

### Consumer Choice Model

The Model of Demand and Supply is usually applied to our examination of market equilibrium. However, it says nothing about how individuals arrive at their decisions which often is the key premise of microeconomics.

#### Framework

1. Individuals face constraints or limits on their choices.
2. Individual tastes or preferences determine the amount of pleasure individuals derive from the consumption of their goods and services.
3. Individuals maximize their welfare or pleasure or happiness through and from consumption of these goods and services, subjected to the constraints they face (Both budgetary constraints due to income limits, and social as a result of legal restrictions or constraints that prevent consumption).

#### Budget Constraint

Before we examine or describe how a consumer chooses what is “best”, we need to describe what defines how much a consumer “can afford”. The answer is quite simple really, their income, given the prices of the goods they consume. How should we name it then, such a constraint? Since it pertains to budgetary constraints arising out of income, we refer to this constraint as budget constraint. However, this is not the only constraint we can imagine, but for all intents and purposes for this course, this is more than good enough.

Let’s focus on the simplest example where the consumer chooses between two goods, 1 and 2, and let the quantities chosen for each be  $x_1$  and  $x_2$  respectively. Further let the prices of each good be  $p_1$ , and  $p_2$  respectively. We are here assuming that there are only 2 goods that a consumer can choose between, 1 and 2. Then the constraint must simply state that the choice of quantities cannot exceed the total level of income,  $y$  available to her. That is

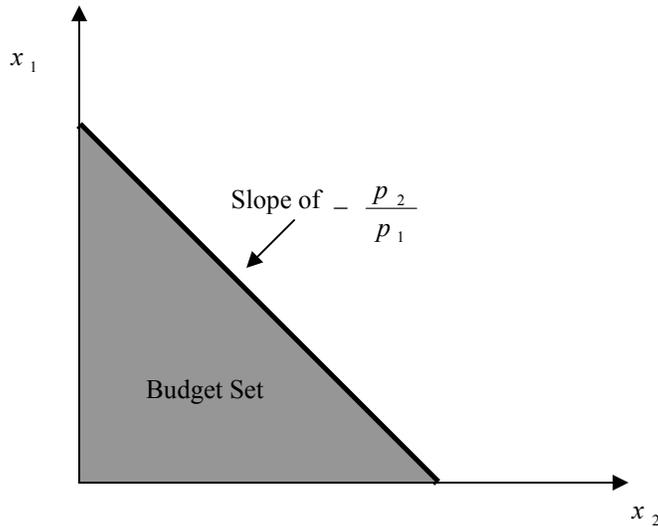
$$p_1x_1 + p_2x_2 \leq y$$

Is this simplification excessive?

Well not really. We can think of say one of the goods say good 1 as the main good, and all other goods as a composite good 2. How can we depict this on a two dimensional diagram? What we want is quantities of good 1, the main good on the vertical axis, and the quantities of the other goods, good 2 on the horizontal axis. Rewriting the budget constraint to depict this,

$$x_1 \leq \frac{y}{p_1} - \frac{p_2}{p_1}x_2$$

This inequality describes then the area or space which is feasible, or the quantities the consumer can choose. Diagrammatically,



That is the inequality describes the area within the triangle above, and we call it the budget set since it describes what the consumer can afford. And the boundary of the inequality is highlighted in bold is called the budget line and is described by the equality

