Labor Demand

1. The Derivation of the Labor Demand Curve in the Short Run:

We will now complete our discussion of the components of a labor market by considering a firm's choice of labor demand, before we consider equilibrium. We will now revisit the production function from your microeconomics course. Let the production function with labor hours (E) and capital (K) as factors of production be

$$q = f(E, K)$$

Where f is increasing and concave in E, and K.

Let's first consider the scenario of a firm in a competitive goods, and factor market. The profit function[1] is then

$$\pi = pq - wE - rK = pf(E, K) - wE - rK$$

The first order condition tells us that the firm will hire labor up to the point where the value of the marginal product of labor (VMP_E) equates with the wage rate.

$$pf_E(E, K) = w$$

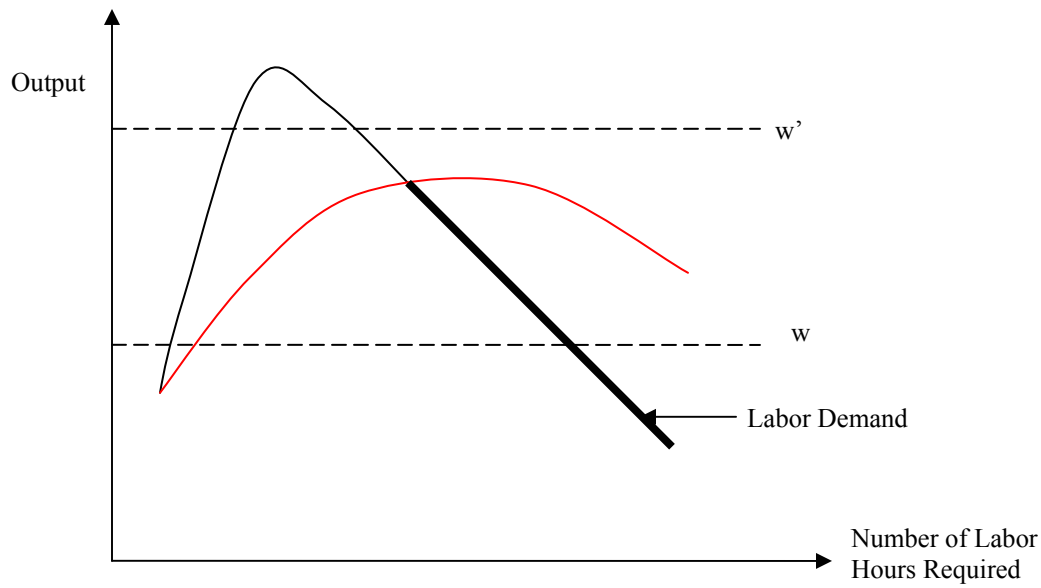
$$VMP_E = w$$

Note that VMP_E is a concave function of E. The same can be said of the choice of capital, but in the long run. How can we depict this choice diagrammatically? First let us describe what is the **value of average product of labor**?

$$VAP_E = p \times \frac{f(E, K)}{E} = p \times AP_E$$

How would the VAP_E look like? Since it is a function of the production function it should have an initially increasing portion, but because it is divided by the amount of labor hours used, and we have assumed that $f(E, K)$ is increasing and concave in E (while E is strictly increasing in E, there will come a point where the denominator is growing at a faster rate that the production function), there will come a point where VAP_E will start decreasing. Further, since $f(E, K)$ is still increasing, when VMP_E starts decreasing, there will be a point where VMP_E intersects VAP_E from the top. Now we are ready to draw VMP_E and VAP_E.

[1] It should be noted that for intents and purpose of our discussion, we assume profit maximization of the firm, reality may dictate that a firm consider other factors that may not be reflected in profit maximization. An example being concerns about damage to reputation, as well as the short run versus the long run needs of a firm. For example, even if the short run maximization dictates that a firm should lay off workers, the firm may choose not to lay off any workers under short run economic downturns if the cost of rehiring, and retraining labor is higher then the immediate need of maximizing profits. What other reasons could there be?



The labor demand is the portion of the VMP_E that is below the VAP_E. Why should that be the case?

