Introduction

This paper summarizes my collection of research findings on the role of shaft flexibility in the golf swing. The research was initiated during my PhD thesis under the supervision of Dr. Eric Sprigings. The goal was to gain a comprehensive understanding of the role that shaft stiffness plays in the golf swing. This paper will provide a rationale for the methodologies I've used, a summary of the important findings, and suggestions for future research on shaft flexibility.

Considering a tee shot with a driver, a primary objective is to determine the shaft stiffness that will result in the maximum shot distance for a particular player. Maximum shot distance is achieved by generating the highest possible ball speed while imparting the optimal launch angle and spin rate for the particular ball speed attained. Shaft stiffness influences all three of these parameters (Figure 1). The predominant factor in generating maximum ball speed is attaining maximum club head speed at impact. Theoretically, club head speed can be increased without changing the golfer's swing mechanics if the behavior of the golf shaft is optimized. During the downswing, the shaft bends backwards storing strain energy. This strain energy could then be converted to kinetic energy at impact. This kinetic energy would result in additional club head speed. The launch angle and spin rate of the golf ball after impact are strongly influenced by the orientation of the club head at impact. The orientation of the club head can be changed by altering the stiffness of the shaft. Therefore, theoretically, the optimal launch angle and spin rate can be generated by finding the shaft stiffness that produces the optimal club head orientation at impact.
Figure 1 Flow diagram illustrating how shaft flexibility influences the distance a golf ball will travel in the air following impact with a driver. Lower factors affect the higher factors of which they are connected. Environment refers to factors such as air temperature and wind. Impact quality refers to the location of impact on the club face. Shaft flexibility may affect the feel of the club during the swing which could alter the golfer’s swing mechanics.

Previous researchers have investigated this area; however, for a number of reasons, it was never thoroughly addressed. Broadly speaking, a researcher can take two approaches to understand the role of shaft stiffness in the golf swing. The first is through live experimental data collection with real golfers (or robotic machines). The second is through the use of computer simulation by employing a forward dynamics model.

Thoughts on live testing

At first consideration, live testing with actual golfers seems the most logical since the researcher can actually measure what is happening during a real swing. In the end, isn’t the goal to perfectly match a real golfer to his or her real equipment? However what the researcher gains in the potential
applicability of his or her findings may be lost due to a lack of control over the experimental conditions and/or the ability to measure the necessary variables in a reliable and valid manner.

In scientific research, perfect control of the experimental conditions means that when the researcher purposefully changes one input variable, they know that any changes in the output are due to their controlled manipulation. This is called internal validity. For example, a golfer hits a golf ball 250 yards with Club A and 275 yards with Club B. The clubs were identical with the exception of shaft stiffness; can the researcher conclude that shaft flexibility resulted in the increased distance? Unfortunately, there are a number of other factors, besides shaft flexibility, that could have been different between the trial with Club A and the trial with Club B (Figure 1). Most of these would be the result of variability in swing mechanics. Even the most consistent pro golfers can only land the ball in the width of a fairway approximately 80% of the time.

The validity and reliability of measurements is always a concern in experimental research. Validity, sometimes known as accuracy, refers to how close a measurement is to the true value. For example, suppose you are trying to determine a golfer’s swing speed by measuring the velocity of the center of the club face at impact during a drive. Unknown to you, the radar measurement system you are using is actually displaying the velocity of the heel of the club face at impact, which is 5 mph slower than the center of the face. Therefore, the swing speed measurement is not valid. Reliability, sometimes known as precision, refers to the repeatability of the measurement. If that same invalid swing speed measurement consistently displays a value that is 5 mph less than the true value, then the measurement is considered reliable. Further, if a measurement is unreliable, than it is also invalid. No measurement method is perfectly valid and reliable. The onus is on the researcher to statistically quantify these values and determine if they are sufficient for their specific research question (MacKenzie & Evans, 2010). Is the measurement method able to measure the differences they need to detect in their study? Suppose the manufacturer of your swing speed measurement system reports an accuracy of ± 2 mph. This system is probably sufficient to use in a study which will compare the swing speeds of professionals to amateurs. However, a system with this level of accuracy may not be sufficient to understand how shaft flexibility influences club head speed, as the differences in club head speeds between two shafts may be smaller than the reported accuracy of the system.