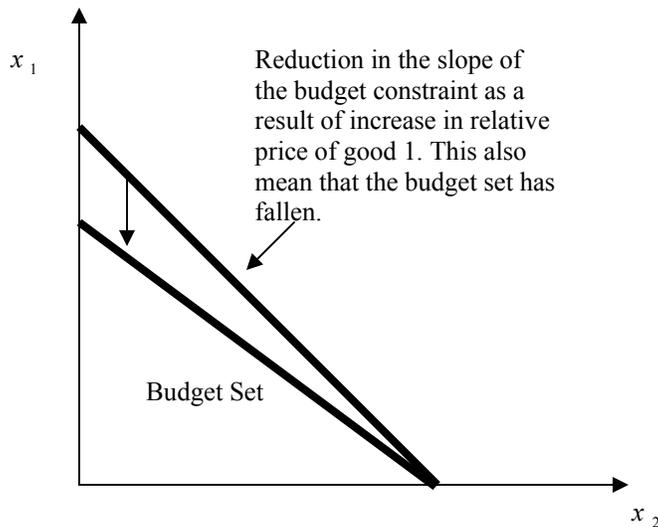
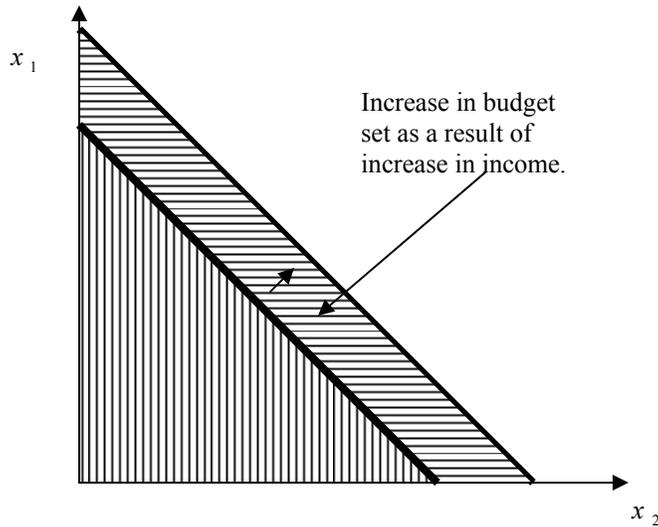
$$x_1 = \frac{y}{p_1} - \frac{p_2}{p_1} x_2 \text{ or}$$
$$p_1 x_1 + p_2 x_2 = y$$

It is easy to see that the slope in the former equation is  $-\frac{p_2}{p_1}$ . Can you use calculus to show that this is true? Note that the slope describes the trade off between the quantities of the two goods, good 1 and 2 since the slope is negative. That is the slope measures the opportunity cost of consuming good 2.

How does the budget constraint change? Looking at the constants that describe the budget constraint, we can say the following.

1. A rise in income raises the budget constraint parallel to the original, and increases the budget set. This is because as income rises the vertical intercept increases, without changing the slope of the budget line. This upward shift in the budget line increases the area from which the consumer can choose her optimal consumption couplet, hence we say the consumption set increases.
2. An increase in the price of good 1 relative to good 2 (this can be brought on by the change in price of either or both goods) reduces the slope of the budget line. What does this mean? When the good 1 becomes relative more expensive relative to good 2, consumers can consume less of good 1, hence the opportunity cost of raising consumption of good 1 is more costly in terms of consumption of good 2 lost.

The two changes are depicted in the diagrams below.



To consolidate the idea about budget constraint, we will examine how the budget constraint changes under different circumstance.

**Taxes:**

How would taxes affect the budget constraint?

Well it depends on what kind of tax we're talking about.

1. Quantity Tax: A quantity tax raises the price per unit consumed. Let the quantity tax  $t$  be applied to the consumption. Then the budget constraint becomes  $(p_1 + t)x_1 + p_2x_2 = y$ . This is equivalent then to an increase in the relative price of good 1, hence reduces the slope of the budget constraint.

2. Value Tax/Ad Valorem Tax: Is a sales tax, and is applied as a percentage of the price of a good. Let the tax raise the price of the good 1 by  $t\%$ , then the budget constraint is  $p_1(1+t)x_1 + p_2x_2 = y$ . This again reduces the slope of the budget constraint. Why? Can you show this? What if the Ad Valorem Tax is applied to all goods, both 1 and 2?

**Draw the shift in budget constraint as result of both types of taxes.**

**Quotas:**

How do quotas affect the budget constraint? Suppose a quota is imposed on the consumption of good 2. How would the budget constraint change? What is the budget constraint equation now? What can you say about the budget set?

**Subsidies:**

Suppose the consumption of good 1 is desirable, and hence the government wants every citizen to consume a minimum of  $\underline{x}_1$ . So that amount below that, the government provides a subsidy say reducing prices by  $s\%$ . That is for consumption levels below  $\underline{x}_1$ , the price of good 1 is reduced to  $p_1(1-s)$ . What is the new budget constraint? Depict this on a diagram?

