

INDUSTRIAL ORGANIZATION  
GAMES AND STRATEGY

A **Game** is a stylized model that depicts situation of strategic behavior, where the payoff for one agent depends on its **own actions** as well as the **actions of other agents**. The most common example (your text has some very good examples which you should read to get a more intuition) being the choice of prices for a firm. If they under price, they stand to gain market share, but lose in terms of possible profit. If they over price, they lose market share.

Consider the following concerns of a player, possibly yourself when you are thinking how hard you need to work for this class. You would like to get the highest mark, and top the class. Yet leave yourself sufficiently enough time to do well in your other classes.

1. The **Optimal Strategy** depends on how you (the agent) believe your fellow classmates (the competition) would perform (act) in class.
2. Since your classmates (other agents or players) act similarly when formulating their believes, you (one) need to formulate an idea about what your classmates believe about you and how you would perform (act) which would determine how they act, and this process goes on.
3. Further, this class has 4 tests in total (including quizzes, mid term test and the final exam) you will have to realize that as time passes, everyone learns more about themselves, and about you and everyone else, and that itself would alter how they behave.

The idea is that the players payoff in the game is interdependent, and it is that which introduces a whole slew of possibilities for strategic behavior, which is the object of **Game Theory**.

A Game consists of the following elements;

1. A set of Players - Such as you and your classmates in the course.
2. A set of Rules (Who can do what, when) - The total number of test involved and the grading system of the school
3. A set of Payoff Function - How you value your outcome or utility or happiness, be it in terms of grades, future earning capacity, or marks.

**An example:**

The game below shows the following; for each combination of strategies by each player, the respective matrix cell shows the payoffs received by each player, where for each cell, the first payoff is that for player 1, and the second element in each cell is the payoff for player 2. Note that each player's payoff is a function of the strategic choice of both players, that is what they both play. A game written in the above form is known as a **Normal Form Game**. We use this form of representation when we have

Table 1: Prisoner's Dilemma Game

		Player 2	
		Left	Right
Player 1	Top	(5,5)	(3,6)
	Bottom	(6,3)	(4,4)

both players moving simultaneously. This assumption is of course not always realistic since typically, strategic moves are sequential, but we could perhaps rationalize this setup as a result informational dissemination lags, though even that is not always justifiable particularly in a world such as today's. **Do you know what is the equilibrium strategy played by each of the players for the above game? What is the equilibrium payoff? Is the equilibrium optimal for each of the players in the sense that it is the best she could have done?**

## 1 Dominant Strategies, Dominated Strategies, and Nash Equilibrium

How do we solve for a normal form game such as that above? Let us examine the payoffs for each player in turn, starting from player 1. Suppose player 1 believes or expects player 2 will always play the strategy of 'Left', then player 1's best strategy is to choose to play 'Bottom' since the payoff of playing 'Bottom' is 6 while that for playing 'Top' is just 5. If instead Player 1 believes player 2 will always choose to play 'Right', player 1 would still benefit from playing 'Bottom'. This than mean that Player 1's optimal choice is 'Bottom' regardless of the choice made by Player 2. This leads us to the following definition;

**Whenever a player has a strategy that is strictly better than any other strategy regardless of what the other player does, or chooses, we say that the player has a *Dominant Strategy***

The concept of **Dominant Strategy** is a very robust concept in the sense that all we need is for the player in question to be rational. We do not even need to know the payoff of the other players.

The game illustrated in table 1 should be a familiar one to you, the **Prisoner's Dilemma Game**. Notice that since the dominant strategy for player 1 is to play 'Bottom' and player 2 is to play 'Right', they will in the end each get a payoff of 4, and yet notice that should they each had played 'Top' and 'Left' respectively, they would have obtained 5, consequent the name of the game or the irony. You can easily modify the game to give you stories such as in class competition, or price competition. An example of the latter goes something like the following, let 'Top' and 'Left' correspond to charging high prices, while the remaining strategy for both corresponds to charging low prices, say a result of price wars. Then the firms could have done better for themselves should they have in some sense "cooperated" with each other (granted that that would have been an violation of Anti-Trust Laws).