- Why not in the upward portion of VMP_E? At that portion of VMP_E, an increase in the number labor hours required would result in greater value attained given the wage rate (i.e. the VMP_E must be greater or equal to the going wage rate, assuming the labor market the firm is in is also competitive). That is the marginal gain in revenue is greater than the cost of hiring an additional hour of labor. Hence the stopping rule for the firm is when the VMP_E equates with the wage rate, and it's on the portion where VMP_E is downward sloping or is decreasing.
- Why must the supply be the segment below the VAP_E? Any wage level above the maxima attained by the VAP_E would mean the average cost is greater than the average benefit, i.e. the firm would be making losses.

Hence the labor demand is the way it is, and describes the amount of labor hours/labor (depending what you have on your horizontal axis) desired at a given wage rate. Note that this is given a particular technology, i.e. this is the short run labor demand curve.

Is there an alternative interpretation to the firm's stopping rule?

First note that

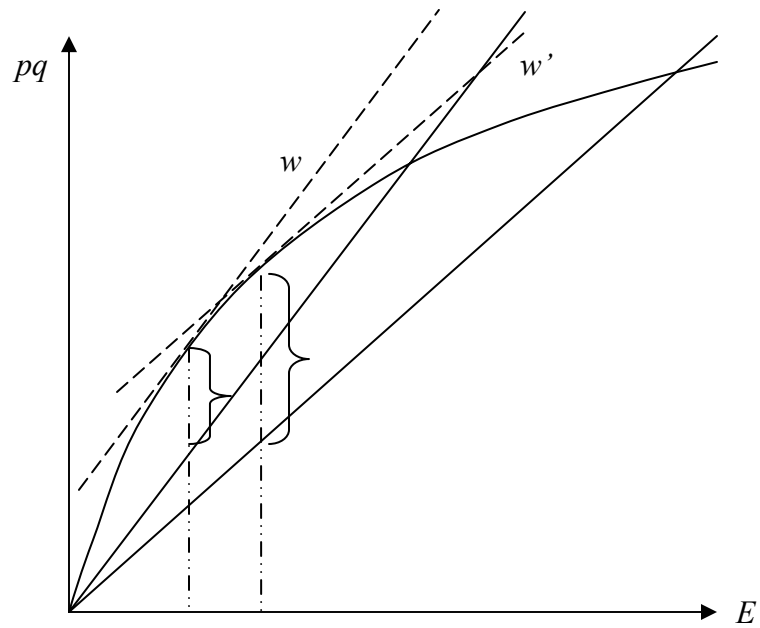
$$pf_E(E,K) = w$$

$$\Rightarrow p = \frac{w}{f_E(E,K)} = \frac{w}{MP_E} = MC$$

Next note that $\frac{w}{MP_E}$'s denominator is increasing but at a reducing rate, while the numerator, the wage rate is strictly increasing. Then MC is increasing, and stopping rule is at the point where marginal cost equates with the price of the product sold.

The market's labor demand in this industry would then be the horizontal sum of these individual demand for labor curves.

Is there another way to think about this profit maximizing strategy besides through the use of isoquants and isocost curves.



Where *w'* is less than *w*.