**Preferences**

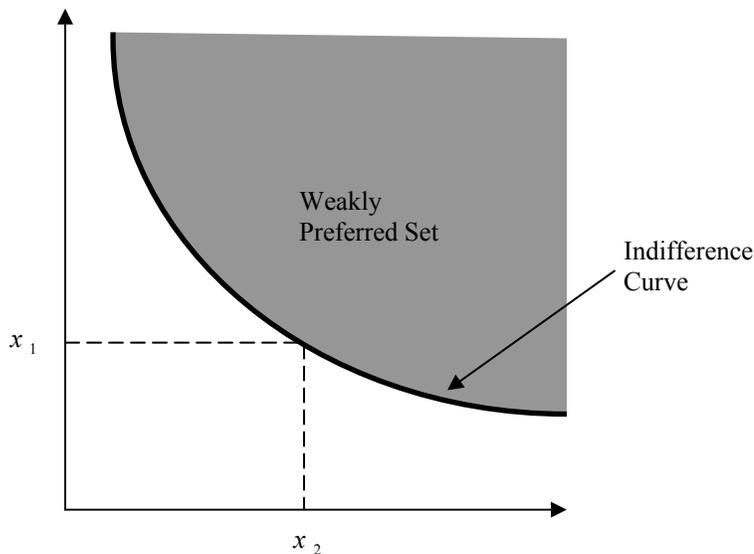
We know individual consumers do not randomly choose between two consumption quantities, or types of goods. Some goods are preferred to others on account of the welfare it gives. For example if you're a snow boarder, being up on the mountains with a top of the line Burton Snowboard is going to give you more joy than a broom for curling. Even if the broom is going to be cheaper than the board, and cheaper to play, you'd still choose the snowboard. The question is how can we study and depict these ideas in a succinct manner. We will start off with some symbols that will tell us clearly what an individual prefers and otherwise. Before we discuss the assumptions required for us to examine how individuals make their decisions, we will again lay down some definitions for symbols we will use.

- The symbol  $\succ$  reads as “strictly preferred to”, and is used as follows: let there be two combination of a basket of goods 1 and 2,  $(x_1, x_2), (y_1, y_2)$ . If  $(x_1, x_2)$  is preferred to  $(y_1, y_2)$ , we write  $(x_1, x_2) \succ (y_1, y_2)$ . Put another way, this says that the level of consumption of  $(x_1, x_2)$  is always preferred to  $(y_1, y_2)$  for both goods 1 and 2.
- The symbol  $\sim$  reads as “indifferent to”, and is used as follows: Again use the same basket of goods. Then if the consumer is indifferent to both quantities, we write  $(x_1, x_2) \sim (y_1, y_2)$ .
- The symbol  $\succeq$  reads as “weakly preferred to”, and is used as follows: using again the same set of quantities for the same basket of goods. We say  $(x_1, x_2)$  is weakly preferred to  $(y_1, y_2)$  when the consumer strictly prefers one of the quantities of the bundle, and indifferent for the other.

### Assumptions

1. **Completeness:** The assumption says that any number of bundles of goods can be compared. Using the two bundles of goods 1 and 2, this assumption says that we can either prefer bundle  $(x_1, x_2)$  to  $(y_1, y_2)$ , or  $(y_1, y_2)$  is preferred to  $(x_1, x_2)$ , or both which means we are indifferent between the two. Using the notation we had before, this is to say that, we either  $(x_1, x_2) \succeq (y_1, y_2)$  or  $(x_1, x_2) \preceq (y_1, y_2)$  or  $(x_1, x_2) \sim (y_1, y_2)$ .
2. **Reflexivity:** Any bundle is as good as itself. That is  $(x_1, x_2) \succeq (x_1, x_2)$ . This is a trivial assumption
3. **Transitivity:** This assumption says that our preference must be consistent. If we prefer  $(x_1, x_2)$  to  $(y_1, y_2)$ , and  $(y_1, y_2)$  to  $(z_1, z_2)$ , then we must prefer  $(x_1, x_2)$  to  $(z_1, z_2)$ . In notation:  $(x_1, x_2) \succeq (y_1, y_2)$  and  $(y_1, y_2) \succeq (z_1, z_2)$ , this implies that  $(x_1, x_2) \succeq (z_1, z_2)$ . Is this assumption a real depiction of reality? Would it surprise you if you know someone would prefer the Z bundle to the X bundle? What is the significance if such a person exists? Can this individual find a bundle he absolutely prefers?
4. **Local Non-Satiation:** This essentially says that more of goods are always better.

