Production and Cost Analysis

The entire production process begins with the supply of factors of production or inputs used towards the production of a final good we all consume in the final good market. Some examples of these factors of production are the labor you will supply when you graduate, machines, raw materials such as pulp, power (such as gas, electricity), machines, factory complex, research laboratory etc. We will begin by assuming that the prices of factors of production are constant first. The reason for this initial simplification is that we want to understand first how and why firms choose a particular way of producing given those prices.

Objectives:
1. Differentiate between economic and accounting profit.
2. Distinguish between long and short run production.
3. Understand what is diminishing marginal productivity.
4. How to calculate the various cost measures, and concepts.
5. Distinguish between various cost curves, and describe their interrelationship, and individual shapes.
6. Understand the firm’s choice using the cost curves.

The role of the firm
The firm as an economic institution (or a component in the market economy machine) transforms or “converts” factors of production into goods and services. Precisely,
Organizes factors of production \(\rightarrow\) Produces the goods/services \(\rightarrow\) Sells to 1.individuals and/or 2.other businesses/firms and/or 3.the government.

The precise combination of activities and process is dependent on the cost of each process. If subcontracting out a process without any loss of quality is attainable, it would be sensible to do so. While for another, where quality and precision might be loss if a particular process is subcontracted out, the firm under consideration would not do so. Let us consider two examples.

1. Consider why most North American “Ready-to-Wear” clothing firms subcontract the clothing manufacturing process to China. The most common reason is often efficiency and cost gains in the process. Do you think there is any other reason for them to do so?
2. Most of your mountain bikes are manufactured by one of a few firms in Taiwan, the largest being Giant. Why do you think Cervelo is still manufactured out of Quebec, and why is Rocky Mountain bicycles are manufactured in British Columbia? Cervelo for example has proprietary rights over their aluminum tube construction, which may make subcontracting costly strategically, or if the suppliers of those tubes are based out of North America.

From the examples, and our earlier examination of demand and supply, we can make the following notes: How an economy operates, which activities are undertaken through the market or the firm through trading
1. depends on transaction costs, and
2. the rent or command over resources.
Consider another simple real life example: From it’s inception, Timbuk2 bags were manufactured out of San Francisco, and the marketing pitch was that it was homegrown, and home built. Visit their website, and see how many of their line of bags are manufactured within San Francisco, and which are subcontracted out to Chinese manufacturing.

**How do we model a Firm’s choice?**

1. **Firms Maximize Profits**

   When you choose to purchase a particular good, you intrinsically choose the object that gives you the greatest enjoyment or joy. We translate the same concept into firms’ choice, but unlike your enjoyment, which is difficult to measure, the conceptualization of the firms’ choice is a little easier. We simply monetize the gains in terms of profits, after netting off wages, and rents. It is a simple idea, the greater the total revenue, the greater the share that can be paid out, and the more that can be reinvested to further develop the product line, and increase market share. Of course not every firm can be described by this idea, for example a non-profit hospital. However, what we will be describing will describe the majority of the firms we see. In terms of an equation, a firm maximizes then the following,

   \[
   \text{Profit} = \text{Total Revenue} - \text{Total Cost}
   \]

   Conceptually, total revenue equals total sales multiply by price. While total cost is the sum of wage rate, rental rates etc multiply by the number of units and time put into used. However, we must note the following finer points,

   1. However in accounting, accountants focus on explicit revenues and costs, or quantifiable revenues and cost that enter into the “books”. In terms of revenue, there is no difference. However, the difference arises when we think of costs. Accountants focus on quantifiable cost, i.e. amounts they receive printed on their receipts.
   2. An economist on the other hand include implicit costs, that is,

   \[
   \text{Total Cost} = \text{Implicit Cost} + \text{Explicit Cost}
   \]

   Implicit Cost includes the opportunity cost of the factors of production.

