Intermediate Microeconomics II, ECON 301
Oligopoly

Our analysis of of firm interaction in the perfective competitive and monopolistic structure has assume two extremes where in the former, we have so many firms to the extent that none exerts any effect on the choice of the other, to the latter where there is none to be concerned with. We will now concern with the intermediate case where we have only a small number of firms who interact with each other, that is an **Oligopolistic Structure**. We are not concerned with how or why firms might want to differentiate themselves one from another, but focus on the strategic choices when they cannot be but the same. In most of our considerations, we will be dealing with the smallest form of an Oligopoly, that of a **Duopoly**, i.e. when there are just two firms.

1 Choice of Strategies

In allowing the 2 firms to interact with one another strategically, we have variables of interest; the price that each set, and the quantity that they each produce.

There are four classfications of how firms can interact with each other, and the consequent choice of the four variables in question. The first two classifications belong to the subgroup of games referred to as **sequential move games**, where firms make their choices one after another in sequence. The next two classifications deal with the class of interaction where firms make their choice variables simultaneously, that is neither knows what the other has done. The last classification does not involve any real strategic interaction, but examines how firms could achieve a higher outcome but behaving as a single unit, and examining when such cooperation is feasible.

1. **Sequential Quantity Setting Game (Quantity Leadership or Stackelberg Model)**: This is the scenario where firms compete on quantity, where one firm makes its choice of quantity first, and the second firm makes its quantity choice with the knowledge of the choice of the first firm. We call the firm who makes the first choice the **quantity leader**, and the second firm, the **quantity follower**.

2. **Sequential Price Setting Game(Price Leadership)**: This is analogous to the above game but where instead we have one firm setting its price first, the **price leader**,

and the second firm making its pricing choice with the knowledge of what the price leader sets, and we call it the **Price Follower**.

3. **Simultaneous Quantity Setting Game(Cournot Model)**: Both firms choose to set quantity simultaneously.
4. **Simultaneous Price Setting Game(Bertrand Model)**: Both firms choose to set price simultaneously.
5. **Collusions and Cooperative Games**: We allow firms to collude with each other, and behave as a single firm, and the firm could maximize their joint profits with respect to either price or quantity choice. This kind of collusive behavior is call a cooperative game.

2 Quantity Leadership

The Stackelberg Model is typically used to describe industries where there is a dominant firm or a natural leader, and other smaller firms being residual claimants to the demand, and becoming followers. The assumptions in this model are as follows,

1. 2 firms in the industry, where without lost of generality, we have firm 1 being the leader, and firm 2 being the follower.
2. Both firms compete by making their choice on quantity. Let the choice made by firm 1 be q_1, and the choice made by firm 2 being q_2.
3. Firms manufacture the same homogenous product.
4. Price in the market depends on the joint output. Let the inverse demand for the market be a continuous increasing twice differentiable function $P(.)$. So that given the quantity choices, the price in the market is $P(Q = q_1 + q_2)$.
5. Let each firm have a different increasing, continuous, convex, cost function $c_i(q_i)$, where $i \in \{1, 2\}$.

The question then is how would each firm make their individual choices given the structure of the game where firm 1 moves first, while firm 2 moves second. Firm 1 being the first mover can rely only on how he believes firm 2 would react to its choice, of course believing that

firm 2 would maximize its profits. Given that reaction, it could then makes its choice that would maximize its profit. This then suggests that the best manner in which to obtain a solution is first to examine the solution of firm 2 given a particular choice of firm 1. Then given that reaction of firm 2 to any choice of firm 1, firm 1 could make the best choice. This sequence of solution is known as **backward induction**.

1. **Follower's Problem**: The follower, firm 2, given a particular choice in price made by firm 1, chooses q_2 to maximize its profits,

$$\max_{q_2} P(q_1 + q_2)q_2 - c_2(q_2)$$

Then the optimal choice of firm 2 can be described by the first order condition to the above problem,

$$\begin{aligned} \frac{\partial P(q_1+q_2)}{\partial Q}q_2 + P(q_1+q_2) - \frac{\partial c_2(q_2)}{\partial q_2} &= 0 \\ \Rightarrow MR_2 = \frac{\partial P(q_1+q_2)}{\partial Q}q_2 + P(q_1+q_2) &= MC_2 = \frac{\partial c_2(q_2)}{\partial q_2} \end{aligned}$$