Another limitation to live testing is that there is no guarantee that the golfer (or robot) is swinging optimally to maximize the potential of a particular shaft. Perhaps after a week of training, and some barely perceptible swing adjustments, the golfer will be able to realize the true potential of the shaft. This is common in the track and field event, pole vault. As pole vaulters increase their strength and speed, they change to poles with different stiffness and length properties to achieve higher jump heights (Liu et al., 2011). However, assimilation to the new equipment requires practice and some minor modifications to the timing of their movements.
Robotic testing machines have been used to circumvent the issue of human variability. However, the forces and torques generated by these machines are not similar to those of a human and they are not capable of dynamically interacting with the club in the same way as a human golfer.

There is also still the issue of optimization.

**Thoughts on forward dynamic simulation**

Forward dynamic simulations provide an alternative to researchers attempting to answer questions about human movement and offer some advantages over live testing. Relative to live testing, there is essentially no error in measuring the outcome of a model, and due to a model’s repeatability in executing a movement; it is very easy to isolate the effects of changing a single variable. However, forward dynamic models are particularly vulnerable to problems of external validity. Do the findings from the model represent what would happen in reality? In general, a more detailed model (incorporating more aspects of the real system) is more resilient to external validity threats. However, a more detailed model is more complex, which inhibits the interpretation of the results. As such, there will always be a subjective estimate as to the best balance between parsimony, on one side of the scale, and goodness of fit on the other. It is easier to interpret the results of a simple model, but the ‘correctness’ of those results depend on how well the model represents the real system. The level of complexity required by a model will depend upon the specific questions the researcher wants to answer. It is the nature of these questions that should dictate how the model is developed. For example, a 2D planar model of the golf swing is not a valid tool for understanding shaft flexibility. In reality, the shaft bends meaningfully in directions that cannot be accounted for in a 2D model; therefore, the model does not represent the actual system. This poor representation would affect the answer to the research question. The onus is on the researcher to demonstrate that the model is capable of representing the necessary characteristics of the real system. This can be accomplished by collecting experimental data from live golfers, or by comparing the model’s results to previously collected experimental data from other researchers.

**Developing the 3D golfer model**

Given the pros and cons just presented, I believed the best way to understand the role of shaft stiffness was through the use of a forward dynamics model that would be adequately validated with experimental means. The model did not have to represent all features of a human golfer. It only had to represent those features that were pertinent to the questions we were asking. Specifically, the pattern of forces and torques being applied to the club needed to accurately reflect those of a real golfer.

A representative mathematical model of a golfer was constructed using a six-segment (torso, arm, and four club segments), 3D, linked system...
(Figure 2). The purpose of the golfer portion of the model was to generate realistic kinetic profiles (forces and torques) on the club segment during the downswing. It was decided that the exclusion of additional segments from the model would not prevent the golfer portion of the model from fulfilling its purpose.

The torso segment was constrained to rotate in a plane angled at 30° above the horizontal. The arm segment was constrained to move in a plane that was angled at 50° above the horizontal (Figure 2). These values were chosen based on static measurements taken from golf professionals featured in Golf Digest Magazine.

**Figure 2 (A)** The torso segment and arm segment were constrained to rotate in specific planes.
Figure 2 (B) Deflection along the Y axis represented lead/lag motion, while deflection along the X axis represented toe-up/toe-down motion.

The torso segment was modeled as a rigid body representing the entire torso rotating about its longitudinal axis. The arm segment was modeled as a rigid body encompassing both the upper arm and forearm with the elbow angle fixed at 180º. In addition to moving in a plane, the arm segment was free to rotate about its longitudinal axis representing both internal-external rotation at the shoulder and supination-pronation at the elbow. The golf club was modeled as four rigid segments with the proximal end representing both the grip end of the club and hand of the golfer. The club segments were connected by spring-damper elements that allowed the shaft to deflect in both the lead/lag and toe up/toe down directions (Figure 2B). The most proximal club segment, connected to the arm, was constrained to have motion only in a representative adduction-abduction plane (ulnar-radial deviation) relative to the arm segment. This is in agreement with anatomical constraints that prevent rotation at the wrist about the
longitudinal axis of the forearm and a proper golf swing in which the lead wrist is kept in an approximately neutral position with regards to flexion and extension. The initial configuration of the golfer-club model for every simulation was as depicted in Figure 2A. An initial angular velocity of 5 rad/s, in the backswing direction, about the torso axis, was also given to all segments of the model, so as to simulate the dynamic transition from the backswing into the downswing.

Four torque generators that adhered to the activation rates and force-velocity properties of human muscle were inserted at the proximal end of each segment, and provided the model with the capability of controlling energy to the system (Springing & Neal, 2000). One torque generator produced rotation of the torso segment (torso torque generator), a second produced extension/abduction of the arm (shoulder torque generator), a third created rotation about the longitudinal axis of the arm (arm rotation torque generator), and a fourth produced adduction of the hand-club segment (wrist torque generator).

