectors $\mathbf{F}_G$ and $\mathbf{F}_L$ measured correspondingly in the global and local reference frames are also related by similar equations:

$$\mathbf{F}_G = [R]\mathbf{F}_L \quad (1.31a)$$

and

$$\mathbf{F}_L = [R]^T\mathbf{F}_G \quad (1.31b)$$

Hence, to transform the force $\mathbf{F}_G$ from the global system of coordinates into the local coordinates, the force vector should be multiplied by the transpose of the rotation matrix, $[R]^T$.

Transformation of moments and couples is done in a manner similar to the transformation of forces. We discuss this issue on the intuitive level, without a strict mathematical proof. Consider a force $\mathbf{F}_G = (F_x, F_y, F_z)^T$ expressed in the global reference system. If $\mathbf{r}_G$ is a vector from the origin of the global coordinate system to the line of action of the force, then the moment of the force $\mathbf{F}_G$ with respect to the origin is given by the vector product $\mathbf{M}_G = \mathbf{r}_G \times \mathbf{F}_G$ (this is essentially equation 1.8). In the local system of coordinates, the moment is represented by

$$\mathbf{M}_L = \mathbf{r}_L \times \mathbf{F}_L \quad (1.32)$$

The vectors $\mathbf{r}_L$ and $\mathbf{F}_L$ are evidently equal to $\mathbf{r}_L = [R]^T \mathbf{r}_G$ and $\mathbf{F}_L = [R]^T \mathbf{F}_G$. Thus,

$$\mathbf{M}_L = ([R]^T \mathbf{r}_G) \times ([R]^T \mathbf{F}_G) \quad (1.33)$$

Because the rotation matrix $[R]$ is orthogonal, the magnitudes of the vectors $\mathbf{r}_G$ and $\mathbf{F}_G$ do not change as a result of the transformation in equation 1.33. For that reason, the magnitude of their cross product also does not alter, $M_L = M_G$. Only the orientation of the moment of force changes. Consequently, the components of the moment in the global and local system differ from each other. The vector of the moment, however, is still normal to the plane containing $\mathbf{r}_G$ and $\mathbf{F}_G$. Because the orientation of this plane in the two reference systems differs by the rotation $[R]$, the difference in the orientation of the normals to the plane is also defined by $[R]$. Therefore,

$$\mathbf{M}_G = [R] \mathbf{M}_L \quad (1.34a)$$

and

$$\mathbf{M}_L = [R]^T \mathbf{M}_G \quad (1.34b)$$

Equations 1.34a and 1.34b are similar to equations 1.30a and 1.30b. Hence, the moment $\mathbf{M}_G$ is transformed to the local reference system according to the same law of transformation as the coordinates themselves.

1.1.4 Replacement of a Given Force by a Force and a Couple

Resume reading here!

Any force $\mathbf{F}$ acting on a rigid body produces two effects: it tends to push or pull the body in the direction of the force, and it tends to rotate the body about any axis that does not intersect the line of force action. According to the principle of transmissibility, the force can be moved along its line of action without changing its effect on the rigid body on which it acts. It cannot, however, be moved away from the original line of action without modifying its effect on

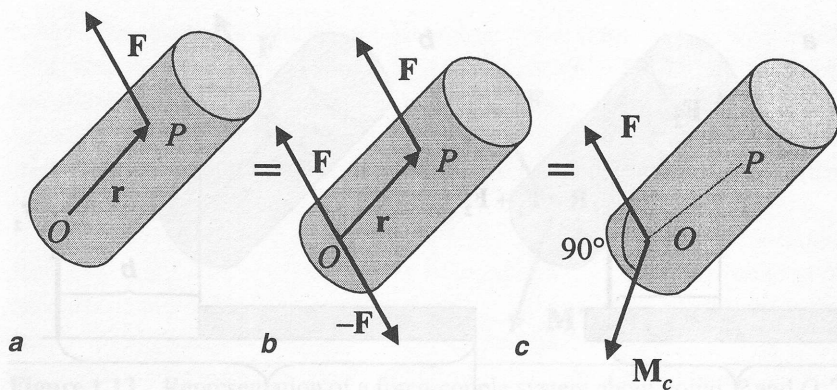


Figure 1.11 A force $\mathbf{F}$ acting on a rigid body at a point P (a) is replaced by an equal force shifted to a point O and a corresponding couple (c). In (b), two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ are added at a point O . In (c), the vector of couple $\mathbf{M}_c$ is normal to the plane containing vector $\mathbf{F}$ and the point P .

the body. Such a parallel displacement changes the moments of force that force $\mathbf{F}$ generates. The parallel displacement of a force can, however, be done if the change in the moments of the force is compensated by a couple (figure 1.11).