### Indifference Curves



The diagram above depicts how economists' typically consider the idea of an individual's preference. The space colored in gray is referred to as the weakly preferred set of consumption bundles to all points on the hull, or at the south west boundary. The boundary formed thus, are consumption bundles where the individual is indifferent between. This is known as the indifference curve.

Consider the rationale for this depiction of preference. Our presumption is a good is that we enjoy its consumption. Consequently our assumption that more is better. If more is better, then as we move north-eastward we are gaining in enjoyment from consumption of the bundle. Hence the points in the gray area are weakly preferred to those in the southwest space. Does this make sense?

We can draw such indifference curves joining all points that yield the individual the same level of enjoyment, i.e. the consumer is indifferent between all points on the indifference curve. What are some shapes the indifference curves can't take?

1. Indifference curves cannot cross.

Proof:

Let there be three bundles X, Y and Z. We will proof by contradiction. Suppose indifference curves can intersect. Let X and Y lies on different indifference curves such that X is prefer to Y, i.e.  $X \succeq Y$ . Next assume that Z is a point on both of these indifference curves. This then mean that  $X \sim Z$ , and  $Y \sim Z$ . However, by transitivity,  $X \sim Y$  which is a contradiction to our setup  $X \succeq Y$ . Hence indifference curves cannot intersect.

2. Indifference curves cannot be upward sloping

3. Indifference curves are not thick

The proof of 2 and 3 are similar. Let there be two consumption bundles X and Y both of which are on the same indifference curve. However, let the quantities consumed in X be greater than Y. But this then contradicts our assumption the more is better. Hence 2 and 3 must be true.

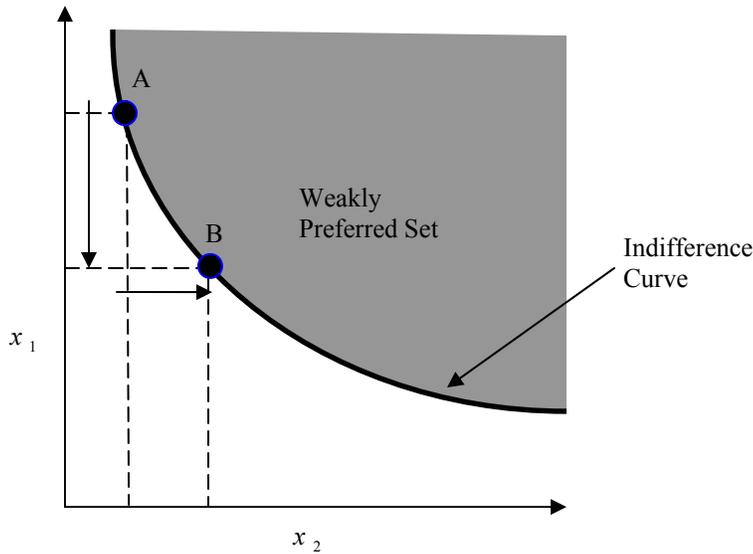
Qualification 1: Indifference curves cannot be upward sloping if we think of goods as good for us, that is we enjoy its consumption. However if one of the goods is a bad, i.e. we do not enjoy its consumption the indifference curves are upward sloping. Why?

Qualification 2: What if we are indifferent to additional units to the consumption of good 1, once we have a level of good 2. That is for a particular level of consumption of good 2, additional units of good 1 do not change your enjoyment. How would the indifference curve look like?

Of course a consumer has a full set of indifference curves to describe various levels of enjoyment. This would be referred to as an indifference map.