Parameter values for segment length, moment of inertia, and mass for a representative golfer with a body mass of 80 kg, and a standing height of 1.83 m, were calculated using the regression equations provided by Zatsiorsky (2002). Parameter values for the club segments were based on taking direct measurements of a standard driver designed in 2001. Moment of inertia values for the shaft segments were determined by modeling the segments as hollow cylinders. The moment of inertia values for the club head were determined geometrically by modeling the head as a semi-ellipsoid.

The forward dynamics model is now functional. Given specific inputs from the muscle torque generators the model can be made to simulate the swing of a real golfer. However, a clear advantage of forward dynamic models is their capacity to be optimized. Optimization is a process that determines the ‘best’ way to do something. To put it mathematically, what are the specific values for the control variables that maximize the objective function? For a golf swing with a driver, control is dictated by the pattern of muscle activity generated by the golfer and the objective is to maximize club head speed. This requires knowing when to activate the muscle torques and if necessary when to deactivate them. This knowledge can be gained by optimizing the forward dynamics model. The best start and stop times for the torque generators were determined using a genetic algorithm optimization scheme. Genetic algorithms are at the forefront of mathematical optimization techniques. Further details on the model and optimization methods can be found from the original sources (MacKenzie, 2008; MacKenzie & Sprigings, 2009a; MacKenzie & Sprigings, 2009b; MacKenzie & Sprigings, 2010).

Can shaft stiffness be customized to increase club head speed?
To address this question, three versions of the model just described were used. They differed only with regards to the constraint parameters governing the maximum torque output from the four torque generators. This allowed the role of shaft flexibility to be evaluated for golfers that generate three different levels of club head speed (i.e. Golfer-Slow ~ 35 m/s, Golfer-Medium ~ 43 m/s, and Golfer-Fast ~ 50 m/s). These club head speed values represent the minimum, average, and maximum club head speeds measured by Brown et al. (2008) on a group of 40 male golfers (age: 20 to 59; handicap: 14 ± 8). Three different levels of shaft stiffness: Flexible, Regular, and Stiff, were used in the model. Their exact stiffness levels were based on direct measurements taken from actual drivers. For reference purposes, a club (Rigid) with a fourth level of shaft stiffness was also modeled. Club-Rigid was not capable of any amount of deflection.

Two separate sets of optimization schemes were conducted to fully explore the concept of determining a precise shaft stiffness for a particular swing. The first scheme involved manipulating the swing of the model to suit a particular level of shaft stiffness. The second scheme manipulated the stiffness of the shaft to suit a particular model’s swing. For both schemes, the objective in optimizing the golfer-club model was to maximize horizontal club head speed at impact with the golf ball.

The objective in customizing the golfer model to fit the club was to maximize horizontal club head speed at impact with the golf ball. The objective function was equal to horizontal club head speed at impact minus any penalty variables accumulated during the simulated golf swing. Penalties were incurred if the model performed movements that were not executable by a human golfer, such as having the arm segment pass through the torso segment. Penalties were also incurred if the model was not in a proper position at impact, such as having the club face misaligned with the target. The control variables were the start and stop times for the four torque generators. This resulted in a total of eight control variables that were optimized to determine maximum horizontal club head speed at impact. The optimization process was repeated for each golfer-club model incorporating each level of shaft stiffness.

It was possible that none of the three simulated clubs used in the previous section were particularly suited to any of the three golfer models. A larger and more complete spectrum of shaft stiffness was investigated in order to obtain a clearer picture of the role that shaft stiffness plays in generating club head speed during the golf swing. The objective in customizing the stiffness of the shaft to the golfer model was also to maximize horizontal club head speed at impact with the golf ball. A single control variable, that regulated shaft stiffness, was used in the optimization scheme. The control variable represented a percentage of the original stiffness of Club-Regular. An exhaustive search was performed by evaluating all stiffness levels from 50 to 150 % in 1 % increments. This range of stiffness was chosen to ensure that it encompassed the maximum range of stiffness values that would be practically developed by manufacturers. For every evaluation in the exhaustive search, the muscle control strategy...
used was that found for the optimization of Golfer-Medium with Club-Regular in the previous section. This procedure was also completed for Golfer-Slow and Golfer-Fast, each with Club-Regular.