Any force $\mathbf{F}$ can be replaced by a parallel force of the same magnitude applied at an arbitrary point O and a couple of magnitude $M_c = Fd$, where d is the moment arm from O to the original position of the force. Such a representation is called a force-couple system.

Consider a force $\mathbf{F}$ acting on a rigid body at a point P (figure 1.11a). To move the force $\mathbf{F}$ away from its original line of action to a point O , we attach at O two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$. These forces can be added because they do not change the state of equilibrium or the motion of the body and cancel each other. The equal and parallel forces $\mathbf{F}$ (applied at P) and $-\mathbf{F}$ (applied at a point O) constitute a couple $\mathbf{M}_c$. The moment of couple $\mathbf{M}_c$ is equal to the moment of force $\mathbf{F}$ about point O . Hence, the force $\mathbf{F}$ (applied at the point P) can be replaced by an equal force applied at an arbitrary point O and a couple $\mathbf{M}_c = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is a position vector of P with respect to O . The couple is added to compensate for the change in the moment of force. The plane of the couple coincides with the plane containing the vectors $\mathbf{r}$ and $\mathbf{F}$. Therefore, the vector of the couple $\mathbf{M}_c$ is perpendicular to this plane. We can conclude that any force $\mathbf{F}$ acting on a rigid body at a point P can be replaced by the same force acting at another point O and the corresponding couple represented by a vector $\mathbf{M}_c$ perpendicular to $\mathbf{F}$. Conversely, if a force $\mathbf{F}$ and a couple $\mathbf{M}_c$ are mutually perpendicular, a single equipollent force (and a couple of zero magnitude) can replace them.

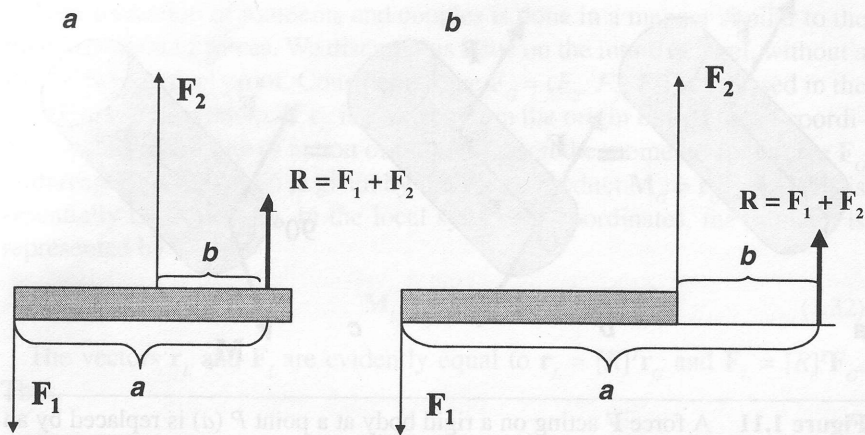


Figure 1.12 Two parallel, opposite, and unequal forces, F_1 and F_2 , are acting on a rigid body. The magnitude of the resultant R equals the algebraic sum of F_1 and F_2 . The distances from the action line of R to the action lines of F_1 and F_2 are inversely proportional to the magnitudes of the forces, $F_1 a = F_2 b$. (The reader is encouraged to prove this statement.) The resultant force R produces the same effect as the forces F_1 and F_2 combined. (a) This resultant force can actually be applied to the body (instead of pulling two ropes in opposite directions, for example, just one rope can be pulled with the same effect). (b) This resultant force R can be computed but cannot be actually applied to the body because the line of its action is outside the body. In general, when two parallel forces have similar senses, their resultant lies between them; when they have different senses, the resultant lies outside the space between them. When the forces are opposite and equal, they form a couple, and their resultant force is zero.

