Intermediate Microeconomics II, ECON 301
General Equilibrium V: Public Goods

In our last discussion on externality, we found that as long as property rights are well defined, individuals could possibly trade towards an equilibrium level of externality. However, the idea is quite a simplification when you realize that consensus may not be that easily achieve in the case when there are more than 2 agents, despite property rights being well defined.

To see this consider the situation of drunk drivers and safe sober drivers. Suppose there are two safe drivers, but one drink in the town in question. As long as the safe drivers do not have common preferences over the externality of safe roads, even if the property rights to safe roads is well defined, they may not and most likely would not agree on the ultimate level of externality, and yet upon the final trade, both of them faces the same probability of encountering the town drunk.

The case above is an example of difficulties generated by a special kind of externality, or more precisely consumption externality, which we call **Public Goods**, which is a good that must be provided in the same amount to all consumers, and the enjoyment or disutility of which cannot be denied to other consumers once produced. That is the fact the one consumer consumes the good, does not mean that another consumer can't consume the same good as well. Some examples of public goods include public walkways, streets, bridges, a armed force etc. When should we provide for a public good? How do we decide how much of a public good to produce?

1 Rationale for Public Goods

Assume that there are two types of individuals in the general population that stand to enjoy the benefits of the country having a standing army. For all intent and purposes we can think of the two groups of citizens as represented by one individual. That way we can abstract away the size of the population at large. So we have two agents. Let the first be called Peace, (denoted by the subscript p), and the other War (denoted by the subscript w). They both know their country needs an army, but under what conditions would and should this be provided. Let the wealth of the respective groups be y_p and y_w. Let the contributions that each of them are willing to pay for the army be a_p and a_w respectively. Let's suppose the army is the only public good they are thinking of providing since some mysterious prior inhabitants of their land had given them paved roads, and a solar powered grid! Don't we all wish! But suppose providence is really that magnanimous. This leaves them with only private goods, which they purchase using the remaining funds available, x_p and x_w. Therefore, the budget constraint is,

$$x_p + a_p = y_p$$

and

$$x_w + a_w = y_w$$

Let the cost of the army, salary, equipment and training, together comes to a, so that $a \leq a_p + a_w$. Let the collective welfare of War be $U_w(x_w, \P_a)$, and for Peace, $U_p(x_p, \P_a)$, where $\P_a$ is an indicator function which takes on the value of 1 if the country gets an army, and 0 otherwise. It also says that each group's welfare is dependent on the amount of consumption of private and the public good. Note further that their individual welfare is dependent on their own level of private good consumption, their consumption of the public good does not deny the other group of its enjoyment. We will adopt the typical assumption, that utility is increasing in a good.

How much would each group be willing to pay for the good? First note that since an army by assumption is a good,

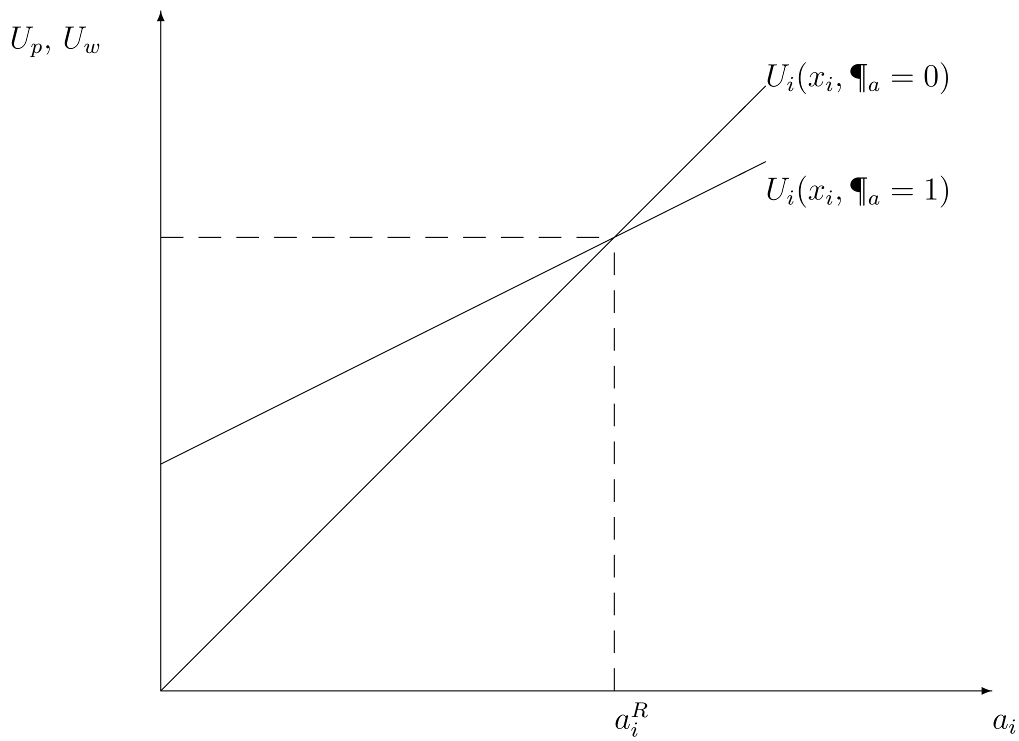
$$U_i(y_i - a_i, \P_a = 1) \geq U_i(y_i - a_i, \P_a = 0)$$