Regardless of shaft stiffness, for a given level of golfer swing speed, the club head speeds attained by the golfer models were similar (Figure 3A). The largest difference in club head speed (0.08 m/s) for a particular level of swing speed occurred with the Golfer-Medium model between Club-Stiff (45.04 m/s) and Club-Flexible (44.96 m/s). The only exception was the lower club head speeds attained with Club-Rigid. It should be noted that Club-Rigid is purely a theoretical construct and cannot exist in reality. However, it does provide an indication that, for swing speeds beyond approximately 50 m/s (~115 mph), any non-rigid shaft contributes upwards of 4% to club head speed. For example, Golfer-Fast generated club head speeds that were approximately 2 m/s faster with the three flexible clubs in comparison to Club-Rigid.

**Figure 3 (A)** Clubhead speed at impact for the 12 optimized golfer-club models (reprinted, by permission, from MacKenzie and Spriggins (2009b), Fig. 6).
Evaluating a complete range of shaft stiffness with a single swing pattern from Golfer-Medium revealed that club head speed increased as shaft stiffness decreased (Figure 3B). The highest horizontal club head speed at impact (46.10 m/s) was attained at the most flexible shaft stiffness level of 50%. This club head speed was approximately 1.1 m/s faster than the club head speed attained when Golfer-Medium was optimized to Club-Regular (44.98 m/s, Figure 3A). However, the shaft stiffness level of 50% also resulted in the largest accumulation of penalties. The increased club head speeds associated with more flexible shafts were accompanied by improper club head orientations and positions at impact as demonstrated by the plot of ‘club head speed – penalties’ (Figure 3B). The value of the objective function (club head speed – penalties) demonstrated that the best golf swing occurred at a stiffness level of 100% (Figure 3B). This finding supports the results of the optimization from the previous section. The optimal shaft stiffness for Golfer-Medium, was the same shaft stiffness it was originally optimized with, namely, Club-Regular.

Based on our simulation results, it would appear that customizing shaft stiffness to a golfer's swing will not meaningfully increase club head speed. Further analysis of the 12 optimized swings will help lead to understanding of this finding. Kick velocity is the mechanism by which a flexible shaft can contribute to club head speed (Figure 1). The simulations revealed that, in general, the faster the swing speed, the greater the kick velocity (Figure 3).
Although not as definitive, it appears that more flexible shafts have the potential to generate greater kick velocities as well (Figure 4A). For all optimized simulations, kick velocity peaked nearly simultaneously with impact (Figure 4B). This suggests that kick velocity played an important role in the overall maximization of club head speed. For example, the Golfer-Medium\Club-Regular simulation demonstrated that kick velocity contributed approximately 7 m/s to the final club head speed (Figure 4).

When the club head speed results (Figure 3A) are compared to the kick-velocity results (Figure 4A) there appears to some discrepancy in the findings. For example, Golfer-Medium attained 1.5 m/s more club head speed with Club-Stiff compared to Club-Rigid (Figure 3A). However, the Golfer-Medium\Club-Stiff combination generated a kick velocity of 6.88 m/s at impact (Figure 4A). One finding suggests shaft flexibility contributed 1.5 m/s to the club head, while the other suggests the contribution was 6.88 m/s. Since the golfer model dynamically interacts with the club model, a rigid club will result in different golfer kinematics compared to a non-rigid club. The angular velocities of the most proximal club segment, about an axis perpendicular to the swing plane, for Golfer-Medium with Club-Stiff and Golfer-Medium with Club-Rigid support this view (Figure 5). At impact, the most proximal club segment of Club-Rigid had a higher angular velocity (+8.95 rad/s) than that of Club-Stiff.