The preceding statement does not necessarily mean that the equipollent force can actually be exerted on the body. If the magnitude of the original couple is large and the size of the rigid body is small, the point of application of the resultant force that corresponds to a zero couple lies outside the body (figure 1.12b). Nevertheless, the equipollent force, which in this case is a purely theoretical construct, can be computed.

May be too intense!

1.1.5 Replacement of a Given Force and Couple by Another Force and Couple: Invariants in Statics

Consider a force F applied at a point P and a couple C that exerts a moment M_C . The moment of couple is a free vector and can be applied anywhere. For convenience, we draw it from the point P (figure 1.13). As we just discussed, when the point of force application is changed from P to another point O , the

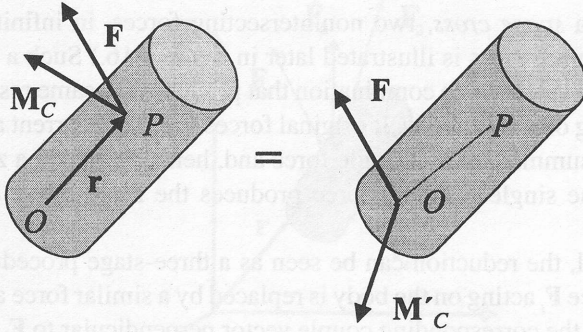


Figure 1.13 Representation of a force-couple system about points P and O . The system is initially given by a force $\mathbf{F}$ and a couple of moment $\mathbf{M}_C$ at P and then moved to a point O .

force $\mathbf{F}$ at P should be replaced by a similar force $\mathbf{F}$ at O and a corresponding couple $\mathbf{C}'$. The moment of couple $\mathbf{C}'$ equals the moment of $\mathbf{F}$ about O . Hence, with the force $\mathbf{F}$ applied at a new location O , two couples, $\mathbf{C}$ and $\mathbf{C}'$ are exerted on the body. Their combined effect is equal to the moment of a single couple $\mathbf{M}'_C$. The moment $\mathbf{M}'_C$ can be found by adding the moment of $\mathbf{F}$ about O to $\mathbf{M}_C$:

$$\mathbf{M}'_C = \mathbf{M}_C + \mathbf{r} \times \mathbf{F} \quad (1.35)$$

where $\mathbf{r}$ is the position vector from O to P . Thus, any force $\mathbf{F}$ and couple $\mathbf{M}_C$ exerted at a point P on a rigid body can be replaced by an equal and parallel force applied at an arbitrary point O and a couple $\mathbf{M}'_C$, provided that equation 1.35 is satisfied.

The magnitude of force F is *invariant* (does not change) under a parallel translation to a new position. The dot product $\mathbf{F} \cdot \mathbf{M}_C$ is also invariant; it does not alter when the point of force application changes, $\mathbf{F} \cdot \mathbf{M}_C = \mathbf{F}' \cdot \mathbf{M}'_C$. Because the vectors $\mathbf{F}$, $\mathbf{r}$, and $\mathbf{F}'$ are in the same plane, the mixed triple product $\mathbf{F}' \cdot (\mathbf{r} \times \mathbf{F})$ equals zero, and therefore

$$\mathbf{F}' \cdot \mathbf{M}'_C = \mathbf{F}' \cdot (\mathbf{M}_C + \mathbf{r} \times \mathbf{F}) = \mathbf{F}' \cdot \mathbf{M}_C + \mathbf{F}' \cdot (\mathbf{r} \times \mathbf{F}) = \mathbf{F} \cdot \mathbf{M}_C \quad (1.36)$$

Hence, the scalars F and $\mathbf{F} \cdot \mathbf{M}_C$ are invariant with respect to the choice of point of force application. If $\mathbf{F} \cdot \mathbf{M}_C = 0$, the system of forces can be reduced to a single force; if $F = 0$, the system is equivalent to a single couple.

Resume reading here!

1.1.6 Equivalent Force-Couple Systems: Varignon's Theorem

Any system of forces acting on a rigid body can be reduced to a single resultant force and a single resultant couple acting at a given point O . (It can also be

reduced to a *space cross*, two nonintersecting forces, in infinite number of ways. The space cross is illustrated later in figure 1.16.) Such a system is the simplest equivalent force combination that produces the same result as the real forces acting on a rigid body. If original forces $\mathbf{F}_i$ are concurrent at O , they can be directly summed up to a single force and, hence, generate a zero moment, $\mathbf{M}_O = 0$. The single resultant force produces the same effect as the forces it replaces.