where $i \in \{w, p\}$. We also assume that the welfare function is not only increasing, but concave in private consumption. Next assume that y_i, $i \in \{w, p\}$ is large enough such that there will be a point where the welfare without the army will intersect the utility with the army once and only once (Single Crossing Property). This would mean that there will be a point where the two welfare functions under different circumstances would meet. It is at this point we have,

$$U_i(y_i - a_i^R, \P_a = 1) = U_i(y_i, \P_a = 0)$$

We call the level of price where an individual is just indifferent between two choices a reservation price, and in our case here, the reservation price to group i being just indifferent to the two choices having the army and not is a_i^R. This is represented below where we assume that having an army alters both the intercept and slope of the utility/welfare function.

Figure 1: Reservation Price of an Army for War or Peace



An allocation is Pareto Efficient if and only if there is no other allocation that can one party better off without making others worse off. If an allocation is Pareto Inefficient, than we have that possibility, or we say **Pareto Improvement** is possible.

In the case at hand, if no army is formed, the allocation is described as $(y_p, y_w, 0)$, while if an army is formed, the allocation is in turn $(x_p = y_p - a_p, x_w = y_w - a_w, 1)$. Starting from the first allocation where there is no army, the question we have to now ask is when should the army be provided? In answering this question, we have to examine whether there is a Pareto Improvement by providing an army, that is,

$$U_i(y_i, ¶ = 0) < U_i(x_i = y_i - a_i, ¶ = 1)$$

We can use the definition to rewrite the above as

$$U_i(y_i - a_i^R, \P_a = 1) = U_i(y_i, \P_a = 0) < U_i(x_i = y_i - a_i, \P = 1)$$

Observing the elements within the utility function at the beginning and end of the inequality, you should realize that since utility is increasing in private good enjoyment, the inequality would be fulfilled if and only if

$$y_i - a_i^R < y_i - a_i$$

$$\Rightarrow a_i^R > a_i$$

which says then that Pareto Improvement is possible only when the reservation price of the public good, which in our story is an army, is greater than the price the individual(s) need to pay to get the public good. Put another way, what this means is that individual(s) would want the public good, if and only if the price they pay is greater than their willingness to pay.

Next note that as long as both individuals utility function meets the above criterion, we have

$$a_p^R + a_w^R \geq a_p + a_w \geq a$$

Then it will always be possible to find a price that both individuals would be willing to pay, and that Pareto Efficiency would be achieved. The above is a **sufficient condition** for the existence of Pareto Improvement. There are some additional notes,

1. The condition for whether there is a Pareto Improvement available is dependent solely on the individuals' willingness to pay, and the total cost of the public good.