During the last third of the downswing, the club head was deflected into a lag position. This lag deflection was created by a force from the golfer in the positive Y direction (Figure 2B), which generated a strain force in the shaft. According to Newton's 3rd Law (action-reaction), the grip end of the club exerts an equal force in the negative Y direction back on the golfer. At this point in the downswing (~ 0.25 s), the model's motion was not noticeably affected by this force from the club (Figure 5). However, near impact (~0.28 s), the shaft recoiled from its lag position into a lead position. This process increased club head speed relative to the grip end of the club; yet, it also served to simultaneously slow down the hands (Figure 5). The force applied by the club to the golfer could not be resisted by the golfer as evidenced by the decreased velocity of the model's hand. A major portion of club head speed can be attributed to the speed of the model's hand.
**Figure 4** (A) Kick velocity at impact for the 9 optimized simulations (reprinted, by permission, from MacKenzie and Sprigings (2009b), Fig. 7)
Figure 4 (B) Kick velocity during the optimized downswing of Golfer-Medium with Club-Regular (reprinted, by permission, from MacKenzie and Sprigings (2009b), Fig. 7)
Increasing club head speed is not the only mechanism by which shaft flexibility can increase the carry distance of golf drive. Shaft flexibility can also influence dynamic loft, which will in turn influence the launch angle and spin rate of the golf ball (Figure 1). Based on the simulation results, for every optimized swing, the maximum lead deflection of the shaft occurred at impact. It was also apparent that lead deflection increased as shaft flexibility increased and that lead deflection increased as swing speed increased. For example, the largest lead deflection occurred with the Golfer-Fast\Club-Flexible pairing (7.20 cm), while the smallest lead deflection was measured when Golfer-Slow was matched with Club-Stiff (5.74 cm) (Figure 6A). The same pattern materialized with the maximum lag deflection data. The largest lag measurement was recorded during the Golfer-Fast\Club-Flexible simulation (-4.79 cm), while the smallest lag value was found during the Golfer-Slow\Club-Stiff simulation (-2.70 cm) (Figure 6A).

**Figure 5** Angular velocity of the grip end of the club, for the optimized simulations of Golfer-Medium with Club-Stiff and Golfer-Medium with Club-Rigid (reprinted, by permission, from MacKenzie and SPrigings (2009b), Fig 8)
Figure 6 (A) Lead deflection at impact for the 9 optimized simulation.
A clear relationship did not materialize for the maximum toe-up measurement across levels of shaft stiffness (Figure 7A). The constraints imposed on the optimization routine by the penalty variables resulted in some imprecise relationships. For example, the optimized muscle coordination strategy that was determined for the Golfer-Medium/Club-Flexible arrangement would have resulted in poor club face alignment at impact for the Golfer-Medium/Club-Regular pairing. Therefore, the optimization algorithm determined a different optimal coordination strategy for that particular pairing, which deflected the shaft differently, but still resulted in a similar club head speed at impact (Figure 3A). If identical swings were used for each shaft, then specific mathematical relationships would have become apparent, but this would have come at the expense of an optimized swing with a correct impact position. The toe-down deflections at impact also failed to demonstrate a clear relationship across conditions (Figure 7B). This was also
related to the slight variations in muscle coordination strategies between conditions.

Figure 7 (A) Maximum toe-up deflection during the 9 optimized simulations.
Figure 7 (B) Toe-down deflection at impact for the 9 optimized simulations. Images of the golf club portray a driver in a toe-up and toe-down orientation, respectively.

The dynamic loft (Figure 8A) and dynamic close data (Figure 8B) revealed the same pattern across conditions as the lead deflection data. This finding was not surprising since dynamic loft and dynamic close are completely dependent upon the amount of toe-down and lead deflection present at impact. The Golfer-Fast\Club-Flexible model produced the largest dynamic loft (6.27°) and dynamic close (5.17°), while Golfer-Slow\Club-Stiff generated the smallest dynamic loft (4.42°) and dynamic close (4.01°). It is important to note that these values of dynamic close were generated without any ability of the shaft to twist about its longitudinal axis.
Figure 8 (B) Dynamic close at impact for the 9 optimized simulations. Images of the golf clubs demonstrate the conventions for measure dynamic loft and dynamic close, respectively.

Understanding how and why the shaft bends throughout the swing

The forces and torques applied by the golfer at the grip end of the club are continually changing in both magnitude and direction in 3D space.
during the downswing. It is solely these two factors, along with gravity, that cause the shaft to bend during the downswing. It is my opinion that a

clearer understanding of the source of shaft bending can be gained by resolving the resultant force, applied at the grip end of the club, into a
tangential component and a radial component. The radial component would act along the longitudinal axis of the grip (Z axis in Figure 2B), while the
tangential component would act in a plane at a right angle to the grip (plane formed by X and Y axis in Figure 2B). The radial and tangential
components are defined by a frame of reference that is fixed in the grip end of the club, and therefore, moves with the club.

