1 Technical Note on National Income Accounting Measures

1.1 How StatCan calculates real GDP.

Let there all the final output produced in an economy be indexed by i which takes on values 1 to N. You can think of $i = 1$ being a particular product, say cars. Further let the time period be indexed by t such that t takes on the values 1 to T (Or you can think of it as running from say 1990 to 2000, or any year or quarter in a year you may wish to consider). The consequent nominal GDP for a set of goods in each period is then the sum of the product of the price of the good and the quantity of the good consumed, of all the goods in that period, say t. In short, it is just,

$$\text{NGDP}_t = \sum_{i=1}^{N} P_{it} Q_{it}$$

The principal use of measuring GDP is to examine the rate of growth of an economy. If we were to use nominal GDP to examine this fact, as noted in class, we cannot discern between how much of the change is due to inflation, and how much due to real changes in productivity in the form of rising output. The most common correction to eliminate inflation, and consequently get a better idea of what is the real increase in value of output, and income (recall the relationship between GDP and National Income) is to use the prices that existed in a particular base year. Let the base year be year 0. Then one possible measure of real GDP is

$$\text{RGDP}_t^0 = \sum_{i=1}^{N} P_{i0} Q_{it}$$

This measure of real GDP however falls short in one respect. Recall from your microeconomic segment where changes in the prices of substitute or complement goods would alter demand. This essentially says the following, just because the price of a good rises, does not mean that a consumer's welfare, or happiness falls by the same proportion, since she can always choose a substitute good, and it is perfectly possible that prices of the substitute good did not change, or change at a slower rate. In that sense, using the above correction over corrects the nominal GDP to obtain the real GDP. One way out is to change the base year periodically, and what was done previously was to reevaluate every 5 years or so. An even more accurate depiction would be for us to use the price in the previous quarter or year, instead of using a base from which to calculate. One possibility would be to use the previous year's prices to calculate real GDP, i.e.

$$\text{RGDP}_t^{t-1} = \sum_{i=1}^{N} P_{it-1} Q_{it}$$

What is the rate of growth of real GDP then? It is just

$$\frac{\text{RGDP}_t^{t-1} - \text{RGDP}_{t-1}^{t-1}}{\text{RGDP}_{t-1}^{t-1}} = \frac{\text{RGDP}_t^{t-1}}{\text{RGDP}_{t-1}^{t-1}} - 1 = \frac{\sum_{i=1}^N P_{it-1}Q_{it}}{\sum_{i=1}^N P_{it-1}Q_{it-1}} - 1 = g_l - 1$$

This brings to mind the following. If all we are doing is to get a better impression of economic growth from the perspective of GDP, why can't we use current prices for current output, but reevaluate previous output at the current prices, i.e. calculate the rate of growth as

$$\frac{\text{RGDP}_t^{t} - \text{RGDP}_{t-1}^{t}}{\text{RGDP}_{t-1}^{t}} = \frac{\text{RGDP}_t^{t}}{\text{RGDP}_{t-1}^{t}} - 1 = \frac{\sum_{i=1}^N P_{it}Q_{it}}{\sum_{i=1}^N P_{it}Q_{it-1}} - 1 = g_p - 1$$

This method however would under state the impact of inflation on GDP, and consequently under correct. So we're in a fix, which price should we use in evaluating GDP growth. If you're thinking, hmmm..., how about some kind of average growth measure, say some kind of average, well, that's exactly what Statcan and economist thought of. To have a decent impression of economic growth, why not calculate a geometric average, i.e.

$$g_{c,t-1/t} = \sqrt{g_{l,t-1/t} \times g_{p,t-1/t}}$$

(Note that $g_{l,t}$ and $g_{p,t}$ are in effect the ratio of current to previous year GDP calculated using reference year and current year prices respectively. This ratio is above 1 if we have growth, and below one if there is a decline. By subtracting 1 from it, we find the percentage change in GDP.) $g_{c,t-1/t}$ is then a number between $g_{l,t-1/t}$ and $g_{p,t-1/t}$, and is known as the FIsher Index. An additional note is that for the reference year, this index is simply 1. How do we use this adjustment factor? Consider the above example, let $t-1$ be the reference year from which we wish to calculate real GDP, then, real GDP in year t using the technique above is just

$$\text{RGDP}_{t/t-1}^{C} = g_{c,t-1/t} \times \text{NGDP}_{t-1} = g_{c,t/t-1} \times \sum_{i=1}^{N} P_{it-1}Q_{it-1} = \sqrt{g_{l,t/t-1} \times g_{p,t/t-1}} \times \sum_{i=1}^{N} P_{it-1}Q_{it-1}$$

Note the error in the appendix of your text: Where it says to get this same real GDP, the calculation is

$$\left(1 + \sqrt{\left(g_{l,t/t-1} - 1\right) \times \left(g_{p,t/t-1} - 1\right)}\right) \times \sum_{i=1}^{N} P_{it-1}Q_{it-1}$$

That is not true. They may be close, but they are not the same.