2. In general, whether it is or it isn't Pareto Efficient to provide a Public Good is dependent on the initial endowment of wealth across the populace. This is because an individuals reservation price is dependent on their endowment of wealth, as well as their preferences. The greater the endowment to someone with a strong preference for the public good would mean that her reservation price would be high, i.e. her willingness to pay would be high. However, if the higher endowment is granted to someone who has little inclination for the public good, than the reservation price would be low. Taken together, this would mean that

under some initial allocations, we would be able to meet the sufficiency condition for public good provision, and others we would not.[1]

2 What happens when Public Goods are provided for Privately?

However, just because it is Pareto Efficient to provide the good does not mean that it will actually be provided! Well, ask yourself the following, if everyone's willingness to pay is greater than the cost of the public good, that is any one person could pay for the provision, what is the incentive to truthfully reveal? You can imagine the typical individual would think twice about revealing his willingness to pay, in hopes that the other individual(s) would pick up the tab! Wouldn't you? We say the individual who doesn't want to reveal her true reservation price so that the other pays for it as **Free Riding** by the individual, and the problem is referred to as the **Free Rider Problem**.

3 Choice of Levels of a Public Good

We have examined the provision of an army as a dichotomous decision, which may be naive, since a country does have a choice as to how large the army should be: A section, platoon, company, battalion, brigade, division, army, number of armies? Suppose we really want just one army, do we arm them with bows and arrows, and a swiss knife, or something more sophisticated? Chainlink suit, or bullet proof vest? You get the idea.

Let us now maintain the previous setup of the model with the exception of the choice of the public good, or army, be $\mathbf{A}$. Further let the cost of the provision now be $c(\mathbf{A})$. This means that the budget constraint facing War and Peace is,

$$x_w + x_p + c(\mathbf{A}) = y_w + y_p$$

[1]However, if you recall the idiosyncratic outcome where externality output is independent of the endowment when utility is quasilinear is applicable here as well when the numeraire good is the private good, over some levels of wealth. The latter caveat is because there is always the binding constraint where the reservation price must be less than actual wealth available.

Without lost of generality, let us examine the problem from the perspective of War. In making her choice, she knows she has to ensure Peace would be happy with the final choice, which we denote at the minimum should be at $\overline{U}_p$. The problem then can be written as,

$$\max_{x_w, x_p, \mathbf{A}} U_w(x_w, \mathbf{A})$$

which is subject to,

$$U_p(x_p, \mathbf{A}) \geq \overline{U}_p$$

$$x_w + x_p + c(\mathbf{A}) = y_w + y_p$$

Since War can always make himself better off by just ensuring Peace gets the minimum level of utility, the first constraint can be written as a equality constraint. The consequent Lagrangian is,

$$L = U_w(x_w, \mathbf{A}) + \lambda_1 \left(\overline{U}_p - U_p(x_p, \mathbf{A})\right) + \lambda_2 \left(y_w + y_p - x_w - x_p - c(\mathbf{A})\right)$$

It is easy to see that the first order conditions are,

$$\frac{\partial U_w}{\partial x_w} - \lambda_2 = 0 \tag{1}$$

$$\frac{\partial U_w}{\partial \mathbf{A}} - \lambda_1 \frac{\partial U_p}{\partial \mathbf{A}} - \lambda_2 \frac{\partial c(\mathbf{A})}{\partial \mathbf{A}} = 0 \tag{2}$$

$$-\lambda_1 \frac{\partial U_p}{\partial x_p} - \lambda_2 = 0 \tag{3}$$

The first order conditions for the multipliers λ_1 and λ_2 are easily found and has not been noted above. You will need it to solve for the actual values of the choice variables if the functional form of the utility and cost functions were given to you.

From equations (1) we have,

$$\frac{\partial U_w}{\partial x_w} = \lambda_2$$

Substituting it into (3) gives the following

$$-\lambda_1 \frac{\partial U_p}{\partial x_p} = \lambda_2 = \frac{\partial U_w}{\partial x_w}$$

$$\Rightarrow -\lambda_1 = \frac{\frac{\partial U_w}{\partial x_w}}{\frac{\partial U_p}{\partial x_p}}$$

Substituting the last equality into (2) gives the equilibrium condition,

$$\frac{\partial U_w}{\partial \mathbf{A}} + \frac{\frac{\partial U_w}{\partial x_w}}{\frac{\partial U_p}{\partial x_p}} \frac{\partial U_p}{\partial \mathbf{A}} - \frac{\partial U_w}{\partial x_w} \frac{\partial c(\mathbf{A})}{\partial \mathbf{A}} = 0$$