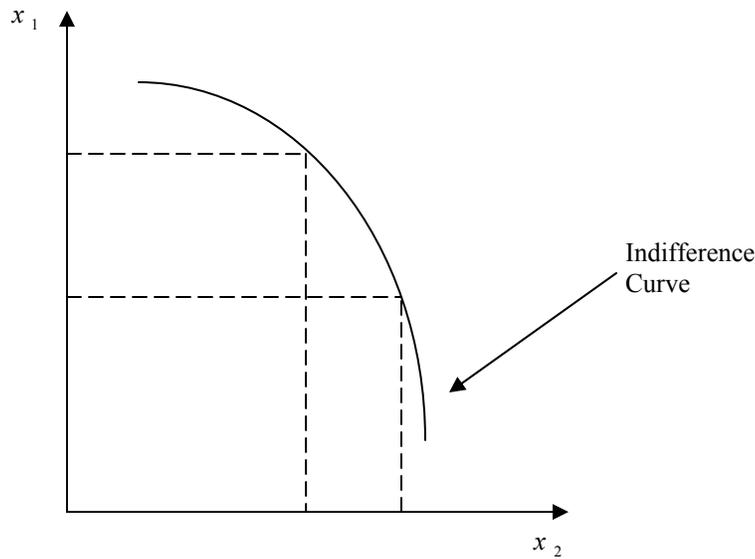
### **What does the slope of the indifference curve tell you?**

Let's start from a point, A, on the top left corner of an indifference curve, and move to point B along the indifference curve. Note the following, 1. The consumer is indifferent between points A and B. 2. Moving from A to B entail the trade of between goods 1 and 2, precisely to get to point B from A, the consumer has to give up some units of 1, to get some units of good 2. This is termed as the **marginal rate of substitution (MRS)**. It is simply the slope of the indifference curve. Why?



**Must the indifference curve be convex?**

In general, there is no need for an indifference curve to be convex, but typically an individual has a convex indifference curve. Why? To answer this question, we can use the marginal rate of substitution idea. From the previous explanation, we can see that if an individual has more of good 1, she is willing to give up more of it to gain additional units of the other good 2. However, as she consumes less of good 1, she would become less willing to give up consumption of good 1 to gain additional units of good 2. This is the idea behind diminishing marginal rate of substitution. This would be usually true for most of us, however, it is perfectly possible that an individual is willing to give up more of good 1 to gain additional units of good 2 even as her consumption of good 1 diminishes, in which case we get increasing marginal rate of substitution, as in the diagram below.

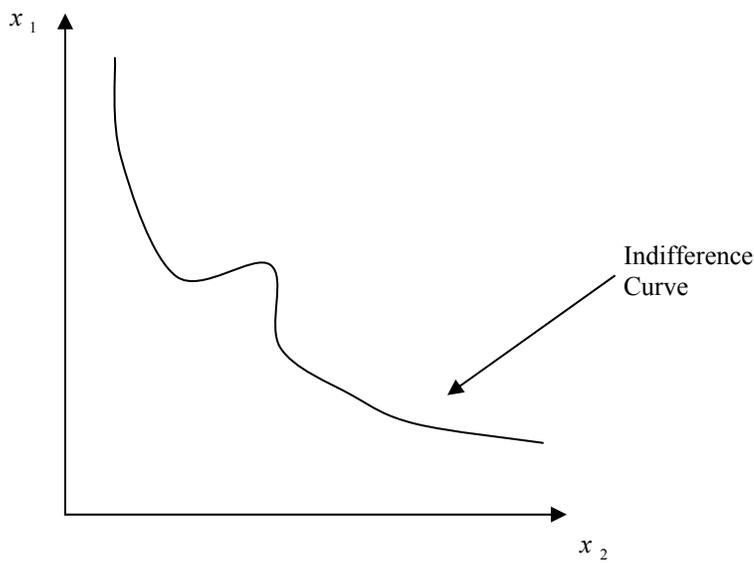


However, such an event is not typical, we will not consider this. To eliminate this possibility, we have to make an assumption about preferences.

