Qualified Equal Opportunity and Conditional Mobility: The Educational Attainment Gender Gap in Canada 1950-2000

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Abstract

Generational mobility policies which follow a Pure Equal Opportunity (EO) social justice imperative seek to reduce the connection between parent and child outcomes. Here it is demonstrated that, in the face of limited ability to raise average offspring ability, singular pursuit of this imperative inevitably involves breaking the connections between high type parents and their children as well as those between low type parents and their children. This does not accord with observed practice wherein EO policies (e.g. Head Start, No Child Left Behind) appear to focus on the latter rather than the former. It is conjectured that policy makers are responding to other imperatives which can be summarized as to not make the inheriting generation of any parental class worse off by the mobility policy. Indeed when this secondary imperative is added to the policy maker's objective function, together with some capacity for improving mean child outcomes, a Qualified Equal Opportunity (QEO) policy emerges, one more akin to observed practice. The consequences of QEO policies for mobility measurement via transition matrices and generational regressions are considerable, mitigating the independence structure in the transition matrix and convexifying the regression relationship. They are explored in the context of the educational attainments of cohorts of Canadian males and females born between the 1920's and 1970's and are generally found to be data coherent.

1 Introduction

"*The conception of social justice held by many, perhaps most, citizens of the Western democracies is that of equality of opportunity. Exactly what that kind of equality requires is a contested issue, but many would refer to the metaphor of "leveling the playing field", or setting the initial conditions in the competition for social goods as to give all, regardless of their backgrounds an equal chance of achievement. A central institution to implement that field leveling is education, meaning education that is either publicly financed or made available to all at affordable costs...* " Roemer (2006)

With roots in recent egalitarian political philosophy[1], the *Equal Opportunity Imperative* sees differential outcomes as ethically acceptable when they are the consequence of individual choice and action, but not ethically acceptable when they are the consequence of circumstances beyond the individual's control. To the extent that an individual's circumstances have to do with the parents they were blessed with, equal opportunity policies have to address the degree to which a child's status when adult is related to the status of their parents at a similar stage in their life cycle. In theory if, as is usually the case, child outcomes are positively associated with parent outcomes, this involves breaking connections between high outcome parents and their children, as well as those between low outcome parents and their children. As Kanbur and Stiglitz (1986) observed, in essence the issue is one of generational mobility and the manner in which it engenders a dynastic aspect to poverty and wealth. The imperative has provoked considerable empirical interest in the extent of generational mobility (or the degree to which a child's parental circumstance conditions his/her outcomes), however systematically low mobility estimates over many studies provide little or no support for the view that the imperative of complete independence of outcome from circumstance has been achieved (see for example Corak (2004) and references therein).

One rationale for a complete mobility policy objective not being met is that, from a policy maker's perspective, other imperatives, not necessarily founded upon social justice sentiments, may be in play. Piketty (2000) noted as much in his interpretation of the conservative – right wing view that, if generational mobility is low (because of the high inheritability of ability) and the distortionary costs of welfare redistributions are high, it is reasonable to argue that low mobility is acceptable[2] Friedman (2005) makes a similar point

[1] See Arneson (1989), Cohen (1989), Dworkin (1981a), Dworkin (1981b), and Dworkin (2000).

[2] Indeed the pursuit of an equal opportunity goal has not been unequivocal, Cavanagh (2002) expresses

in conjecturing (with a considerable amount of supporting evidence) that economic growth has facilitated the equalizing of opportunities (amongst other improvements in social justice) in effect allowing the poor to catch up without disadvantaging the rich.

A policy which makes some inheriting groups worse off than they would have been absent the policy, may not be politically viable (the parents of such groups would almost certainly not vote for such a policy) so that policy makers responding to the median voter (Downs 1957) or probabilistic voter (Coughlin 1992) mandates may wish to avoid this part of the package. Affirmative action policies are very much in this vein since they incorporate normative objectives that weigh policies in the favour of "poorly" endowed, focusing on improving the life chances of the "inherited poor" rather than diminishing the life chances of the "inherited rich"[3]. In effect the policy maker's second imperative is a sort of Pareto condition wherein the lot of the poorly endowed is improved without diminishing the lot of the richly endowed.