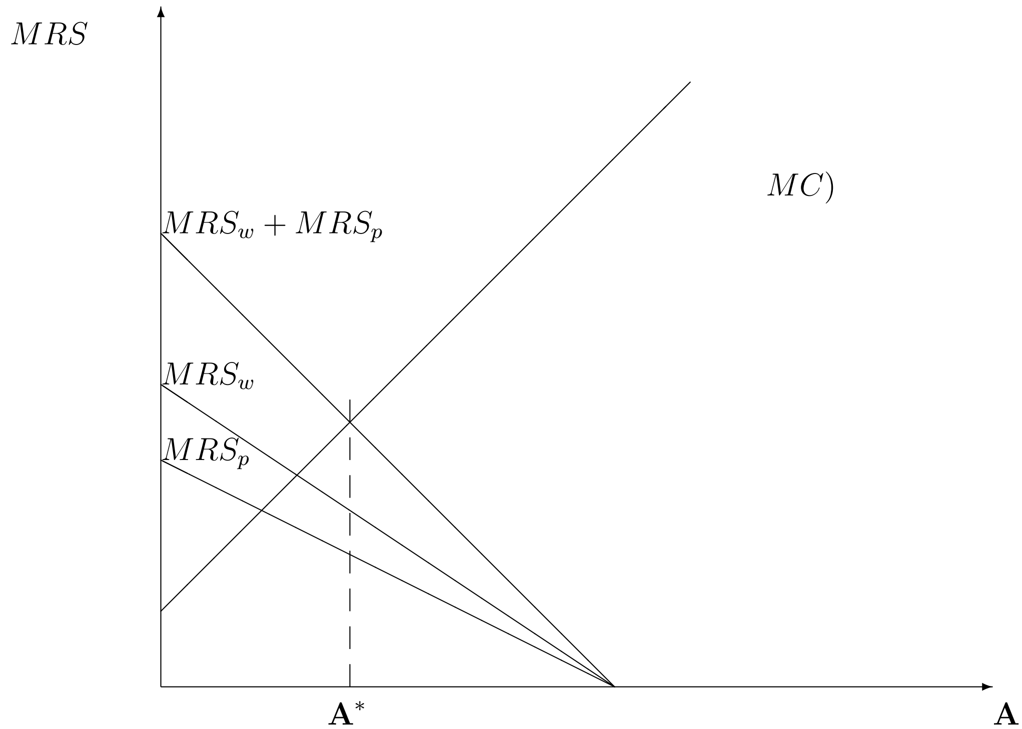
$$\Rightarrow MRS_w + MRS_p = MC(\mathbf{A})$$

or

$$\frac{\Delta x_w}{\Delta \mathbf{A}} + \frac{\Delta x_p}{\Delta \mathbf{A}} = MC(\mathbf{A})$$

Intuitively, what the condition is saying that Pareto Efficiency is achieved only when the sum of the marginal willingness to pay for an additional unit of army in exchange with the private consumption good is just equal to the marginal cost of providing the army at the optimal level of the army. If the sum of the marginal willingness to pay for the army exceeds the marginal costs of its provision, Pareto Improvement is possible by increasing its provision. Notice that in equilibrium the marginal rate of substitution can be different for all individuals for the consumption of the public good/army. However, recall the equilibrium condition for private good in general equilibrium (The Exchange Economy), which would have the individual marginal rate of substitution of the constituents being the same, failing which there would be gains to trade. Note further in private goods, consumption levels are in general different, but in public goods, the consumption levels are the same. Diagrammatically, the choice of public good is illustrated in the following diagram,

Figure 2: Efficient Level of Public Good Consumption



4 Quasilinear Preference & Public Goods

Just as in the case of externality, when agents have quasilinear utility, there will be a unique level of public good regardless of the allocation of private good, if the private good is the numeraire good. To see this, maintain the same example as before, but giving the utility function a functional form of the following,

$$U_i(x_i, \mathbf{A}) = x_i + \nu_i(\mathbf{A})$$

, where ν_i is a increasing, and concave function. This means that the marginal utility of private good consumption is always one, so that the marginal rate of substitution will be dependent solely on the level of **A**. That is,

$$MRS_i = \frac{\Delta U_i(x_i, \mathbf{A})}{\Delta \mathbf{A}}$$

This means that the Pareto Efficient level of the army is

$$MRS_w + MRS_p = \frac{\Delta U_w(x_w, \mathbf{A})}{\Delta \mathbf{A}} + \frac{\Delta U_p(x_p, \mathbf{A})}{\Delta \mathbf{A}} = MC(\mathbf{A})$$

Since ν_i is increasing and concave, and the above equilibrium is not dependent on x_p nor x_w, it uniquely determines the Pareto Efficient level of the army provided. This means that to find the Pareto Set merely involves maintaining the public good level which is defined above, and redistributing the endowment of the private goods.