Which gives nothing but the usual marginal revenue equal to marginal cost condition. However, this is not a solution because this only describes what firm 2 would optimally do upon seeing what the quantity leader does. You can see this since the quantity of firm 1 is in the condition, q_1. That is the above equation provides the following relationship,

$$q_2 = R_2(q_1)$$

That is it tells us how firm 2 would react to the quantity choice by firm 1 or the quantity leader, and we consequently call this a **reaction function** of firm 2. But how does this reaction function look like in relation to the choice of quantity made by firm 1. That is what is the slope of the reaction function with respect to q_1. We can discover this by differentiating the first order condition with respect to q_1

$$q_2\frac{\partial^2(P(Q))}{\partial Q^2}\left(1+\frac{\partial q_2}{\partial q_1}\right) + \frac{\partial P(Q)}{\partial Q}\frac{\partial q_2}{\partial q_1} + \frac{\partial P(Q)}{\partial Q}\left(1+\frac{\partial q_2}{\partial q_1}\right) - \frac{\partial^2 c_2(q_2)}{\partial q_2^2}\frac{\partial q_2}{\partial q_1} = 0$$

$$\Rightarrow \frac{\partial q_2}{\partial q_1} = \frac{-\left\{\frac{\partial^2 P(Q)}{\partial Q^2} + \frac{\partial (P(Q))}{\partial Q}\right\}}{\left\{\frac{\partial^2(P(Q))}{\partial Q^2} + 2\frac{\partial(P(Q))}{\partial Q} - \frac{\partial c_2(q_2)}{\partial q_2}\right\}} \leq 0$$

The last inequality follows since for the numerator the inverse demand is decreasing in Q, and it is has to be concave for total revenue to be concave, while for the denominator,

it has also got to be concave since otherwise the profit maximizing problem is not well behaved. This then means that the reaction function for firm 2 will be negatively related to the choices made by firm 1.

2. Leader's Problem: Given the reaction function of firm 2, and assuming there is complete information, i.e. no asymmetry in information, the choice that firm 1 makes will then have to account for the reaction coming from firm 2. Then firm 1's problem is to solve the following,

$$\max_{q_1} P(Q)q_1 + c_1(q_1)$$

but it must be subject to the additional condition of the manner in which firm 2 would behave in reaction, that is the choice by firm 1 is constrained by $q_2 = R_2(q_1)$, and we can rewrite the problem as a unconstrained problem of

$$\max_{q_1} P(q_1 + R_2(q_1))q_1 + c_1(q_1)$$

A Linear Demand Example: We can see the implications more clearly by assuming a particular functional form for the demand, specifically assume that $P(Q) = a - b(q_1 + q_2)$, and using backward induction, we first solve for the reaction function for firm 2 by maximizing the profit function for firm 2 with respect to q_2 (assume for simplicity that the cost function for both firms be zero),

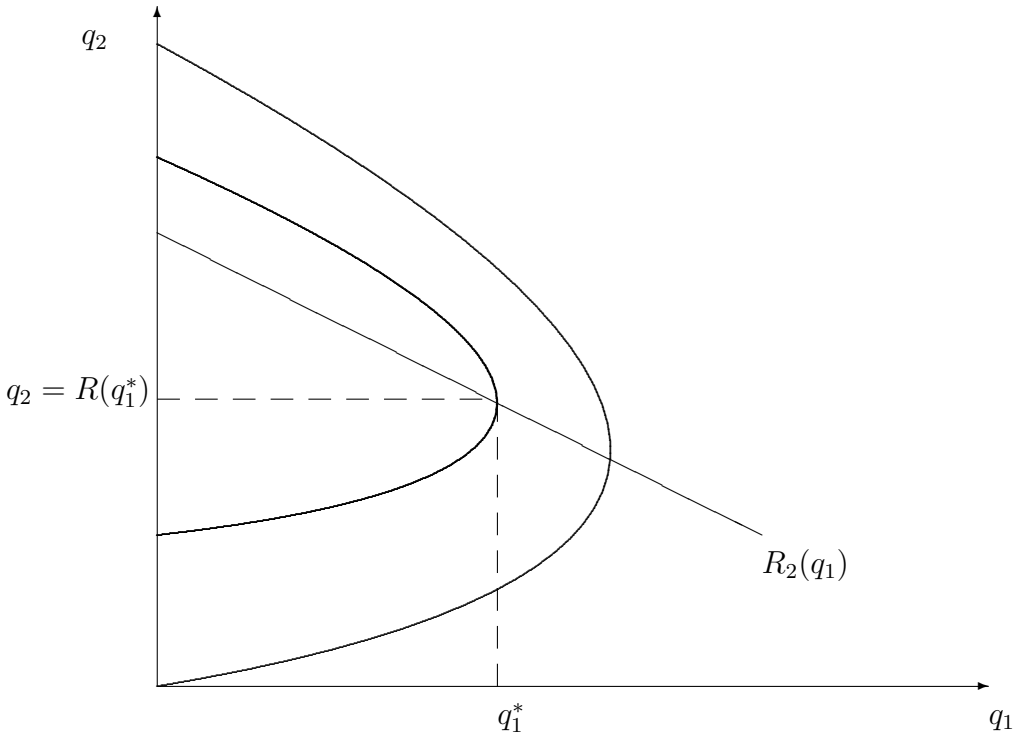
$$\max_{q_2}(a - b(q_1 + q_2))q_2$$

So that the reaction function is,

$$\begin{aligned} -bq_2 + (a - b(q_1 + q_2)) &= 0 \\ R_2(q_1) = q_2 &= \frac{a - bq_1}{2b} \end{aligned}$$