2. The Employment Decision in the Long Run

From the above first order condition of the profit maximizing problem, we have

$$pf_E(E,K) = w$$
$$VMP_E = w$$

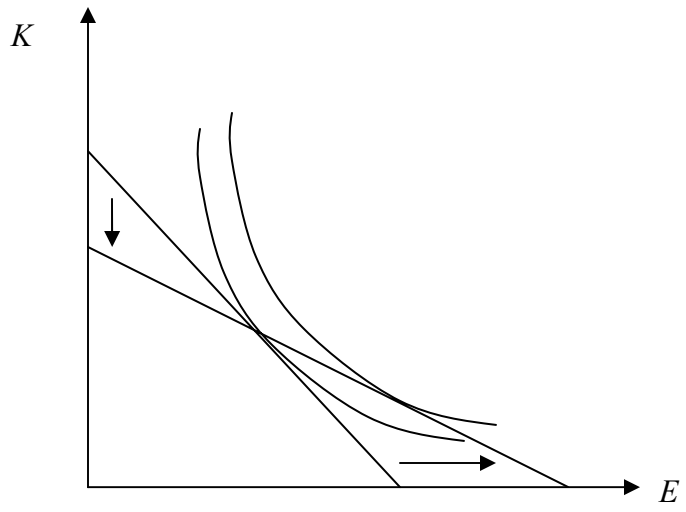
The same is true for capital,

$$pf_K(E,K) = r$$
$$VMP_K = r$$

Then in long run equilibrium,

$$\frac{r}{f_K(E,K)} = \frac{w}{f_E(E,K)}$$
$$\Rightarrow \frac{r}{w} = \frac{f_K(E,K)}{f_E(E,K)} = MRTS$$

How would you depict this equilibrium using isoquant and isocost curves?



Is the labor demand more or less elastic in the long run?
The decomposition of a change in wage rate, and cost of capital is similar to that in your consumer theory, and our analysis of labor supply. Income effect is now replace by scale effect (As a result of a uniform rate of reduction in cost, or as a result of technological change that affected the cost of production uniformly), while substitution effect remains the same (and is dependent on the change in the relative cost of the factor inputs).

3. Elasticity of Demand for Labor

The **elasticity of labor demand** is just

$$\delta_{SR} = \frac{\text{Percent change in employment}}{\text{Percent change in wage}} = \frac{\Delta E_{SR}}{\Delta w} \times \frac{w}{E_{SR}}$$

What are some determinants of this elasticity? These are also known as Marshall's Rules of Derived Demand

a. Availability of Substitute Inputs: The greater the number of substitutes, the more elastic the demand.
b. Elasticity of Supply of Substitute Inputs: The more elastic the supply of substitute inputs, the more elastic labor demand is.
c. Elasticity of Demand for Output: If demand for output is inelastic, the demand for labor would also be inelastic. The converse is true.
d. Ratio of Labor Cost to Total Cost: The smaller the proportion to total cost, the more inelastic is labor demand.

Application of the above to Union Behavior:
A union's war cry is generally for the increase in collective wages for unionized members through the power of collective bargaining. However, as we know the greater the wages above the equilibrium level, the more labor would go unemployed. A union would then be most effective if the industry under consideration faces an inelastic demand for labor. It is then in the interest of a union to lower the firm's elasticity of demand for labor. How can this be achieved?

- It is then not surprising the unions typically resist technological advances that increase the possibilities of substituting between labor and capital. Point a and b above.
- Unions often want to limit the availability of goods that compete with the output of unionized firms. A good example was the resistance to the entry of Japanese cars into the U.S. market. This relates to point c above.
- Unions are more successful when their wage bill makes up a small proportion of total cost, such as electricians, and carpenters.

For a more detailed description, see Borjas page 131-132.

4. Changing Demand Conditions and Global Competition

Since labor demand is a derived demand, derived from the demand for a firm's product, changes in the product's demand will affect the labor demand for the firm.
How has global competition affected Canadian Labor demand?
With the advent of global competition and trade, there is greater availability of substitute products. This competition drives prices down, reducing the derived labor demand. Can we prove such a conclusion based on our simple model of firm profit maximization? From the first order conditions, we can differentiate E implicitly with respect to p to obtain the following implicit relationship between labor demand, and price of product manufactured;

$$f_E(E,K) + pf_{EE}(E,K)\frac{\partial E}{\partial p} = 0$$

$$\Rightarrow \frac{\partial E}{\partial p} = -\frac{f_E(E,K)}{pf_{EE}(E,K)} \geq 0$$

This means that as competition drives down prices of Canadian goods, labor demand falls, since by concavity $f_{EE}(E,K)$ is less than zero.