5 The Free Rider Problem

The next question to ask is how we can achieve the Pareto Efficient level of Public Good Provision in general. We know in the case of private goods that the market mechanism will take the economy to a competitive equilibirum which by the welfare theorems are equivalent to Pareto Efficient allocations. Does it apply in the case of public goods.

An individual thinking about his contribution to the provision of a public good, and keeping with the example thus far, the army provision, must formulate a believe system about his fellow citizen's contribution choice or strategy as well. Without lost of generality, let's focus on the choice from the perspective of Peace now. Suppose Peace believes that War's contribution to the army is $\overline{a}_w$. Then a selfish Peace, who disregards the effect her choice has on War, would make her optimal choice by solving the following problem,

$$\max_{x_p, a_p} U_p\left(x_p, \mathbf{A} \equiv \mathbf{A}(a_p + \overline{a}_w)\right)$$

subject to,

$$x_p + a_p = y_p$$

This problem can be more easily solved by reducing it to a unconstrained problem,

$$\max_{a_p} U_p\left(y_p - a_p, \mathbf{A} \equiv \mathbf{A}(a_p + \overline{a}_w)\right)$$

And the first order conditions is,

$$\frac{\partial U_p}{\partial x_p} = \frac{\partial U_p}{\partial \mathbf{A}}$$

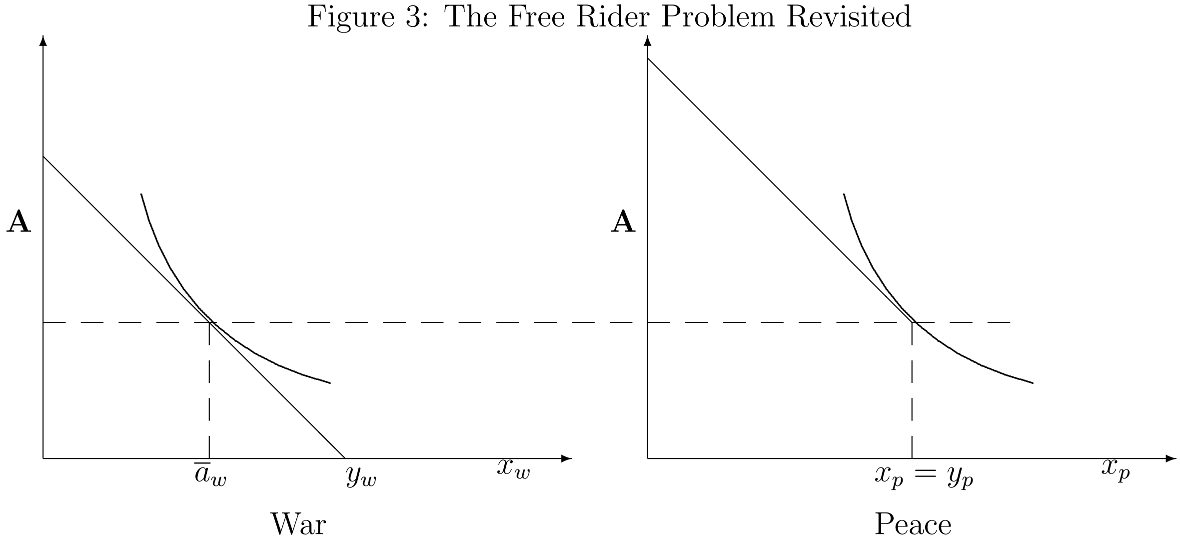
$$\Rightarrow MRS_p = 1$$

It is easy to see that the condition for War is likewise,

$$MRS_w = 1$$

Thus together, all we get is the competitive equilibrium. However, there is an additional idiosyncracy about public goods that this model does not capture. The solution provided ignores the fact that the solution needn't be an interior solution. What is $a_p = 0$ or $a_w = 0$. That is we have assumed $a_i > 0$, where $i \in \{w, p\}$. The structure of such a problem would require knowing Kuhn-Tucker Conditions which is outside of the curriculum at the moment. (Ask me if you wish to know more). Intuitively, what we need to know is the following, suppose the solution to the above problem for peace is a_p^*. How does this solution compare to the situation if Peace freerides off of War, which given their names, isn't far fetched. When