Or in more succinct terms the *Prisoners' Dilemma* illustrates the conflict between individual versus joint incentives.

Table 2: Iterated Elimination of Dominated Strategies, Part 1

		Player 2		
		Left	Center	Right
Player 1	Action/Strategy			
	Top	(1,1)	(2,0)	(1,1)
	Middle	(0,0)	(0,1)	(0,0)
Bottom	(2,1)	(1,0)	(2,2)	

Using the same procedure we used above to derive a dominant strategy, you should realize that there is no dominant strategy in the game of table 2. Does that mean that this game has no equilibrium, or some such stable situation? However, although neither player has a dominant strategy, player 1 does have a dominated strategy, a strategy that given the normal representation above, he would not play, to play 'Middle'.

**A Dominated Strategy is a strategy that yields a payoff that is inferior to that of another strategy, regardless of what the other player does.**

But unlike the idea of a dominant strategy, a dominated strategy does not yield an action that we know the player would play, just one that we know he wouldn't play. However, if we believe that a rational player would never play a dominated strategy, we might be able to eliminate it from our consideration, and thereby examine if the game may change. Assuredly, if we can conceive of this, so could player 2. The game would then change to the following;

Table 3: Iterated Elimination of Dominated Strategies, Part 2

		Player 2		
		Left	Center	Right
Player 1	Action/Strategy			
	Top	(1,0)	(2,0)	(1,1)
	Bottom	(2,1)	(1,0)	(2,2)

In this new game, and examining now the payoffs to the strategies of player 2, you would realize that 'Center' is a dominated strategy (as mentioned before). However, strictly speaking, 'Center' is not a dominated strategy in the original game, it will be chosen if 'Middle' is played by player 1. This in turn means that we can alter the game to the following representation,

Table 4: Iterated Elimination of Dominated Strategies, Part 3

		Player 2	
		Left	Right
Player 1	Action/Strategy		
	Top	(1,0)	(1,1)
	Bottom	(2,1)	(2,2)

If we keep applying the same rationale, we will eventually arrive at the payoff of (2,2) where player 1 plays action 'Bottom' while player 2 plays 'Right', which is the solution to the game. This is an example of the process called *Iterated Elimination of Dominated Strategies*. Note that the assumptions are far more stringent in the use of this technique than that for dominant strategy where all we needed was for each player to be rational utility maximizing agents. Here, we needed in addition that each player believe the other is rational and in turn believe that the other believes the other believes he is rational.

**It is not only important whether players are rational, it is also just as important that players believe that other players are rational**

To see the importance of this last assumption, we examine the following game;

Table 5: Dubious Application of Dominated Strategies

		Player 2	
		Left	Right
Player 1	Action/Strategy		
	Top	(1,0)	(1,1)
	Bottom	(-100,0)	(2,1)

Using the previous reasoning, we would say that a solution to the above game is for player 1 to play 'Bottom', while player 2 plays 'Right' (note that here the choice of 'Left' by player 1 is the dominated strategy, and that playing 'Right' is a dominant strategy as well.). If player 1 does not adhere to the belief that the other is rational, and that player 2 may play 'Left'. If player 1 sticks to his choice of playing 'Bottom', then it is possible that he stands to lose -100, consequently choosing not to play 'Bottom'.

Table 6: Nash Equilibrium

		Player 2		
		Left	Center	Right
Player 1	Top	(2,1)	(2,2)	(0,1)
	Middle	(1,1)	(1,1)	(1,1)
	Bottom	(0,1)	(0,0)	(2,2)

In the above game, note that using the prior concepts and solution strategies, there are neither dominated nor dominant strategies. What would the players choose? What might be a likely solution? If you look hard, you'd realize that what one player plays is dependent on what they conjecture the other player plays. A solution would then be such that,

1. Players choose an optimal strategy given their conjectures of what the other players do, and
2. that such conjectures are consistent with the other player's strategy choices.