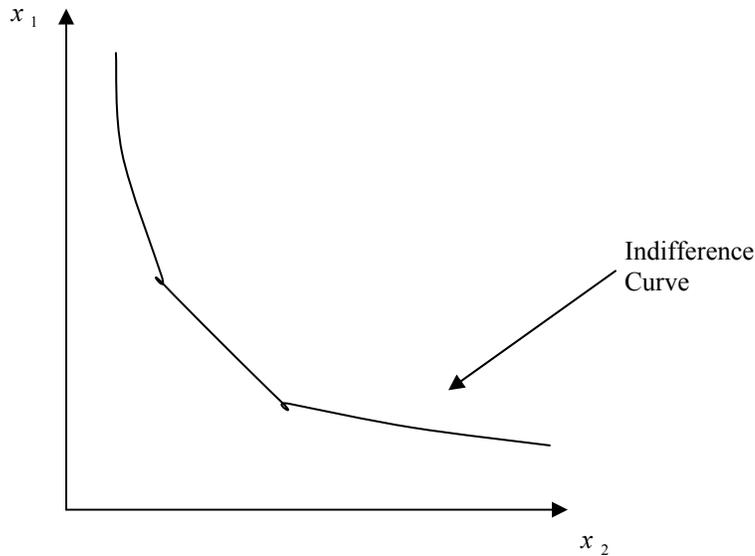
Let there be two bundles,  $(x_1, x_2)$  and  $(y_1, y_2)$  on the same indifference curve. Let  $\alpha$  be a number between 0 and 1. For us to ignore all concave indifferent curves, we need the following assumption,

$$(\alpha x_1 + (1 - \alpha)y_1, \alpha x_2 + (1 - \alpha)y_2) \succeq (x_1, x_2) \text{ and} \\
 (\alpha x_1 + (1 - \alpha)y_1, \alpha x_2 + (1 - \alpha)y_2) \succeq (y_1, y_2)$$

This hence removes the above type of indifference curves. How about something like the following:



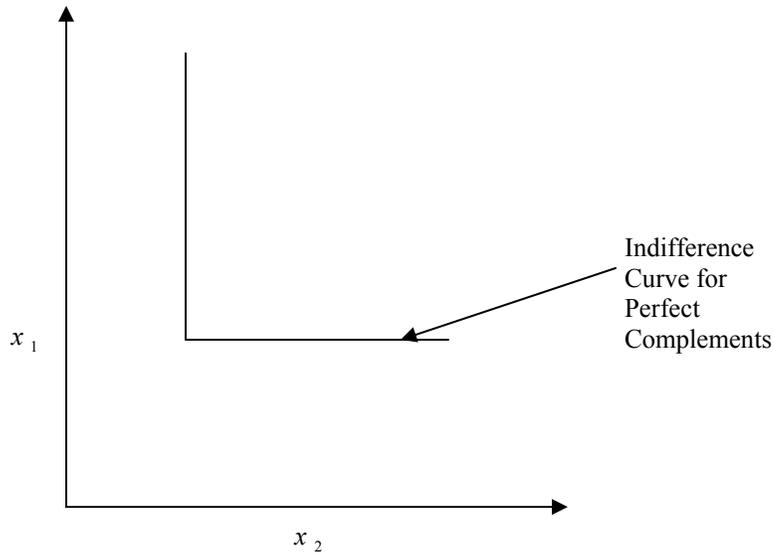
Does the assumption we made allow such an indifference curve?  
How about the following?



That is there is a flat segment. If yes why, if no, why not?

What are some general shapes and functional forms that the indifference curves take

1. **Substitutes:** What happens when the substitution between the goods follow the same rate as consumption of good 1 falls. This is a possibility which you may see. The geometric representation for such a case is simply a straight downward sloping line. When the substitution is 1 for 1 for example, we get perfect substitution, and the intercepts are the same, when we draw the axes with the same measure of quantity.
2. **Perfect Complements:** When goods we are perfect complements, it would mean that there is only one combination of the two goods (or more generally any number of goods consumed in fixed combinations.) that yields one particular level of enjoyment. By raising consumption of either away from the perfect combination yields no additional gain in enjoyment.