an individual contributes to the level of the army, she cannot deny the other agents from consuming it. Consequently, she has to consider if it is really necessary to increase it. Whether the current level given War's contribution is sufficient? That is Peace needs to ask if $U_p(x_p^*, \mathbf{A}(a_p^* + \overline{a}_w))$ is greater than $U_p(y_p, \mathbf{A}(\overline{a}_w))$, which is a legitimate concern given the characteristic of a public good. Diagrammatically, this is represented below,

Figure 3: The Free Rider Problem Revisited

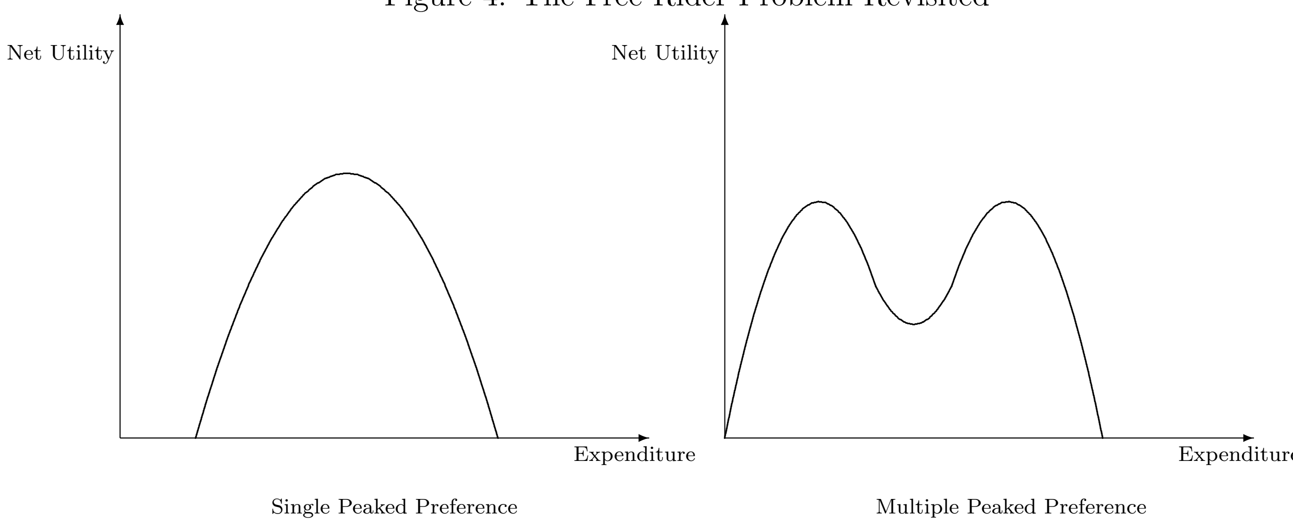


This essentially say that the market competitive mechanism where we use prices as signals will generally not lead to Pareto Efficient allocations. How do we then achieve Pareto Efficiency. This might be achievable via Social Mechanisms such as **Command Mechanisms** where power of the choice is vested with a small group, or a single individual. Another method is through the **Voting System**.

6 Voting

A full discussion is not possible, but we'll try to glean some insights. We have already found out about the **Paradox of Voting**. We will discuss briefly how by making some assumptions we could avert the primary problem with voting, the issue of that social preferences isn't transitive. It turns out that if we can restrict individual preferences over the expenditure due to her as **Single Peaked**, the former would be averted. This condition is represented below.

Figure 4: The Free Rider Problem Revisited



This assumption is quite natural, since we can imagine an individual initially preferring to increase her expenditure on the public good but she will realize that each increase in public good involves more taxes on her (and consequently we refer to utility as net utility). As long as preferences are single peaked, voting will lead to **Median Expenditure**, where we have half of the individuals preferring to increase expenditure, and the other half preferring to decrease it. However, this does not in general imply that the allocation of public good in the society will be Pareto Efficient. Further, individuals may still choose to vote differently from their preferences so as to manipulate the final outcome.