Two sets experiments were conducted with the golfer model to gain a comprehensive understanding of shaft deflection. In the first experiment, the
location of the center of mass of the club head in both the front-to-back and heel-to-toe directions was manipulated, which provided the six conditions
shown in Figure 9. Radial force causes the shaft of a driver to bend due to the fact that the center of mass of the club head is not inline with the shaft.
Therefore, moving the center of mass should lead to predictable outcomes in the direction and magnitude of shaft deflection. As expected by theory,
the normal condition showed the greatest lead deflection at impact (6.25 cm) followed by the in-line condition (3.97 cm) and then by the reversed
condition (1.21 cm) (Figure 10A). Changing the location of the center of mass along the club head's X axis had a similar effect on the toe-up/down
deflections at impact (Figure 10B) The normal center of mass position resulted in a toe-down deflection (-2.26 cm) at impact. The in-line condition
approached zero, but finished with a small toe-up deflection (0.69 cm) at impact. Reversing the center of mass position along the X axis resulted in a
toe-up deflection (3.60 cm) at impact.
Figure 9 These six images represent possible club designs, which would result in changes to the position of the center of mass as described in the text. The top row demonstrates repositioning the center of mass along the X axis, which would primarily affect toe-up/down deflection (see Figure 10b). The bottom row demonstrates repositioning the center of mass along the Y axis, which would primarily affect lead/lag deflection (see Figure 10a). The reversed condition in the top row is analogous to hitting a ball off the face of a left handed driver while using a right-handed swing. Reprinted, by permission, from MacKenzie and Sprgings (2010), Fig 5)
**Figure 10 (A)** Lead/lag deflection at impact for all simulated downswing conditions.
Manipulating the position of the club head's center of mass certainly provided some valuable information on how radial force influences shaft bending. However, even if the center of mass is geometrically placed in-line with the shaft at the start of the simulation, the shaft can still bend due to tangential forces. This would result in the club head center of mass no longer being collinear, and radial force would once again exert its influence. Therefore, a second methodology was implemented, which allowed both the complete removal and complete isolation of radial force throughout the entire downswing. Complete removal of the radial force component from the optimized swing of the golfer model had a simple effect on club head deflection in both the lead/lag and toe-up/toe-down directions at impact. Lead deflection at impact remained positive (4.72 cm) but was reduced from its value under normal conditions (6.25 cm) (Figure 10A). Toe-down deflection at impact remained negative (-1.02 cm) but was also reduced in magnitude in comparison to the normal condition (-2.26 cm) (Figure 10B). This reduction seems reasonable when the effect of radial force on shaft deflection was isolated (see ‘Only Radial’, Figure 10A and 10B).
Was the model valid?

As mentioned earlier, forward dynamic models are susceptible to external validity concerns. How well does the model represent the movements of a real golfer? How well does the model represent the forces a real golfer applies to the club? Most importantly, for this study, how well does the model represent the deflection of a real golf shaft?

The ability of the model to move and apply forces like a real golfer was tested by instructing the model to match the swing pattern of a real golfer and measure the resulting fit. Kinematic data was collected from a male golfer (3 handicap) swinging a driver. An optimization scheme was employed which minimized the root mean square error (RMSE) between the absolute angular displacement of the model's segments and that of the live golfer. The optimization scheme functioned by manipulating the control variables, which consisted of the onset and duration times for the four torque generators. The model was able to closely match the kinematics of the live golfer. The RMSE values for the torso (1.25°), arm (0.66°), and club (1.53°) absolute angular displacement curves revealed the high level of agreement. The R2 values for the associated model and live golfer displacement curves were all greater than 0.99. The radial force acting on the club also showed close agreement (RMSE = 14.4 N, R2 = 0.98) and improved as impact approached. The club head speed curves for the simulated and live swings were also in close agreement (RMSE = 1.32 m/s and R2 > 0.99). The live golfer generated a club head speed at impact that was only 0.04 m/s faster than that of the model.