3. Indifference curves generated by Cobb-Douglas Utility and the Quasi Linear Utility functions

### Utility

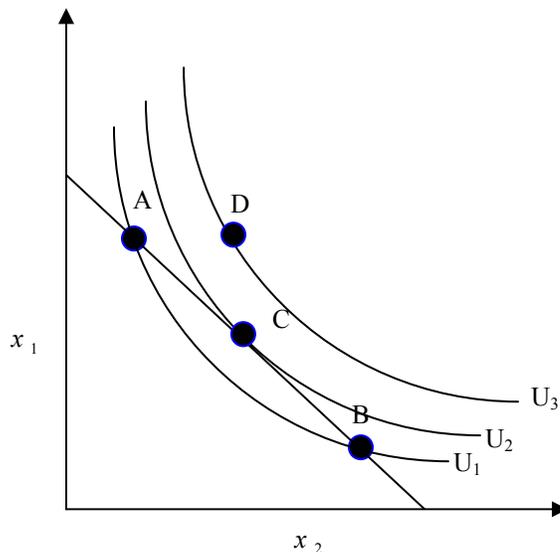
So far we have talked about indifference curves representing individuals' preferences. However, what if we have more goods we want to consider. Then the depiction on a diagram wouldn't be sufficient. How do we rate the degree of enjoyment as depicted by the indifference curve? We do so using the idea of utility which is just a ranking of the various indifference curves in a strictly increasing (monotonically increasing) manner. That is we give arbitrary value to an individuals enjoyment or welfare. All we need is to show that as the indifference curve moves away from the origin, the individual gets more welfare.

We can now talk about the functional forms for different types of indifference curves.

1. Perfectly substitutable: The functional form of such a indifference curve is  $U = ax_1 + bx_2$ , where  $a$  and  $b$  are positive constants, and  $U$  is just the level of utility.
2. Complements: The functional form is  $U = \min\{ax_1, bx_2\}$ .
3. The most common ones are the Cobb Douglas function  $U = x_1^\alpha x_2^\beta$ , or the Quasi Utility function,  $U = ax_1 + f(x_2)$ . The quasi linear utility function has a linear in good 1 and concave in good 2 here.

Intuitively, the choice the consumer chooses is determined by the point where the indifference curve just meets the budget constraint, i.e. say  $U_1$ . Why? Consider the scenario when the individual's equilibrium is characterized by the diagram below. What does that mean?

1. Would the consumer choose bundles between A and B within the budget constraint space? Well she wouldn't since she could do better right on the boundary of the budget constraint by using all her income. On such a equilibrium, she has two choices, A and B. Which should it be?
2. Further, she can always do better by moving from A and B to point C since she'd be on a higher indifference curve.
3. She cannot consume at point D on indifference curve because it is above the budget constraint, and is hence not attainable.



We say equilibrium is attained when the indifference curve is just tangent to the budget constraint. This gives us an idea what is equilibrium condition: It's just the point where the slope of the indifference curve is equal to the slope of the budget constraint. We know the slope of the budget constraint is  $-\frac{p_1}{p_2}$ . The slope of the indifference curve is just

$\frac{dx_1}{dx_2}$ , which describes how the individual trades off between his consumption of 2 goods.

Note that her utility remains the same. She is not gaining any additional welfare by moving along the indifference curve. Note further that we can rewrite the slope of the

indifference curve as  $\frac{dU}{dx_2} = \frac{\text{Marginal Utility of } x_2}{\text{Marginal Utility of } x_1} = \frac{MU_2}{MU_1}$ . Marginal utility describes

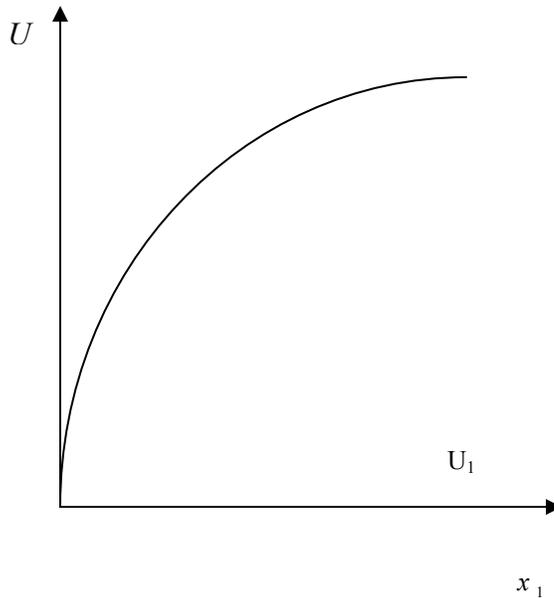
the change in individual welfare with a change in the level of consumption. The in equilibrium, the equilibrium condition is