Of course this argument negates the possibility that wages may be driven down to maintain Canadian firms' competitive edge. That is we could have wages falling to meet the fall in prices due to competition. Can you prove this conclusion from our profit maximizing condition?

$$\frac{\partial w}{\partial p} = f_E(E,K) \geq 0$$

Another approach to understanding how trade and competition may affect domestic demand for labor is the following:

With competition, and freer trade, prices in both domestic and foreign markets must equate, else there would be arbitrage possibilities (i.e. if the product is cheaper in one market than another, someone could just buy from the cheaper market, and resell the items in the more expensive market for nonzero profits). Let the superscript for domestic firms be denoted by *d*, and *f* for foreign. Then in equilibrium

$$\frac{MP_E^d}{w^d} = p = \frac{MP_E^f}{w^f}$$

Now if both economies have the same marginal product of labor, with the exception that the Canadian labor commands higher wages, open trade would require wages to fall to that attained in the foreign economy. This argument could also be extended to the case when wages are the same, with the exception that marginal product are different. Suppose the sole difference is the Canadian labor is less productive. To maintain equality in the condition, we could either raise marginal productivity, or lower wage rates.

Your text has a good sample of where Canada stands in terms of labor cost to both Newly Industralized Economies, and developed western economies in table 5.1 on page 160, while table 5.2 on page 161 reveals Canada in terms of productivity. An interesting question to ask yourself in all these comparisons is the following: How does Canada's industrial composition compare with the other advanced economies? That is without full knowledge of this composition, those numbers reveal little about how capable an economy will be able to meet the challenges of changing taste in consumption.

Read exhibit 5.1 on page 165 about the Free Trade Agreement between US and Canada, and how it affected Canada in the short-run and long run.

5. What is the Relationship between the Firms Production Function and the Demand for Labor? (This section is just good to know, but not necessary to know)

Let start by assuming the typical production function, but now with only labor as the sole input,

$$q = f(E) = E^{\alpha} K^{\beta}$$

The profit function is then,

$$\pi = \max_{E,K}(pq - wE - rK) = \max_{E,K}\left[p\left(E^{\alpha} K^{\beta}\right) - wE - rK\right]$$

Assuming we would like to find the long run labor demand, that is K is variable as well. The first order conditions for E and K are respectively,

$$\alpha p\left(E^{\alpha-1} K^{\beta}\right) = w$$

$$\beta p\left(E^{\alpha} K^{\beta-1}\right) = r$$

Combining these two first order conditions we can find the equilibrium condition.

$$E = \left(\frac{\alpha p K^{\beta}}{w}\right)^{\frac{1}{1-\alpha}}$$

$$K = \left(\frac{\beta p E^{\alpha}}{r}\right)^{\frac{1}{1-\beta}}$$

Therefore,

$$K = \frac{\beta w}{\alpha r} E$$

However note that the firm first chooses its optimal output given the competitive price. Let this choice of output be y.

$$y = E^{\alpha} K^{\beta}$$

$$\Rightarrow K^{\beta} = \frac{y}{E^{\alpha}}$$

We can now substitute this condition into the demand function for labor, E

$$\therefore \alpha p\left(E^{\alpha-1} \frac{y}{E^{\alpha}}\right) = w$$

$$\Rightarrow \frac{\alpha p y}{w} = E$$

$$\Rightarrow \ln E = \ln \alpha + \ln p + \ln y - \ln w$$

Which is just your long run labor demand function in natural logarithm. Note the relationship between price of output, wage rate, and total output. Do the signs concur with your expectations?

6. **What if we have Imperfect Competition in the Product Market?**
Suppose the firm who is doing the hiring operates as a monopoly in the goods market. Again let us fix the output of this monopoly firm at y which it has chosen as its profit maximizing level. Recall then its profit function is as follows,

$$\max_E \pi_m = \max_E p(f(E,K))f(E,K) - wE - rK$$

Thus it is not a price taker, unlike the competitive good firm. Its demand function of labor is thus,

$$(p_q f(E,K) + p)f_E = w$$
$$MR_q \times MP_E = MRP_E = w$$

Note the difference in the implicit demand function for labor. Instead of equating the value of marginal product of labor to wages, we now equate it to the marginal revenue product of labor. This is because the monopoly is a price setter, as opposed to a price taker.

In and of itself what does this mean for the labor market?
If the firm's labor skills are not specific, that is if the labor market is competitive, whether the firm is a monopoly or otherwise makes no difference, since its demand is aggregated into the market labor demand. However, the following section deals with the possibility that the firm may in fact be a monopoly labor demander in the labor market, monopsony.