In general, the reduction can be seen as a three-stage procedure: (1) Each original force $\mathbf{F}_i$ acting on the body is replaced by a similar force at an arbitrary point O and the corresponding couple vector perpendicular to $\mathbf{F}_i$. (2) The concurrent forces at O are added according to the parallelogram rule and reduced to one resultant force. (3) The couple vectors are also added according to the parallelogram law and thus reduced to one resultant couple vector (we can do this because the couple vectors are free vectors and can be moved to one point). The magnitude and direction of the resultant force are the same regardless of the selected point of force application. In contrast, the magnitude and direction of the resultant couple depend on the particular point selected.

Usually, the resultant force vector and the resultant couple vector are not mutually perpendicular. If they are, the resultant force-couple system can be reduced to a single equivalent force or, if the resultant force is zero, to a single couple. The resultant force vector and resultant couple vector are perpendicular to each other when all the original forces are either coplanar (act in the same plane) or parallel.

By the definition of equivalent force systems (see Mechanics Refresher, p. 2), the equivalent sets of forces acting on a rigid body produce the same effect. Therefore, they exert the same moment of force about any arbitrarily chosen point in space. In particular, *if a set of forces acting on a rigid body is reduced to one resultant force, the moment of the resultant force about any point O is equal to the moment of the original force system about O .* This statement is credited to the French scientist Pierre Varignon (1654–1722) and is known as Varignon's theorem, or the *theorem of moments*. For easy memorization, the theorem can be simplified: *the sum of moments equals the moment of the resultant*. Varignon's theorem is valid for both coplanar and parallel force systems. It is also valid for concurrent forces (figure 1.14). The orthogonal force components of a force $\mathbf{F}$ applied at a point P are the concurrent forces. Therefore, the moment of a force $\mathbf{F}$ about O is equal to the sum of the components of that force, F_x , F_y , and F_z , about O .

Varignon's theorem is widely used to analyze parallel force systems. For parallel forces, the moment of a resultant force about any point is equal to the sum of the moments of the original forces about the same point. Because the moment of the resultant force about its line of action equals zero, the sum of the moments of all the contributing forces about this line is also zero. This

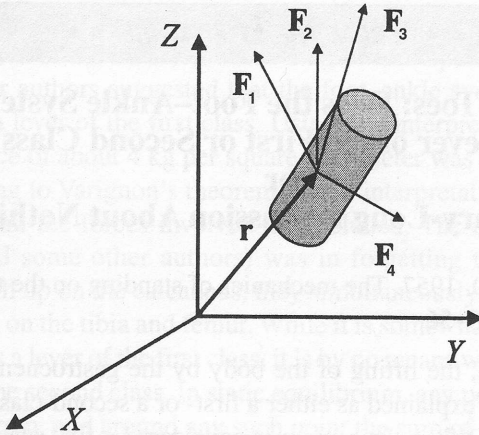


Figure 1.14 Proof of Varignon's theorem for concurrent forces. Consider several forces $\mathbf{F}_i$ ($i = 1, 2, \dots, n$) exerted on a rigid body at a point P . Denoting the position vector of P as $\mathbf{r}$ and applying the distributive property of vector products, we obtain $\mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \dots + \mathbf{r} \times \mathbf{F}_n = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n)$, which proves the theorem.

property of the resultant force of parallel force systems is used to find the location of the resultant force. In the biomechanics of human motion, the most common parallel forces are the gravity forces that act on different parts of the body. Varignon's theorem provides a tool for determining the center of gravity, a point at which the resultant gravity force acting on the entire body is exerted. When external contact forces are parallel, Varignon's theorem can be used to determine the point of application of the resultant.