Note that the reaction function is decreasing in q_1. We can also intuitively derive the reaction function by noting that the profit function describes a sequence of isoprofit curves that are concave in q_2 for the couplet (q_1, q_2). This is depicted in the figure below. Next note that the value of the isoprofit is increasing as it moves to the left since that is the direction where q_1 is decreasing. For a given quantity choice by firm 1, q_1, firm 2 wants to pick the isoprofit that is furthest to the left of the vertical line extended from firm 1's choice of q_1. The optimal choice for firm 2 occurs when this vertical line is just tangent, which is the equivalent to our usual tangency condition with respect to cost.

Figure 1: Derivation of a Reaction Function



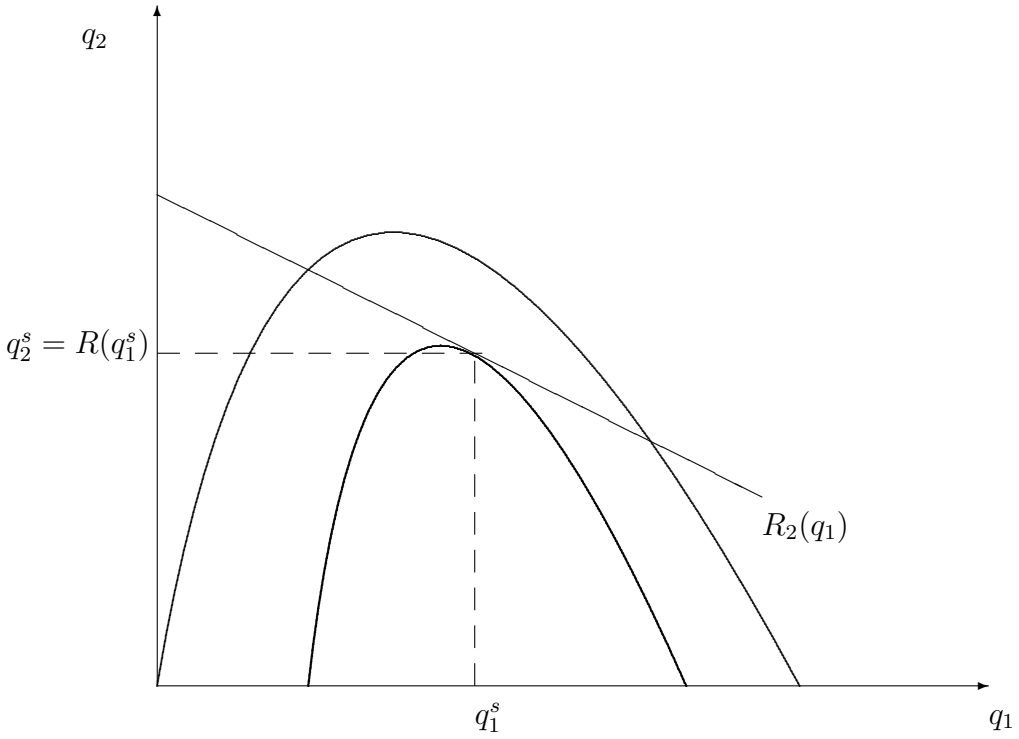
To solve for the optimal choice of firm 1, all we need to do as noted above is to substitute the reaction function of firm 2 into firm 1's profit function,

$$\begin{aligned} & \max_{q_1}(a - b(q_1 + q_2))q_1 \\ \Rightarrow \quad & \max_{q_1}(a - b(q_1 + \frac{a - bq_1}{2b}))q_1 \\ \Rightarrow \quad & \frac{aq_1}{2} - \frac{bq_1^2}{2} \\ \Rightarrow \quad & q_1^s = \frac{a}{2b} \end{aligned}$$

Substituting the above solution back to firm 2's reaction function, it is easy to see that $q_2^s = \frac{a}{4b}$, and the total output for this industry is $\frac{3a}{4b}$. Diagrammatically, this solution is represented below. As before the profit function allows us to derive the set of isoprofit curves, which is concave in q_1. The solution is consequently at the tangency between the reaction function of firm 2 to one of the isoprofit lines in the set. Note again the lower isoprofit curves correspond with higher profits, that is as it shrinks, it in effect is tending towards the monopoly choice since q_2 is tending towards 0. The tangency is intuitive since

the reaction function is given to firm 1, so that given the reaction function, the best firm 1 can do is to keep moving to a lower level of the isoprofit, which corresponds to a higher level of profit until tangency is reached as in the diagram below.

Figure 2: Stackelberg Equilibrium



An important insight from the Stackelberg Model is that there is a **First Mover Advantage** in the sense that the first mover obtains a larger share of the market, and consequently a greater level of profits.