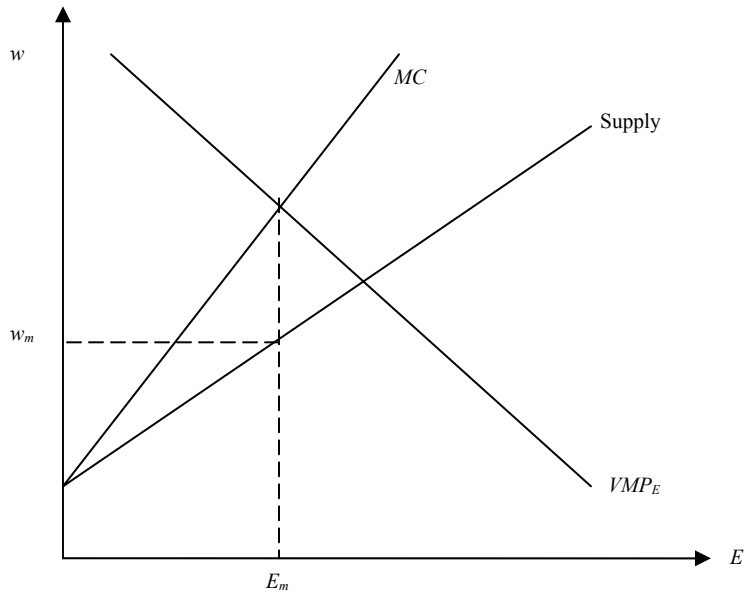
7. **What if the firm has Monopoly Power in the Labor Market, i.e. is a Monopsony.**
If a firm is a monopsonist in the labor market, it is effectively the only demander of labor, and hence dictates the going wages. How would the profit maximizing problem for a monopsonist look like? First note that instead of being a wage taker, it now sets the wages.

$$\max_E \pi_m = \max_E pf(E,K) - c(E) - rK$$

where c is the increasing and convex cost function with respect to labor E. The demand function now for labor is

$$pf_E(E,K) = c_E(E)$$
$$\Rightarrow pMP_E = c_E(E)$$

That is the amount of labor hired is attained when the value of marginal product of labor equates with marginal cost, i.e. wage is not a horizontal line at a specific wage rate for the firm, but a upward sloping curve. Diagrammatically,



Let consider the simplest case where the monopolist's cost function is $c(E) = \alpha E^2$ where the labor supply function is $w = \alpha E$. Then marginal cost is just $c_E(E) = 2\alpha E$. Then the labor supply curve is below the marginal cost the firm faces, which in turn determines the amount of labor it hires. Further, since there is no necessity for the firm to pay a wage higher than what is desired on the labor supply curve, a monopsonist hires less labor and at a lower wage rate than a competitive labor market.

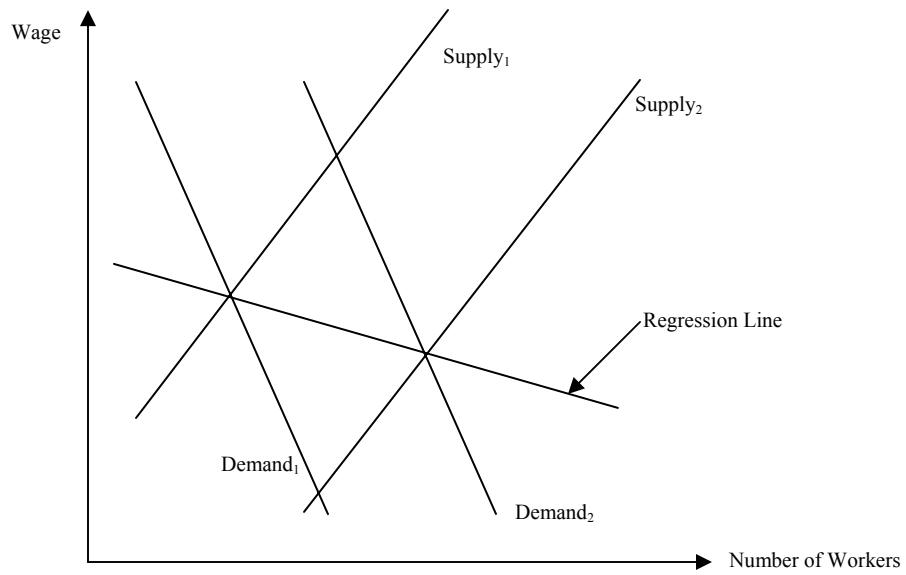
8. Estimating Labor Demand when we have Endogeneity Problem? The method of Instrumental Variables.