But what do we do if the year is more than 1 year away from the reference year. That is what if the reference year is $t-1$, and the current year is $t+1$. In

that case, the chain weighting procedure is used by Statcan, and we calculate

$$\begin{aligned}
g_{c,t+1/t} &= \sqrt{g_{l,t/t-1} \times g_{p,t/t-1}} \times \sqrt{g_{l,t+1/t} \times g_{p,t+1/t}} \\
&= \sqrt{\frac{\mathrm{RGDP}_t^{t-1}}{\mathrm{RGDP}_{t-1}^{t-1}} \times \frac{\mathrm{RGDP}_t^{t}}{\mathrm{RGDP}_{t-1}^{t}}} \times \sqrt{\frac{\mathrm{RGDP}_{t+1}^{t}}{\mathrm{RGDP}_t^{t}} \times \frac{\mathrm{RGDP}_{t+1}^{t+1}}{\mathrm{RGDP}_t^{t+1}}} \\
&= \sqrt{\frac{\sum_{i=1}^N P_{it-1}Q_{it}}{\sum_{i=1}^N P_{it-1}Q_{it-1}} \times \frac{\sum_{i=1}^N P_{it}Q_{it}}{\sum_{i=1}^N P_{it}Q_{it-1}}} \times \sqrt{\frac{\sum_{i=1}^N P_{it}Q_{it+1}}{\sum_{i=1}^N P_{it}Q_{it}} \times \frac{\sum_{i=1}^N P_{it+1}Q_{it+1}}{\sum_{i=1}^N P_{it+1}Q_{it}}}
\end{aligned}$$

So what is the GDP in period $t+1$? Well it's just

$$\mathrm{RGDP}_{t+1/t-1}^{C} = g_{c,t+1/t-1} \times \mathrm{NGDP}_{t-1}$$

So the greater the distance from a reference year, the more times we would have to multiply the different geometric measures calculated from adjacent years then. This is what is noted on the website of Statcan.

However, is this what is really done. Well, not exactly! The problem arises because in truth, we do not have prices and quantities for every product, but the value transacted. Well, we can manipulate what we have right? Let us imagine all we have are nominal consumption values C_{it} for the period and each product we have been refering to, and next recall we have the consumer price index. Can we then transform the formula. Well, yes of course! Economists are one of the most resourceful bunch. Let us start from the nominal GDP again,

$$\mathrm{NGDP}_t = \sum_{i=1}^N P_{it}Q_{it} = \sum_{i=1}^N C_{it}$$

And g_l and g_p can be rewritten as

$$g_l = \frac{\mathrm{RGDP}_t^{t-1}}{\mathrm{RGDP}_{t-1}^{t-1}} = \frac{\sum_{i=1}^N P_{it-1}Q_{it}}{\sum_{i=1}^N P_{it-1}Q_{it-1}} = \frac{\sum_{i=1}^N \frac{P_{it-1}}{P_{it}} P_{it}Q_{it}}{\sum_{i=1}^N C_{it-1}} = \frac{\sum_{i=1}^N \frac{P_{it-1}}{P_{it}} C_{it}}{\sum_{i=1}^N C_{it-1}}$$

and

$$g_p = \frac{\mathrm{RGDP}_t^{t}}{\mathrm{RGDP}_{t-1}^{t}} = \frac{\sum_{i=1}^N P_{it}Q_{it}}{\sum_{i=1}^N P_{it}Q_{it-1}} = \frac{\sum_{i=1}^N C_{it}}{\sum_{i=1}^N \frac{P_{it}}{P_{it-1}} P_{it-1}Q_{it-1}} = \frac{\sum_{i=1}^N C_{it}}{\sum_{i=1}^N \frac{P_{it}}{P_{it-1}} C_{it-1}}$$

From here on everything else stays the same.

1.2 GDP Deflator and CPI

As noted during the class, the are good measures of the prices that agents in the economy faces. Why do we want to know these prices, and how they vary. Well, excessive inflation may mean a reduction in purchasing power for some, depending on whether you're on the losing end of the bargain. It gives a rough

gauge essentially. But how do we know which to use. Well each measure gives a different story. We will consider each in turn. First, GDP Deflator, measured as

$$\text{GDP Deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100$$

Nominal GDP is as we have considered, while real GDP is as we have discussed above as well. Compare this with the Consumer Price Index for a current year (this uses a base year basket of goods, and base year quantities of goods consumed)

$$\text{Current Year CPI}=\frac{\text{Cost of Base Year Quantities at Current Prices}}{\text{Cost of Base Year Quantities at Base Year Prices}} \times 100$$

What are the differences between the two then? Well GDP measures all final goods and services transacted in the economy and includes final goods and services consumed by firms, while the latter, CPI, includes only final goods and services transacted by the average consumer. That is the latter is not as broad based as the former. Or put another way, the former might be too broad based compared to the latter. What should be used then. Well, it would depend on what we are interested in, be it something general, or strictly to do with the average consumer. Even then, the further removed in time we are from the base year, the less applicable would CPI be. Further, note that because of the detailed and correction performed on the real GDP measure, it would seem that the GDP deflator is the better measure. Another plus for the GDP deflator is the fact that because it evolves with the goods that are produced in the economy, it more accurately parallels what is available for consumption then what we see in the CPI. But what neither measure is able to solve is the idea pertaining to quality changes over time.

Is this the be all and end all for discussions in National Income Accounting. Of course not! But we'll leave it when you move on.

1.3 Exercise

Suppose an economy produces only cheese and wine, and the table below depicts the valued consumed in a particular year, and the average price of all the brands of cheese and wine. Calculate using the method above the nominal GDP, the Fisher Index for each year using the previous year as the reference year, the Fisher Chained Index, Real GDP, and the GDP Deflator. Keep your final answers up to 3 decimal places.

		1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Cheese	Value Consumed	1,500	1,680	1,944	2,240
	Price	15	16	18	20
Wine	Value Consumed	550	600	608	600
	Price	22	20	16	12