3 Price Leadership

If we adopt a similar assumption as in sequential quantity consumption such that the firms produces the same homogenous product and where if we further assumed the same marginal cost for both firms at c, but that firm 1 moves first by choosing prices and suppose that the price chosen is $p_1 > c$, then it is always optimal for firm 2 to undercut firm 1 when it comes its turn, and thereby obtaining the entire market. Given this reaction, it is only optimal for

firm 1 to always choose to set $p_1 = c$, and firm 2 would do likewise. Consequently, there is a **Last Mover Advantage** here. In general it is not fruitful to examine price choices in homogenous products since the competition is so intense (as you will notice in Bertrand Model) that we always obtain the competitive equilibrium. However, if instead we assume the goods are differentiated, and substitutable, we can then perform a similar analysis as in Stackelberg Model, but what will be discerned is that there will be instead a **Last Mover Advantage**, highlighting the intensity of price competition. **Make it a point to develop such a model yourself, using linear demand of the form,** $q_i = a - bp_i + dp_j$**, where** $i \neq j$ **and** $i, j \in \{1, 2\}$.

4 Simultaneous Quantity Setting: Cournot Competition

Cournot competition is one where firms simultaneously choose their optimal quantity produced instead of prices. The manner in which we derive a solution is through examining what the **best strategy each has given <u>their believes</u> in what their competition would do**.

Before we begin, as usual we have to stipulate the assumptions:

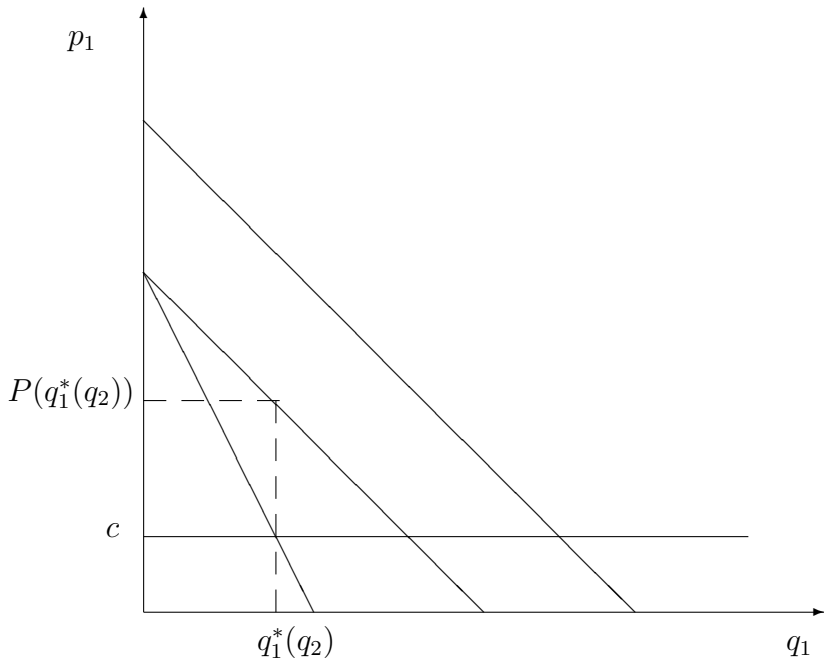
1. There are two firms (though the problem can be generalized to the mulitple firm case), $i \in 1, 2$.
2. Firms produce a homogenous product.
3. Firms choose optimal quantity produced simultaneously.
4. Marginal Cost of production are the same for both firms, c.

4.1 A Description of the Process

Let the output of each firm be q_i. The price that is sold is ultimately dependent on the joint choices of both firms, i.e. $P \equiv P(q_1 + q_2)$. That is given what firm j chooses, firm i's choice will ultimately affect the prices of the market. If we were to plot this, what we will derive is the residual demand of the firm in question. Essentially, given this residual demand, each

firm will then make their choices as if they were a monopoly in order to maximize their profit, i.e. by setting marginal revenue equal to marginal cost.