Suppose that player 1 conjectures that player 2 plays 'Right', and similarly player 2 conjectures that player 1 chooses 'Bottom'. The player 1, given his conjecture, has the optimal choice of playing 'Bottom'. Similarly, player 2, given his conjecture, his optimal choice is to play 'Right'. Then player 1 expects player 2 to choose what in fact he chooses, and likewise for player 2. This is known as a **Nash Equilibrium**. (There is more than one Nash Equilibrium in the normal form game of table 6, that of player 1 playing 'Top' and player 2 playing 'Center'. Do you agree? Can you make your point using the same technique.)

**A pair of strategies constitutes a *Nash Equilibrium* if no player can unilaterally change its strategy in a way that improves its payoff**

The application of **Nash Equilibrium** always produces an equilibrium (although the existence of Nash Equilibrium applies to most games, it does not apply to all. Further, when we say always, it includes the situation when players randomize on their strategies, what is typically terms, mixed strategies. What we have dealt with thus far has been pure strategy equilibrium, however it is possible that a **Pure Strategy** Nash Equilibrium might not exist, but there is a **Mixed Strategy** Nash Equilibrium. We will talk about this in subsequent classes), and in fact may produce more than 1 Nash Equilibrium.

Games where there are more than 1 equilibrium are like games where there is an impetus for all players to coordinate to a collective choice, but there is more than 1 choice and that players disagree over which choice is the better. Consider the problem of a lazy student who wants everyone not to work so hard, so that he stands a chance to get a passing grade, so that one possible equilibrium is

where everyone does poorly, everyone is reasonably happy with that equilibrium since no one worked hard for the course, and had more time with private pursuits. However, for the smart and ambitious student, he knows that his future is riding on him working hard, and would rather have a high GPA, this would mean the lazy student would have to suffer the indignation of working hard, and thinking more. How do they coordinate to their respective preferred Nash Equilibrium?

## 2 Sequential Games: Commitment and Backward Induction

With the advancement of communication technology, the transmission of information now is so fast that almost every game seems to be like a simultaneous move game. However, in some situations, there is indeed a long lag between action, and the time it takes for what has occurred to be disseminated. In those situations, we may be better served to consider sequential decision making.

Consider the aggressive pricing decisions of monopolies, or monopolistically competitive firms facing the threat of entry by new firms. The best way to depict sequential games is to use **Game Trees**, which depicts the choices of each players sequentially. At the end of each branch of the tree, after all the players have moved, we can see the payoffs from everyone's choices. Such a game tree is depicted below for a entry-retaliation game. A circle in the tree that is not filled,  $\bigcirc$ , is a decision node.

A game always starts with a decision node, which in the sequential game of entry is first made by the potential new firm. Here firm 1, the potential new firm, either chooses to enter or not. If it chooses not to enter, the game ends, and firm 1 gets nothing in the form of profits, while firm 2, the incumbent firm gets the highest profit of 50. If however firm 1 chooses to enter, the next decision node is for firm 2 to move, and it chooses whether to retaliate or otherwise. Such games as depicted below is also often referred to as an Extensive Form Game.

The game depicted below has two Nash Equilibrium, (Enter, Do Not Retaliate) and (Do Not Enter, Retaliate). To see that, we can examine the veracity of each in turn. First, suppose firm one chooses to enter, then the choice is vested with firm 2, whether to retaliate or otherwise. From the lowest two payoff in the diagram, it is clear that firm two would always choose not to retaliate, and get a payoff of 20 instead of -10. Similarly, given that firm 2 never choose to retaliate, firm 1's best strategy is to choose to enter.

For the second case, suppose that firm two always chooses to retaliate, given this strategy, firm 1 would always choose not to enter, since 0 is greater than -10, which is what firm 1 gets. Going the other way around, given that firm 1 does not enter, it does not matter what firm 2 chooses since it always gets the payoff of 50.

Although there are two equilibrium, the second outcome does not make sense. Consider the

following, if firm 1 were to disregard the threat made by firm 2 that it would retaliate and enter. Upon entry, would firm 2 really choose to retaliate. Since we have already found that firm 2 would not, the equilibrium does not make sense.