What is endogeneity? It is bias in our estimated coefficient of interest, which in our estimation of say, the demand of labor, or the supply of labor may be the elasticity of demand, and supply respectively. Consider a simple regression for labor supply,

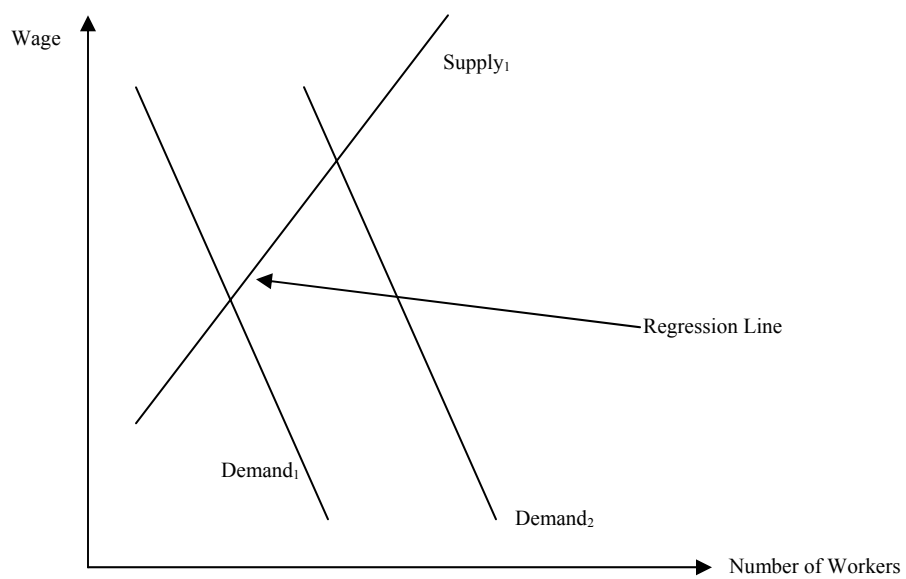
$$\ln E_i = \alpha + a \ln p_i + b \ln y_i - c \ln w_i + \varepsilon_i$$

Think about what our data might be telling us. The observations of labor supply involves typically equilibrium points at different times. These equilibrium points may in fact involve shifts in both demand, and supply labor as in the figure below. Then at least from the diagram, we would be biased to find elasticity of supply that may in fact be negatively sloped, when we were expecting a positively sloped labor supply!

The bias is created because there are other variables affecting labor supply, such as demand conditions, which is captured in the error term.



What can we do then? The Method of Instrumental Variables
One strategy is to find a situation where some underlying factor is shifting the demand without shifting the supply. In econometrics, a variable that causes a shift in one curve without another, a instrument, or instrumental variable, and the technique used to perform such a regression as **Method of Instrumental Variables (IV)**. Diagrammatically,



That is the technique is able to isolate the labor supply curve. (What do we need to estimate demand?) However, this is conditional us being able to find a good instrument, and it is usually highly debatable whether an instrument is good, or whether one is better than the other.

As an example, consider the estimation of women's labor supply during the periods when women were increasingly empowered through the legalization of abortion, or the availability of contraceptive technology. The typical argument of these technologies affecting women is through allowing them to complete their desired human capital investment, hence increasing their skills, and the demand for female workers. However, these technologies also affected women through permitting to entering the labor force hence increasing labor supply. How then can we estimate the true elasticity of labor supply for women during these periods, and separate between what was the previous value of elasticity, and what was caused by the advent of these technologies? Can you think of an instrument?

For an example on the estimation of labor demand, read Borjas, section 4.12, pages 153-158.