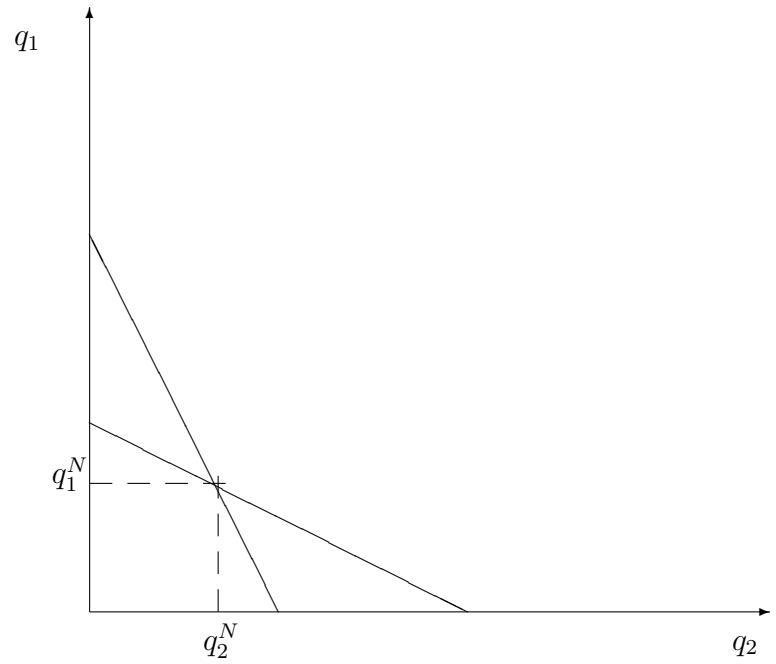
Figure 3: Quantity Choices



Considering some extreme considerations; suppose firm 2 chooses to produce nothing, then the best that firm 1 can and would do is to produce the monopoly quantity. On the other hand, if firm 2 chooses to produce at the competitive level, in which case, the best that firm 1 can do is to produce nothing. This illustrates how each firms choices are tied to each other. We call, just as in the case of Bertrand competition, $q_i(q_j)$ a reaction function of i, where $i \neq j$, $i, j \in 1, 2$. The relationship, as you may discern is decreasing in the choice of the other firm, since the more the other firm chooses, the **Residual Demand** would be smaller, i.e. limiting the choices of the firm in question.

If we were to plot the choices of each firm given the other's choices, we would get a reaction function, as in the Bertrand case. Whereas in the latter, the reaction function is upward sloping, the case for Cournot competition is downward sloping since as noted before, the greater the choice of the competition, the smaller the residual demand.

Figure 4: The Cournot Equilibrium



4.2 A Simple Algebraic Model

Given the intuition and insights we can now examine a algebraic example. Let the demand of the market be $P(Q) = a - bQ$, where $Q = q_1 + q_2$. Each firm would choose to maximize their profit which given constant marginal cost is,

$$\pi_i = Pq_i - cq_i$$

$$\Rightarrow \pi = aq_i - bq_i^2 - bq_iq_j - cq_i$$

Their first order condition would be,

$$a - 2bq_i - bq_j - c = 0$$

$$\Rightarrow R_i(q_j) = q_i = \frac{a - bq_j - c}{2b}$$

where $i \in 1, 2$. In equilibrium, since all firms are **symmetric**, $q_i = q_j$, which means that

$$\Rightarrow R_i(q_j) = q_i = \frac{a - c}{2b} - \frac{q_j}{2}$$

$$\Rightarrow q_i^* = \frac{a - c}{2b} - \frac{q_i^*}{2}$$

$$\Rightarrow q_i^* = \frac{a-c}{3b}$$

And the equilibrium price is,

$$P^* = a - \frac{2(a-c)}{3}$$
$$\Rightarrow P^* = \frac{a+2c}{3}$$

which is greater than the marginal cost of c. Note further that this duopoly's output is greater than the monopoly's but less than it would have been under perfect competition. Consequently, duopoly's prices are greater than perfect competition, but less than monopoly's. **Can you show this is true? How does the equilibrium quantity and prices change as the number of firms increase? What if the marginal cost of the firms are not the same, that is** $c_1 \neq c_2$

5 Simultaneous Price Setting: Bertrand Competition

Firms can compete on several variables, and levels, for example, they can compete based on their choices of prices, quantity, and quality. The most basic and fundamental competition pertains to pricing choices. The Bertrand Model is examines the interdependence between rivals' decisions in terms of pricing decisions.

The assumptions of the model are:

1. 2 firms in the market, $i \in \{1, 2\}$.
2. Goods produced are homogenous, $\Rightarrow$ products are perfect substitutes.
3. Firms set prices simultaneously.
4. Each firm has the same constant marginal cost of c.

What is the equilibrium, or best strategy of each firm? The answer is that both firms will set the same prices, $p_1 = p_2 = p$, and that it will be equal to the marginal cost, in other words, the perfectly competitive outcome. This is a very powerful model in that it says that price competition is so intense that all you need is two firms to achieve the perfect competitive outcome. We will show this through logical arguments and contradictions, as well as through the use of a diagram.

Using logical arguments:

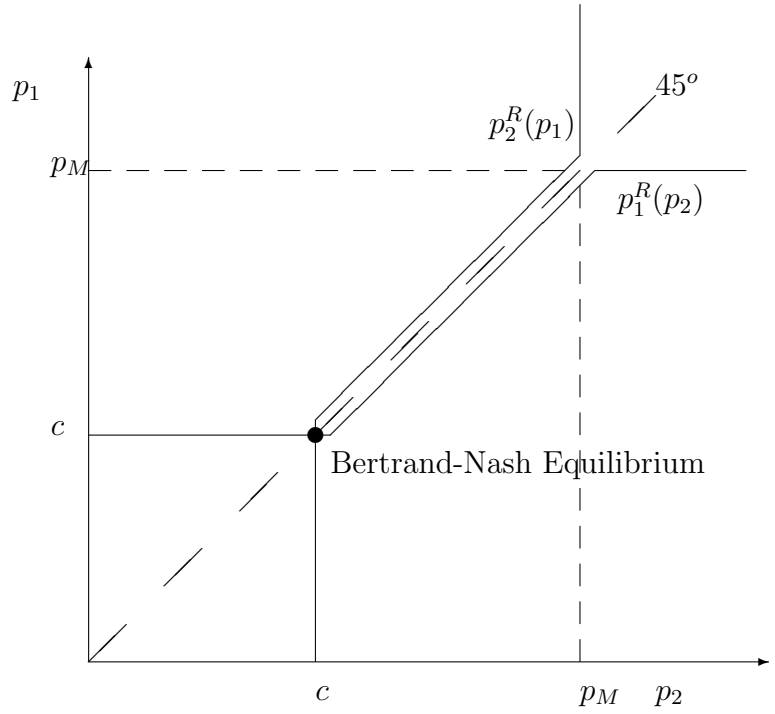
1. **Firm's will never price above the monopoly's price**: Suppose not. And suppose firm 1 believes that firm 2 would choose a price p_2 above the monopoly's price, then the **best response** of firm 1 is to price at the monopoly's price since at that point, its profit is maximized. And firm 2 would be driven out of the market. Therefore no firm would ever price above the monopoly's price.

2. **In equilibrium, all firm's prices are the same**: Suppose firm 2 chooses to price at the monopoly's price, what is the best response of firm 1? Firm 1 would realize that by pricing at a slightly lower price, it would be able to capture the entire market since the goods are perfectly substitutable, that is $p_1 = p_M + \epsilon$, where p_M is the monopoly's price , and $\epsilon > 0$. Then only one firm is left. Therefore the equilibrium where firms charges a different prices cannot be an equilibrium, $p_1 = p_2 = p$.

3. In equilibrium, prices must be at the marginal cost: Suppose not, than $p_1 = p_2 = p > c$. However, either firm would always find it is in their best interest or their best response to under cut its competition and obtain the entire market for itself, by reducing its prices a little bit more, say $\epsilon > 0$. By induction, it is in fact not possible then to have an equilibrium above the marginal cost, since it is only at the marginal cost that firms have no incentives to deviate from the equilibrium prices.

$\therefore$ in equilibrium, $p_1 = p_2 = p = c$. Notice that in making the arguments we have always stated the firm's choice as a function of the other firm's choice, $p_i^*(p_j)$, where $i \neq j$, and $i, j \in \{1, 2\}$. This is known as a reaction function. Depicting our argument on a diagram with prices on both the axes. It is obvious that equilibrium is achieved only at the point where the reaction functions meet, since it is only at the intersection that each firms best response corresponds with the other's. Any other point cannot be an equilibrium since the actions that one believes the other would do would never be realized. Only at c does their expectations match, and the equilibrium is sound since both firms are the same, symmetric.

Figure 5: The Bertrand Model and Equilibrium



6 Collusion and Punishment Strategies

We have thus far treated firms' choice independently, in the sense that they act to optimize their own welfare or profit. But as we have seen, if they act as one, then the choice of monopoly or collusion outcomes are typically far better for the collective. When firms form into a collective unit, we call it a **Cartel**. We will now show that although profts are always higher, the firm will always have incentives to unilaterally deviate from the stated strategy of optimizing joint profits. Consider the simple case of two firms in a market, where the inverse demand is $P(Q = q_1 + q_2)$ for the entire market. Further let their individual cost of production be the different, $c_1(q_1)$ and $c_2(q_2)$ respectively for firm 1 and 2. Then cartel maximizing quantities are,

$$\max_{q_1,q_2} P(q_1 + q_2)(q_1 + q_2) - c_1(q_1) - c_2(q_2)$$

Then the optimal choice are just,

$$\begin{aligned} \frac{\partial P(q_1^c + q_2^c)}{\partial Q}(q_1^c + q_2^c) + P(q_1^c + q_2^c) &= \frac{\partial c_1(q_1^c)}{\partial q_1} = MC_1^c \\ \frac{\partial P(q_1^c + q_2^c)}{\partial Q}(q_1^c + q_2^c) + P(q_1^c + q_2^c) &= \frac{\partial c_2(q_2^c)}{\partial q_1} = MC_2^c \end{aligned}$$

The interesting interpretation of the optimization condition is that instead of just being concerned with the effect on themselves of raising an additional unit of quantity, the firm(or the cartel proper) is also concerned with the effect on the other firm, for example for the optimizing choice of firm 1, this "care"is represented by $\frac{\partial P(q_1+q_2)}{\partial Q}(q_2)$. Also note that the quantities are the same if and only if the marginal cost of production of both firms are the same, and that if they are different, the firm with the lower marginal cost will be allocated the higher production.