A particular way of getting around this extra nuisance equilibria is to solve the game backward, a principle commonly referred to as **Backward Induction**. So we first consider the second node, after firm 1 enters. In that case, we know that firm 2 would never retaliate. Given that firm two never retaliates, we are left with the solve Nash Equilibria of (Enter, Do Not Retaliate).

Solving a game backward need not always be this easy and clear cut. For example, if the game after entry is a simultaneous move game such as we have considered earlier, then we would have to first solve the latter first, before solving for the entire game. The smaller game, the simultaneous move game within the context of the entire game, is called a **Subgame** of the larger one. Equilibria derived in the manner described here is referred to as a **Subgame Perfect Equilibria**.

In the first extensive form game above, the equilibrium of (Do Not Enter, Retaliate) was rejected on account of the "incredible" commitment required of the incumbent firm for the equilibria to stick. We say that the threat of retaliation is not a **credible** threat. We will now modify the game such that firm 2 now formulates a binding none renegotiable contract such that if firm 1 chooses to enter, firm 2 would definitively retaliate. Let the contract be such that were firm 2 to choose not to retaliate, it would incur a penalty of 40 so that the middle payoff in the first game becomes (10,-20). The new game is depicted below.

Figure 1: Extensive Form Representation of the Sequential Entry Game

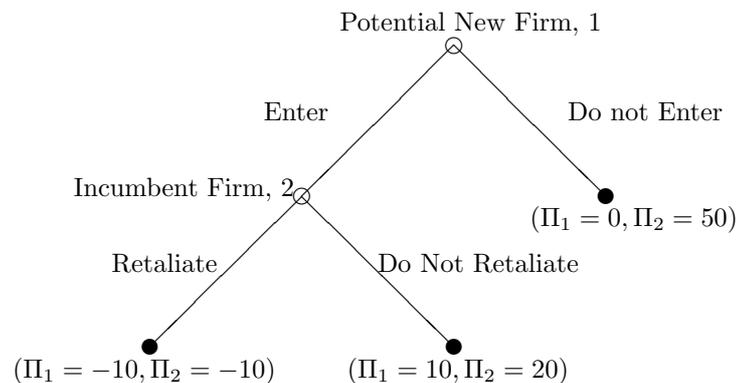
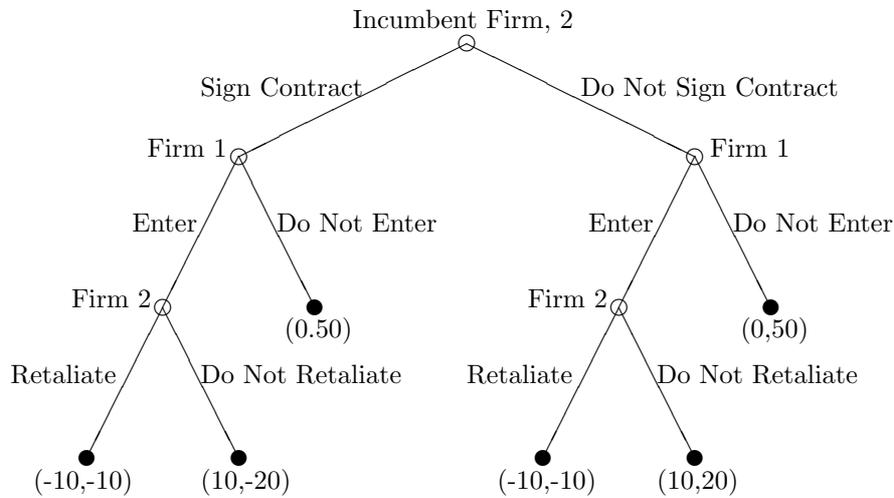


Figure 2: Value of Commitment



What the above figure now adds is the additional stage where firm 2 decides whether to sign on the contract and commit itself or not (Note that the contract is costless. Of course if the cost of contracting is high, the payoff may be altered such that at the end of the day, we arrive back at the original game). If the contract is signed, the firms play the game on the left, otherwise they play the game on the right.