$$\frac{MU_2}{MU_1} = \frac{p_2}{p_1}$$

That is in equilibrium the ratio of the marginal gain in welfare in the consumption of goods must equate with the price ratio of the two goods. This is a very intuitive explanation for how individuals make their choices. To proof that this is derived formally: Let the individual's utility function be the following  $U = u(x_1, x_2)$ , and her choice is constrained by the standard budget constraint. We can then rewrite the utility as

$U = u\left(x_1, x_2 = y - \frac{p_1 x_1}{p_2}\right)$ . Then the choice of good 1 is the point where the individuals

welfare is the highest. Before we go on, it should be noted that we typically require that utility functions that describe an individual welfare is concave. We say we want it to be well behaved. Consider a simple diagram where we depict utility with respect to consumption level. In such a depiction, the relationship between the levels of consumption of good 1 is positive with the levels of utility, however, as it increases we would tend to want less of it. You can think of it as it being the point where we just start getting sick from excessive consumption, of say donuts (Though I don't see how). This means there is a point where consumer's appetite is satiated (That is why we say we assume local satiation, as opposed to global satiation). If the relationship is a straight line, there is no point where the marginal gain in utility starts decreasing, and in fact if the relationship is convex, the marginal gain in utility increase with consumption. Summarize, the concavity assumption says we want it to be possible for the consumer to have a choice.



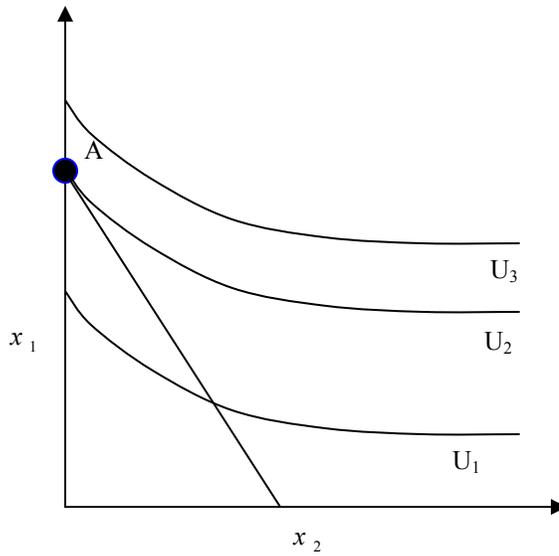
The point then is when the marginal gain in utility is zero. Let's focus on good 1:

$$\frac{\Delta u \left( x_1, x_2 = y - \frac{p_1 x_1}{p_2} \right)}{\Delta x_1} = \frac{\Delta u}{\Delta x_1} + \frac{\Delta u}{\Delta x_2} \frac{\Delta x_2}{\Delta x_1} = \frac{\Delta u}{\Delta x_1} - \frac{\Delta u}{\Delta x_2} \frac{p_1}{p_2} = 0$$

$$\frac{\Delta u}{\Delta x_1} = \frac{\Delta u}{\Delta x_2} \frac{p_1}{p_2} \Rightarrow \frac{\frac{\Delta u}{\Delta x_1}}{\frac{\Delta u}{\Delta x_2}} = \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

Which is the condition we noted earlier. Note that the condition is exactly the same for good 2.

The above depiction of consumer choice involves positive levels of consumption for both goods. This is typically referred to as an interior solution. A variation is known as a corner solution.



The above point  $A$  is known as a corner solution, where the individual consumes only a positive amount of good 1, but none of good 2 in equilibrium. Of course another corner solution is when the consumer consumes good 2 but none of good 1.