The swing plane is also often used as a way to describe the movement pattern of a golfer. As such, the swing plane of the model was compared to that of an expert golfer measured by Coleman and Rankin (2005). Coleman and Rankin showed that, for a 5 iron swung by a professional golfer (Participant 1 in their study), the left arm swing plane showed a slight increase in angle before peaking at approximately 138° and then steadily decreasing to an angle of approximately 103° at impact (Figure 11). Thus the swing plane for their Participant 1 increased in steepness by approximately 35° throughout the downswing. The swing plane angle for the model in our study, using a driver, also increased in steepness by approximately 35° as it decreased from a maximum angle of 165° to an angle of 130° at impact (Figure 11). The swing plane angle followed the same general pattern as presented by Coleman and Rankin (for Participant 1 who had a 0 handicap), but was consistently ‘flatter’ at all points during the swing because of the added length of the driver over a 5 iron. The driver swing plane angle at impact for our study (130.1°) was also in the range reported by Williams and Sih (2002) through the impact area (131.5 ± 3.9°); Williams and Sih did not report swing plane data for the entire downswing.
Figure 11 (A) Comparison of the model’s swing plane with a driver to that of Participant 1 (0 handicap) from Coleman and Rankin 2005 with a 5-iron.
The ability of the model to represent the deflection of the shaft during a real swing was the most important aspect of the validation. Acquiring excellent experimental data on shaft deflection is not easy; however, previous researchers have presented some clear experimental results. Perhaps the best experimental data was presented by Butler and Winfield (1994). We decided it was unnecessary to expend time and energy replicating their results, so we compared their shaft deflection results to the model. As shown by the results of Butler and Winfield (1994), it is difficult to compare the exact magnitudes of shaft deflection during the downswing between golfers, or between an optimized model and a golfer. Butler and Winfield reported differences in the magnitude of shaft deflections during the swing of three golfers using the same club and who all generated the same club head speed at impact (46 m/s). However, there were certain characteristics of each swing that held true. The highest magnitude of shaft deflection occurred in the toe-up direction at the start of the downswing. Maximum toe-up deflection occurred before maximum lag deflection. Also, the shaft was always deflected in the toe-down and lead directions at impact. The patterns of shaft deflection reported by Butler and Winfield have been included in the graphs below for comparison to the shaft deflections generated by the optimized simulation of Golfer-Medium swinging Club-Regular. Given the

Figure 11  (B) Convention for measuring the swing plane angle.
similarities, it would appear that the model's representation of shaft behavior is valid.

**Figure 12** (A) Lead/lag deflections during the downswing of Golfer-Medium with Club-Regular plotted with the same measurement from three golfers that all penetrated the same clubhead speed of 46 m/s.
Figure 12 (B) Toe-up/down deflections during the downswing of Golfer-Medium with Club-Regular plotted with the same measurement from three golfers that all generated the same clubhead speed of 46 m/s. Live golfer data was taken from Butler and Winfield (1994).

The reader might wonder why a more direct validation of shaft deflection was not conducted. For example, an inverse dynamic analysis could have been performed on a golf swing using a sophisticated 3D camera system whilst simultaneously capturing shaft deflection data. The kinetic pattern captured through the inverse dynamic procedure could then be used to control the movements of a forward dynamics model with a flexible shaft. The simulated shaft deflections could then be compared to the actual deflections. This seems ideal; however, inverse dynamics is a very blunt instrument. The errors in the calculation of the joint toques from a live golfer (which will be meaningful) will result in corresponding errors in the predicted club head deflection values. The known existence of these errors would make it pointless to compare the model's shaft deflection to that of a live golfer. It is for this reason that we have focused on the experimental results of previous researchers in order to compare our model's shaft deflection behavior.

**Practical applications**

The simulation results indicated that customizing the stiffness of a golf club shaft to perfectly suit a particular swing will not increase club head speed (and therefore ball speed) enough to have any meaningful effect on performance. No single shaft stiffness out-performed the other two at any
level of swing speed. At any given level of swing speed, the difference in club head speeds across levels of shaft stiffness did not exceed 0.1 m/s. An attempt was made to fine tune shaft stiffness even further by matching all possible levels of shaft stiffness with a single swing. The results indicated that regardless of swing speed, club head speed increased marginally as shaft flexibility decreased. Unfortunately, the marginal increases in club head speed were also accompanied by unacceptable configurations of the golfer-club models at impact.

Previous studies have shown that shaft flexibility can increase club head speed via the contribution from kick velocity (Butler & Winfield, 1994; Miao et al., 1998). Even in the simulation described in this summary paper, a kick velocity of 10.51 m/s at impact was recorded for Golfer-Fast with Club-Regular (Figure 4A) suggesting that club head speed at impact would have been reduced by 10.51 m/s if the shaft were perfectly rigid. Yet when Golfer-Fast was matched with Club-Rigid, club head speed was only reduced by 1.87 m/s in comparison to Club-Regular. Also, kick velocities at impact for Golfer-Fast/Club-Flexible (10.51 m/s) and Golfer-Fast/Club-Stiff (9.55 m/s) differed by approximately 1 m/s, yet both simulations resulted in the same club head speed of 52.94 m/s at impact. These findings show that kick-velocity can be a misleading variable. This is true because the shaft can dynamically interact with the golfer.