A special case of a parallel force system called a *lever* deserves particular mention. A lever is a rigid body revolving about an axis, the *fulcrum*. Levers involve a system of three forces: a resistance force, an effort force, and the force that is exerted on the fulcrum. In human movement, the effort force is the force generated by a subject or a muscle. Levers are classified by the position of the fulcrum with respect to the resistance and effort forces. In *first-class levers*, the resistance force and the effort force are on opposite sides of the fulcrum, as in a pair of scissors or in a seesaw. In *second-class levers*, the resistance force is applied between the fulcrum and the effort force, as in a nutcracker or wheelbarrow. In *third-class levers*, the effort force is applied between the fulcrum and resistance. Such an arrangement is typical for muscles in the human body; the point of muscle insertion is between the joint (fulcrum) and the point of application of the resistance force, for example, the external force acting on the *end effector*. Levers are just systems with three parallel forces; hence, Varignon's theorem can be applied. In levers, the moments are

■ ■ ■ FROM THE LITERATURE ■ ■ ■

Rising on the Toes: Does the Foot–Ankle System Work As a Lever of the First or Second Class?

or

A Century-Long Discussion About Nothing

Source: Fenn, W.O. 1957. The mechanics of standing on the toes. *Am. J. Phys. Med.* 36: 153–56.

In earlier research, the lifting of the body by the gastrocnemius–soleus muscle group was explained as either a first- or a second-class lever system (figure 1.15). When the system is considered a first-class lever, the ankle joint is regarded as a fulcrum, the ground reaction force is treated as the resistance force, and the force exerted by the *Achilles tendon* on the calcaneus is the effort force. When the system is analyzed as a second-class lever, the fulcrum is at the ball of the foot, and the resistance (body weight) acts at the talus, that is between the fulcrum and the point of exertion of the Achilles tendon force.

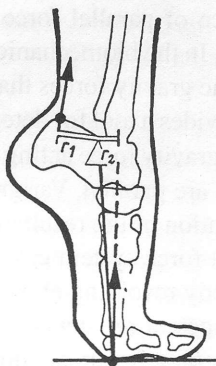


Figure 1.15 Rising on the toes can be modeled as a lever of the first or second class. Either approach is correct.

The discussion of this movement has a long history, starting with a paper by E. Weber published in 1846. Weber's problem was calculating the force exerted by the triceps surae when rising on the toes. For this analysis, he weighted the body using a lever attached to the belt of the subject and determined the load on the lever when the subject rose on his toes. In calculating the results, Weber considered the foot as a lever of the second class, with the fulcrum at the ball of the foot. He obtained a sur-

prisingly low value for the absolute muscle force (force per unit of cross-sectional area of the muscle), about 1 kg of force per square centimeter. Later, other authors suggested that the foot–ankle system should be regarded as a lever of the first class. Using this interpretation, an absolute muscle force of about 4 kg per square centimeter was obtained.

According to Varignon's theorem, either interpretation is correct, provided that all the forces involved are included. The real error made by Weber (and some other authors) was in forgetting that when the calf muscles pull up on the calcaneus, they simultaneously pull down with an equal force on the tibia and femur. While it is somewhat simpler to regard the ankle as a lever of the first class, it is by no means wrong to regard it as a lever of the second class. In static equilibrium, any point can be considered a fulcrum, and around any such point the sum of the clockwise moments equals the sum of the moments acting in the counterclockwise direction. The discussion of whether rising on the toes should be modeled as a first- or second-class lever system is beside the point.

usually computed with respect to the fulcrum, although this is not necessary for an equilibrium equation. The theorem is valid for moments computed about any point.

As mentioned previously, a force system acting on a rigid body can be reduced to a single resultant force if, and only if, the vector of the resultant couple $\mathbf{M}_c$ is perpendicular to the vector of the resultant force $\mathbf{F}$. In the three-dimensional case, this usually does not happen, and the original force system commonly cannot be reduced to a single resultant force or a single couple.

1.1.7 Wrenches No thanks I prefer using a hammer!

Representative paper: VanSickle et al. (1998)

In three dimensions, an arbitrary set of forces can be reduced to a resultant force and a corresponding couple. Innumerable equivalent force-couple representations are possible. These resultant forces would have the same magnitude and direction but different points of application, which can be selected arbitrarily. When the point of force application is changed, the corresponding couple also changes. Consider, as an example, figure 1.16a, where two forces, a vertical force $\mathbf{F}_v$ and a horizontal force $\mathbf{F}_h$, are exerted on a rigid body at points A and B, correspondingly (a space cross). These forces can then be reduced to a force $\mathbf{R}$ at A and a couple $\mathbf{M}_c$ acting in the horizontal plane (figure 1.16b) or to force $\mathbf{R}$ at B and a couple $\mathbf{M}_c$ in the vertical plane (figure 1.16c). All three representations are equipollent.