2. **The Production Process**

   Recall that when we consider supply curve that the long run supply is more elastic than the short run supply curve. The rationale we had was that in the short run, certain factors of production or inputs into a process are not variable. Hence the supply is inelastic, or quantities supplied is unresponsive to changes in own price changes. Note again, when we say short and long run, it does not necessarily mean a particular duration or number of years. It just differentiates between when some factors of production are not variable (Short Run), and when it is all variable (Long Run).
a. **Production Tables and Production Functions**

Production tables and functions are an easy for us to conceptualize how firms choose their optimal combinations of inputs. Our description here does not describe the whole process, but a simplification. You have to be mindful of this. This is done in detail in subjects such as Operations Research.

The first two columns of the following table tells you something important. It tells you how much input, in this case, the number of workers, need to be hired to produce a certain amount of product. In other words, it tells you the relationship between the inputs (factor of production) and output. Of course a real firm needs more factors of production that just labor.

What other information can be gleamed from the table. First note that at any time, a firm has some variables to meet immediate changes in demand, but others it can not. Examples of the former are labor and materials, while examples of the latter are factory floor space, number of machines used in a manufacturing process. There is a very interesting notion that can be introduced here, and that is the concept of **marginal product**, which means *the additional output that will result from an additional input (say labor), holding the level of other inputs constant.* Note that this concept is different from **average product**, which means the *output per worker*. A variation of figure 9-1 on page 203 of your text shows you how marginal and average product is calculated and graphed. What can you say about the changes in the marginal and average product of labor?

<table>
<thead>
<tr>
<th>Number of Workers (L)</th>
<th>Total Output</th>
<th>Calculation for MP_L</th>
<th>Marginal Product (MP_L)</th>
<th>Calculation for AP_L</th>
<th>Average Product (AP_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>8-0</td>
<td>8</td>
<td>8/1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>20-8</td>
<td>12</td>
<td>20/2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>34</td>
<td>34-20</td>
<td>14</td>
<td>34/3</td>
<td>11.33</td>
</tr>
<tr>
<td>4</td>
<td>46</td>
<td>46-34</td>
<td>12</td>
<td>46/4</td>
<td>11.5</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
<td>56-46</td>
<td>10</td>
<td>56/5</td>
<td>11.2</td>
</tr>
<tr>
<td>6</td>
<td>62</td>
<td>62-56</td>
<td>6</td>
<td>62/6</td>
<td>10.33</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>64-62</td>
<td>2</td>
<td>64/7</td>
<td>9.14</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>64-64</td>
<td>0</td>
<td>64/8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>60-64</td>
<td>-4</td>
<td>60/9</td>
<td>6.67</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>50-60</td>
<td>-10</td>
<td>50/10</td>
<td>5</td>
</tr>
</tbody>
</table>

Putting the numbers on a diagram may help you see the relationship.
The main idea is the following:
1. That marginal product initially rises, but diminishes eventually. The term of this is diminishing marginal productivity. This is often referred to as the **The Law of Diminishing Marginal Productivity**, and states that *as more and more of a variable input is added to an existing fixed input in production, eventually the additional output produced by an additional unit of the variable input will fall.* Note that this idea relates to our conception of our short and long run supply and production.
2. The average product rises at a slower rate, and intersects the marginal product curve as the latter is falling.

b. **The cost of Production**

Here are some cost concepts which we will use, and whose interrelationship we will examine. Be patient, their significance will be apparent shortly.

**Fixed Cost (FC):** Cost that are spent and cannot be changed in the period of time under consideration.

**Variable Cost (VC):** Costs that change as output changes.

\[
\text{Total Cost} = \text{Fixed Cost (FC)} + \text{Variable Cost (VC)}
\]

**Average Total Cost or Average Cost (ATC):** Is just the cost of producing a unit of a product. Let \( Q \) be the quantity supplied or produced. As its name suggests, it is calculated as

\[
\text{ATC} = \frac{TC}{Q} = \frac{(FC + VC)}{Q}.
\]

Average Fixed Cost (AFC): Is just the fixed cost involved in producing a unit of a product.

\[
\text{AFC} = \frac{FC}{Q}.
\]

Note the following, even without drawing a graph, or using a table, the concept of FC would inform you that AFC falls the more of a product is produced.