Starting from the game on the right, from where the firm 2 does not sign the contract, then we already know that the subgame perfect equilibrium is (Enter, Do Not Retaliate) and the payoffs to the game is (10, 20). The subgame is the game on the right that begins with firm 1 choosing whether to enter or not.

However, if we were to examine the subgame, beginning from firm 1's decision node, after firm 2 signs the contract to commit. The sole difference between this subgame and the one on the right, is that the payoff after firm 2 chooses not to retaliate is now punitive for firm 2 since it is -20 instead of 20 due to the lost of 40. It is clear, using Backward Induction that firm 2 will choose to always retaliate, and given this choice, firm 1 will always choose not enter since the payoff after retaliation is -10 for it, while if it had chosen not to enter, the payoff would have been just 0. That is the equilibrium in this subgame is where the respective players play (Do Not Enter, Retaliate).

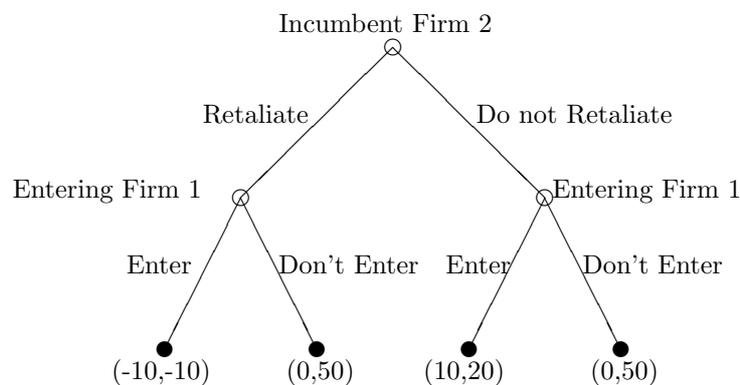
Comparing the two payoffs, when firm 2 signs and when it does not sign the contract, it is clear that the payoff to choosing to sign the contract and consequently, should entry take place to retaliate is the subgame perfect equilibrium since firm 2 is the first mover, it would be comparing the payoff of 50 from signing the contract, and that of 20 if it did not. This leads to the following point;

A **Credible Commitment** may have significant strategic value.

There is also a point as noted in your text on methodology that should be noted. If there is some possibility that the incumbent player may commit, and that there is a choice open to contract, we should include it in the model.

There is yet another manner we can augment the original model, that of switching the order of move. The new model is depicted below;

Figure 3: Capacity to Precommit



The idea here is that even in time, we actually observe entry by firm 1 first, if firm 2 can precommit to retaliation, this would be tantamount to firm 2 moving first. Solving this new game using Backward Induction, you should now find that the **Subgame Perfect Equilibrium** is where firm 1 chooses not to enter, and firm 2 choosing always to retaliate, and the equilibrium payoff is (0,50).

Another instance where sequence of moves matter is when the game depicts long term situations where players choose both long run and short run variables, such as a firm's capacity versus pricing strategy, another couplet could be product positioning versus pricing, and yet another entry versus output and pricing decisions. In all these couplets, the first is a long run variable, while the latter is a short run. When modelling this sort of choices, we typically model the choice in the long run variable first before the short run choices, this is because the short run choices, are typically made given the values on the long run variables.

### 3 Repeated Games

Although it is true that typically reality dictates that strategic behavior occurs over extended periods, sometimes it is possible and advisable to simplify our analysis and abstract from the complexity of reality, so that we can glean the key ideas those interactions tell us. Sometimes we can do so use a simple static (one period) normal representation to see the key ideas (Our previous discussion where we suggested the shrinking of the horizon of long and short term strategic choices into a two period extensive form game is also another example).

However, that need not always be the case, a case in point, consider the situation where a player changes its strategic variable in response to a rival's action. The idea here is that a player is changing its strategic variable, which obviously a static model cannot account for. However, it is possible to consider **Repeated Games or Stage Games**.

Consider the following simultaneous move game,

Table 7: Stage Game

		Player 2		
		Left	Center	Right
Player 1	Top	(5,5)	(3,6)	(0,0)
	Middle	(6,3)	(4,4)	(0,0)
	Bottom	(0,0)	(0,0)	(1,1)

Because this game only allows the players to choose an action only once, we call such game a **One Shot Game**. A repeated game then can be defined as a one shot game repeated a number of times (either finite or infinite). Let us repeat the above game twice, and call it a two period game.