However, is there an incentive for firms to unilaterally deviate from this choice? That is will the Cartel unravelled in the face of its own greed! Let us just simply concern ourselves that only firm 1 is considering this possibility. Its profit under the collusion is

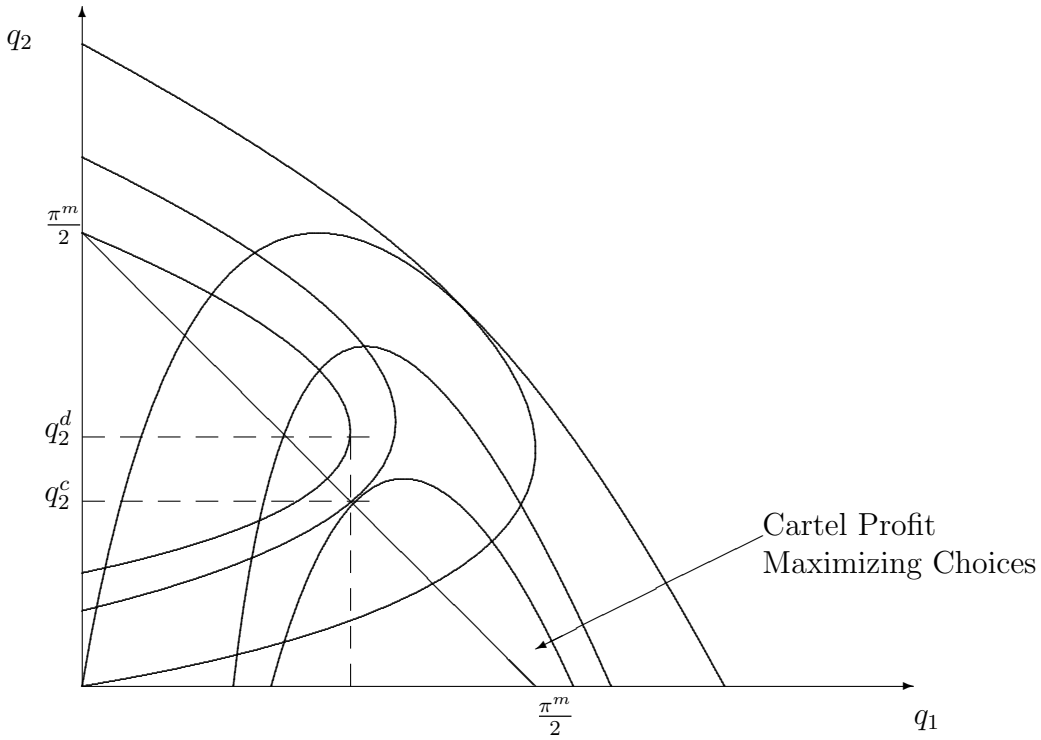
$$\pi_1^c = P(q_1^c + q_2^c)(q_1^c) - c_1(q_1^c)$$

To see if there is any incentive to cheat on the cartel deal, we need to show that there is marginal gains to profit if it did, that is $\frac{\partial \pi_1^c}{\partial q_1}$. We can do so, we can then examine the following,

$$\frac{\partial \pi_1^c}{\partial q_1} = \frac{\partial P(q_1^c + q_2^c)}{\partial Q}(q_1^c) + P(q_1^c + q_2^c) - \frac{\partial c_2(q_2^c)}{\partial q_1}$$

What we have to note then is that since from the cartel's first order condition with respect to q_1, that $\frac{P(q_1^c+q_2^c)}{\partial Q}(q_2^c)$ is greater than zero, which in turn implies $\frac{\partial \pi_1^c}{\partial q_1} > 0$, which implies it will always pay the firm to deviate. What this says intuitively is that if one firm knows the other will stick to its strategy of collusion, it will always pay to deviate, and increase production unilaterally. However, that is not all, in fact, if it believed that the other firm would deviate, it would try to deviate first to gain more profits first! To see this diagrammatically, assume a linear demand function. The result is below.

Figure 6: A Cartel



The line connecting the two points where we have both firms producing at the monopoly level reflect differing shares of the monopoly profits, the share of which would depend on their relative cost advantages. In the diagram above, the equilibrium corresponds to firm 1 producing more, which then implies that firm 1 would have a higher level profits, i.e. a higher share of the monopoly profits. That equilibrium corresponds to the point where the isoprofits are just tangent to each other. However, if one firm, say firm 1, sticks to its strategy of collusion, then firm 2 will always find it beneficial to deviate, which in the above diagram is q_2^d, when it deviates, instead of the collusion level which is q_2^c, and note that $q_2^d > q_2^c$. **Can you show the same ideas using instead Bertrand Competition?**

But we know it is mutually beneficial to stay the course in collusion, so the next question to ask is whether under what circumstances could collusion be maintained, and what kind of strategies must be played for the collusion to be sustainable. We will examine this next.