Average Variable Cost (AVC): Is just the variable cost involved in producing a unit of a product.

\[
\text{AVC} = \frac{VC}{Q}.
\]

The following relationship follows:

\[
\text{TC} = \text{FC} + \text{VC}
\]

Which implies that

\[
\frac{TC}{Q} = \frac{(FC + VC)}{Q}
\]

Which gives

\[
\text{ATC} = \text{AFC} + \text{AVC}
\]

One final cost concept is the following: Marginal Cost (MC), which is the cost involved in the production of an additional unit of a product. In other words,

\[
\text{MC} = \frac{\text{Change in Total Cost}}{\text{Change in Quantity Produced}} = \frac{\Delta TC}{\Delta Q}
\]
Idiosyncrasies of the relationship between the various cost curve.

1. The difference between the TC and VC is precisely the fixed cost. That is there is no difference in the slope of the TC and VC, and the sole difference is in the intercept of the TC which is just the FC.

2. The ATC and AVC is a U shaped curve. Why? We know that as quantity supplied increases, every additional unit of those quantities require the same or more input or factors of production due to the Law of Diminishing Marginal Productivity. Put another way, although as long as the firm manufactures at or within it’s capacity, cost would fall as more is produced, since it is “within it’s capacity”. But when those capacities are reached, an additional unit of input produces less than the last input. That is it starts becoming more expensive to produce the additional unit, since it requires more of the variable inputs. The firm would need more variable input to manufacture that additional unit, and if each unit of input must be paid the same rate, the cost would increase faster than the rate of increase in quantity supplied. That point at which this capacity constraint is incurred is where ATC and AVC starts rising.

3. The point where the ATC and AVC starts rising is also when MC intersects it. Why is this so? When the marginal cost (MC) of an additional unit of product is still below the ATC or the AVC, it means that the firm can still increase production by an additional unit and have it shared among all its quantities produced without raising the per unit total and average cost. However, once marginal cost is sufficiently high, the firm would not be able to raise in production without impunity. That is it will not be able to increase production without increasing per unit cost it faces, and hence the intersection. The reason for the intersection of AVC to be sooner and ATC is because of the fixed cost element in total cost. But can we prove this without a shadow of doubt. Well we
can, and the following shows it. Let \( C(.) \) be the cost dependent on the quantities supplied, \( Q \). **Note: This will not be tested.**

\[
ATC = \frac{C(Q)}{Q}
\]

\[
\frac{\partial ATC}{\partial Q} = \frac{QC'(Q) - C(Q)}{Q^2} \begin{cases} 
\leq 0 &\iff C'(Q) - \frac{C(Q)}{Q} \leq 0 \Rightarrow MC \leq ATC \\
\geq 0 &\iff C'(Q) - \frac{C(Q)}{Q} \leq 0 \Rightarrow MC \leq ATC
\end{cases}
\]

The prove for variable cost is similar.

4. Why is the AFC downward sloping? The reason is simple. Fixed Cost is a constant, or a unvarying number, while its divisor, quantity is increasing. So as the quantity supplied rises, the divisor increases, but the numerator stays the same, and AFC is strictly decreasing, and is asymptotically 0. And by virtue of this ATC and AVC tends towards each other as quantity supplied increases since \( ATC = AFC + AVC \).
The relationship depicted above parallels our description of why the MC and AVC, and ATC are U shaped, and the juncture where they intersect. To reiterate our point;

1. MC starts rising only when MP starts falling.
2. The point where MP equates with AP (point where AP and MP intersects) is the point where AVC and MC intersect. The point where AP intersect MP is the point where the additional unit of output by the factor under consideration can no longer be shared among all the other units of the factor under consideration (say labor) so that the per unit production of the factor input starts falling (AP). Since all units of the factor input are paid the same rate, it would also describe the point where AVC starts rising. Note that this diagram does not describe the relationship between productivity and the ATC, since ATC includes the AFC which pertains to the non-variable fixed factors.