In a single one shot game such as in discussed in section 1, the action of a player corresponds to her strategy. However, in a repeated game this is not true. Consider the repetition of the above game in table 7 twice. The actions available remain the same in each repetition. However, in stating their individual strategies, each player needs to specify what action she would choose in the first period, and what she would choose in the second period as a function of what she chose in the first period. In general, a strategy in a repeated game is defined as a player's **Complete Contingent Plan of Action** for all possible occurrences in the game. Each player in the above repeated game in table 7 has then  $(3 \times 3)^9 = 59,049$  strategies available.

Will a repeated game yield significant insights not seen in a one shot game? In most instances, that is possible. Considering the game proper, lets focus on the simultaneous game in period 1. You should be able to see that the two possible equilibrium are (Middle,Center) and (Bottom,Right). Note however, that the best total payoff is for both of them to play (Top,Left), though it is not a Nash

Equilibrium. The best Nash equilibrium in terms of individual and total payoff is (Middle,Center).

What is the Nash Equilibrium in the two period repeated game then? One possible equilibrium is the repeated play of the same strategies of the one shot game. That is they could play (Middle,Center) or (Bottom,Right) twice. The strategy that would lead the players to the first equilibrium is that player 1 plays Middle in period 1, and plays Middle in period 2 regardless of what happened in period 1, and the same for player 2. This is known as a **History Independent Strategy**.

The interesting question is however, is it possible to the players to achieve an equilibrium of the repeated game where the single period incarnations do not correspond with the one shot game equilibria. Consider the following strategy for player 1: Play Top in period 1, and play Middle in period 2 if period 1 actions were (Top,Left), otherwise play Bottom. For player 2: Play Left in period 1, and play Center in period 2 if period 1 actions were (Top,Left), otherwise play Right.

Does the above strategies yield a Nash Equilibrium? Consider each player in turn. Using Backward Induction again, suppose the previous period 1 yielded the equilibrium play of (Top,Left), which means the respective players play (Middle,Center) and since we know that that is a Nash Equilibrium, there is no impetus for either party to deviate. Similarly, if in period one, the action played was not (Top,Left), then the action of both players would yield the payoff from (Bottom,Right), and we know that since that is also a one shot Nash Equilibrium, neither players have the incentive to deviate.

We can now move on to the first period. Considering player 1 first; If she plays Top, and by assumption, player 2 plays Left, which yields the payoff of (5,5) to the players respectively. Given this play, they know that their period equilibrium would be (4,4) with the equilibrium actions played of (Middle,Center). Each of them thus have a lifetime payoff of 9.

We now ask, Is there an impetus for either player to deviate from this stated plan. Consider player 1, if he believes that player 2 will play Left, he could raise his period 1 payoff to 6 by playing Middle, which then yields the play in period one of (6,3), with the actions (Middle,Left). But we know that since in the second period given this history, player 1 and 2 would play (Bottom,Right) yielding a payoff of (1,1), which means that the lifetime payoff to player 1 to deviating from the stated strategy to be 7, lower than 9. Therefore there is no incentive for player 1 to deviate from her strategy. Doing the same for player 2 where given player plays Top, Player 2 could raise her payoff in period 1 by playing Center, yielding the equilibrium payoff associated with (Top,Center). The payoffs and conclusions remain the same as that for player 1 (verify for yourself). Therefore we can conclude that the designated strategies constitute a Nash Equilibrium (**Notice that we have not consider that the discount value for the period 2 payoff may not be 1. What is the critical value of the discount factor that would maintain this Nash Equilibrium**). The reason this new equilibrium is possible is that because the players could use their period 2 strategies to punish deviations.

**Since players can react to other players' past actions, repeated games allow for equilibrium outcomes that would not be an equilibrium in the corresponding one shot game.**

The ability to punish is the key ingredient in the operation of cartels, and collusive behavior, something we will examine more of later.