Consider the following model which for all intent and purposes is a Bertrand model setup, with the sole exception that the game is repeated and infinite number of times.

1. Homogenous Product

2. Duopoly, $i \in \{1, 2\}$
3. Firms set prices simultaneously
4. Constant marginal cost of c.
5. Firms engage in **Repeated Game** where they set prices in each period t, where $t \in \{1, 2, ..., \}$.

Such a model would be a more realistic depiction of the economy at large. The question then is what possible equilibrium would arise. We know from our previous discussion that the Bertrand-Nash Equilibrium for a one short game described by points 1 to 4 is where the firms price at their marginal cost. This would mean that one possible strategy by both firms is to price at marginal cost all the time, so that both firms earn zero profits in perpetuity. Since we know that in each period, such a strategy is yield a Nash Equilibrium, there is no possibility that either firm would deviate. However, just as we did when we examine Game Theoretic Concepts, we may want to consider the possibility that firms might want and be able to arrive at a Nash Equilibrium that would allow both firms to achieve higher profits.

Another possible strategy would be for the firms play a **Grim Trigger Strategy**. Each firm would price at the monopoly price in each period, p_m, so that the market as a whole earns a monopoly profit, π_m, which they agree to share equally, $\frac{\pi_m}{2}$. This strategy would continue as long as the other firm abide by the agreement, whether tacitly, or explicitly arrived at. However, if in the previous period the other firm were to choose not to price at p_m in one period, the firm would punish the other by pricing at c in perpetuity so that the aggregate profit for both firms is zero in perpetuity. We now need to understand under what conditions would this collusive agreement stand. We can do so by examining whether the payoff from playing the strategy survives at least **one deviation** from the strategy.

Let the discount factor of the firm be δ, and determines the value of each dollar in the following period in the current period. If both firms play the strategy, the payoff to each in perpetuity is,

$$\frac{\pi_m}{2} + \delta\frac{\pi_m}{2} + \delta^2\frac{\pi_m}{2} + ...$$
$$= \frac{\pi_m}{2}\left(1 + \delta + \delta^2 + ...\right) = \frac{\pi_m}{2}\frac{1}{1-\delta}$$

Alternatively, consider what the payoff to each player would be should they deviate while the other stays the course. Since both firms are the same, we only need to consider the case

of one firm. Should one firm deviate, it would realize that just by pricing $p_m - \epsilon$ would allow it to capture the entire market in that period, but it would in turn earn nothing in all subsequent periods in perpetuity, which would give it a lifetime payoff of π_m. Now we can bring the two payoff together. The strategy to achieve Collusion can be a Nash Equilibrium if and only if,

$$\begin{aligned} \frac{\pi_m}{2}\frac{1}{1-\delta} &\geq \pi_m \\ \Rightarrow \pi_m &\geq 2\pi_m - 2\delta\pi_m \\ \Rightarrow \delta &\geq \frac{1}{2} \end{aligned}$$

This means that as long as the **discount factor** is sufficiently large, the collusion equilibrium is sustainable. Typically, we think of the **discount factor**, $\delta \in [0, 1]$. The reason is as follows; the value of a dollar saved in a period yield $1 + r$ in the following period, where r is the **interest rate** for the saving. So that in turn, the value of a dollar tomorrow is the inverse of that, i.e. $\frac{1}{1+r} = \delta$. However, there may be other factors that can determine the discount rate which we will discuss shortly. But the crux of the matter is that in deciding whether to enter into a collusive agreement, the firms weight the long run gains versus the short run losses, or the argument is reversed when you think in terms of deviation.

Interest rates are typically thought of as a annual rate, however, **price changes can be more than once a year**, and the preceding discussion has left the duration of a period deliberately ambiguous, so that if the number of price changes per period is more than once a year, we have to adjust the true interest rate. Suppose the price changes f times a year, then the effective rate per period is $\frac{r}{f}$. This then mean that $\frac{1}{1+\frac{r}{f}} = \delta$.

Another possible factor is that of stochastic termination in each period, that is the market can seize to exist in any future period. Suppose the probability that the market can become obsolete is b. That is with probability b that the firm will get $\frac{1}{1+\frac{r}{f}}$ and probability $1 - b$ the firm would get zero. This means that the dollar tomorrow is worth $\frac{1}{1+\frac{r}{f}}b$.

The opposite of the market seizing to exist is that the market grows, such as the software industry. Suppose the market grows at a rate of g, then each dollar of profit is worth $1 + g$ in the following period.

Taking all this factors into considerations, the definition of δ can be expressed as,

$$\frac{1+g}{1+\frac{r}{f}}b$$

We can then say the following about the likelihood of a successful collusion;

Collusive Pricing is more likely if,

1. Frequency of price changes are high, that is a high f since in means increased interactions between the firms.
2. There is a high probability that the market will continue to survive, that is a high b.
3. The growth rate of the industry is high, that is g is high.

Since all the factors ensure a higher discount factor.