Based on the previous arguments it should not be inferred that certain levels of shaft stiffness cannot outperform others for a particular swing when considering increased driving yardage. Rather it should be understood that if driving distance was found to be meaningfully different across levels of shaft stiffness, then that increased driving yardage would probably be a result of factors other then differences in ball speed. Aside from golf ball speed, which is primarily a function of club head speed, ball launch angle and spin rate are the only other two factors under the control of the golfer that can affect driving distance (Figure 1). For a perfectly impacted golf ball, no side spin is imparted to the ball and the flight of the ball does not deviate from a path directly above the target line. Under these conditions, launch angle refers to the angle between the velocity vector of the ball and the horizontal immediately after impact, while spin rate refers to the amount of angular velocity (back spin) imparted to the ball. With respect to the execution of a golf drive, spin rate is primarily a function of club head loft (including dynamic loft), while launch angle is a function of both club head loft and club head path (Winfield & Tan, 1994). If a golfer has a consistent swing, then the path of the club head relative to the ball will not change from swing to swing. Therefore, by adjusting the amount of club head loft relative to the ball, an optimal ball launch angle and ball spin rate can be found for achieving maximum driving distance. Results from the computer simulations demonstrated that club head loft can change by as much as 0.65° depending on shaft stiffness for a golfer with a swing speed of approximately 45 m/s (101 mph) (Figure 8A). For example, the Golfer Medium Club-Stiff simulation resulted in 4.82° of dynamic loft at impact, while the Golfer Medium/Club-Flexible simulation resulted in 5.47° of dynamic loft (Figure 8A). The results from the optimization study conducted by Winfield and Tan suggest that a loft change of this magnitude may be enough to
have a meaningful influence on driving distance. However, a golfer must swing very consistently to take advantage of these changes. Also, larger differences in club head loft could be expected when comparing the non-optimized swings of many amateur golfers.

It should also be pointed out that the loft increases brought about as a result of shaft flexibility could also be made simply by changing the static loft of the club. For example, club head loft dynamically increased by 0.65° when Golfer-Regular was matched with Club-Flexible (5.47°) in comparison to the Golfer-Regular/Club-Stiff (4.82°) combination (Figure 8A). These two simulations would theoretically produce different ball flight conditions because of the discrepancy in club head loft at impact. If the face angle of Club-Stiff was geometrically changed to increase the static loft of the club by 0.65°, both conditions would then produce the same theoretical ball flight. This would be the case because the club head paths between conditions were already not meaningfully different (not reported), and the club head speeds were already very similar (44.96 m/s vs. 45.04 m/s). The effect of statically changing the loft of club face was confirmed using the model. Statically increasing the loft from 10° to 10.65° resulted in the loft at impact increasing by 0.6496°. Physically altering the loft of the club face will not change the position of the center of mass of the club head and therefore, will have no influence on shaft deflections during the swing.

However, there are two other factors to consider before defining the role of shaft stiffness. First, perhaps solely manufacturing a more complete spectrum of club head geometries with a single shaft stiffness is not practical. It may be more economically feasible for club manufacturers to provide a range of both club head geometries and shaft stiffness that produce the desired impact conditions. Second, the influence of feel cannot be easily dismissed. Certainly, the feel of the driver is linked to the golfer’s confidence in executing the shot, which will inevitably weigh heavy on the final outcome of the swing. It is also possible that, early in the swing, the feel of the club associated with shaft flexibility may subconsciously influence the golfer’s swing mechanics. In summary, club head path, loft and speed influence ball flight. As a variable, shaft stiffness was found to only have a meaningful effect on club head loft which can also be changed by altering the physical geometry of the club.

**Future research**

Using valuable empirical experience and a thoughtful process of trial and error, a club fitter may be able to determine a level of shaft stiffness that works better than most for a particular golfer. Research can also be conducted at this superficial level; have many golfers hit numerous shots, with a few drivers covering a range of shaft stiffness, and determine if particular shafts performed better for particular golfers. But researchers should be able to confirm why a certain shaft stiffness appears to work better for a particular golfer? Specifically through what mechanism was club head speed increased if at all? Did the golfer swing the club differently based on how it felt during the backswing and transition into the downswing? If the study used different club heads, were the masses and locations of the center of mass of the club heads identical? Although a particular shaft stiffness
produced the best results, was it actually being used in the optimal manner? For example, did kick velocity peak at impact? A sound experimental study on the role of shaft stiffness must be able to accurately partition club head speed into the following sources.

1. Contribution from the linear velocity of the lead wrist joint

2. Contribution from the angular velocity of the club about the lead wrist joint

3. Contribution from the kick velocity of the club.

Measuring these variables will allow the researcher to determine if any changes in club head speed can be directly attributed to shaft flexibility. Further, these three variables should add up to the measured club head speed. If the researcher cannot do this, then the experimental methods are not reliable enough to address the question.

References