Should the policy maker follow the dual mandates of equal opportunity guided by this Paretian imperative, a qualified equal opportunity program emerges with asymmetric mobility aspirations for increasing the mobility of the poorly endowed, and not increasing the mobility of the well endowed when it involves a loss of their wellbeing. The extent to which these objectives are fulfilled is bounded by the capacity in the system. Such policies can no longer be characterized as unqualified moves towards the independence of outcomes and circumstances for all groups. They are rather equivocal moves, modifying the joint distribution of outcomes and circumstances differentially toward independence for the poor in circumstance and independence for the rich in circumstance only if their material wellbeing is not diminished. Evaluating policy outcomes under this type of regime requires rethink-

some philosophical reservations, Jencks and Tach (2006) question whether an equal opportunity imperative should require the elimination of "..all sources of economic resemblance between parents and children. Specifically ...(it)... does not require that society eliminate the effects of either inherited differences in ability or inherited values regarding the importance of economic success relative to other goals.". In a similar vein Dardanoni et al. (2006) question how demanding the pursuit of equal opportunity should be in terms of the feasibility of such a pursuit.

[3]As a matter of casual empiricism, equal opportunity programs observed in "Liberal" societies do seem to be of this flavour. For example, when questioned on the widening gap between the rich and poor, the British Prime Minister responded that "... the issue is not in fact whether the very richest person ends up be ing richer. The issue is the poorest person is given the chance they don't otherwise have. The most important thing is to level up, not level down." Interview with the Prime Minister on BBC News Newsnight on June 4, 2001. Transcript available from `http://news.bbc.co.uk/2/hi/events/newsnight/1372220.stm`

ing current empirical approaches to mobility measurement which has been the realm of two techniques, transition matrix and generational regression analysis.

The implications of intergenerational mobility on the *transition matrix* have been examined in van de Gaer, Schokkaert and Martinez (2001). Their aim was to axiomatically develop measures which could distinguish between (1.) mobility as *movements* of the constituents of a society, (2) as an indicator of *equal opportunity* and (3) as an *indicator of life chances* which they showed to be incompatible concepts. Whereas van de Gaer et al. (2001) considered differences between various conceptions of intergenerational mobility and its implication on the *transition matrix*, this paper considers in stead the consequence for the *joint density matrix* generated by social policies flavoured with a Paretian imperative vis-à-vis the common conception of intergenerational mobility, defined by van de Gaer et al. (2001) as *equal opportunity*[4]. Interest in the Paretian imperative is not guided by any normative judgement but rather by a belief that this may well be how policy is formulated. This paper has two main contributions, firstly it provides the implications on the joint density matrix and regression approach to intergenerational mobility should the social planners' objectives be influenced by such Paretian issues (as seems to be evident in current social policies), and secondly to provide a simple measure of intergenerational mobility that focuses on the measurement of the proximity between the empirical versus the hypothesized *joint density matrices*.

In Section 2 it is formally shown that when the policy maker faces the Pareto improvement constraint of not making the children of specific socioeconomic groups materially worse off under an equal opportunity policy, a *Qualified* Equal Opportunity Policy emerges. The extent to which this can be achieved is limited by the degree of flexibility in the system (represented in the model presented by potential growth in average child outcomes, much along the lines of Friedman (2005)). Mobility improvements are qualified by their circumstance source in some sense, and implications for the way in which such mobility is measured are then developed for both transition matrix and regression analyses. A means of evaluating the success of

[4] The Overlap Measure proposed in this paper can be adapted to the three conceptions of intergenerational mobility suggested by van de Gaer et al. (2001), since each transition matrix has an implied structure on the joint density matrix, which the empirical joint density can be measured against. See Anderson, Ge and Leo (2010) for the discrete variable case, and Anderson, Linton and Whang (2009) for the continuous variable case. Further, the third mobility measure for Markov chains proposed by van de Gaer et al. (2001) is related to the Overlap measure in the sense that it measures the *complement* to the overlapping region of the conditional probabilities.

mobility policies differentially is developed in Section 3 where a *Qualified* or *Conditional* Mobility measure is proposed which is simple to employ and permits the identification and examination of group specific mobility changes in the sense that the mobility of the "poor" or "rich" in circumstance can be addressed separately. Implications for the way in which regression measures of mobility are used and interpreted are also examined.

To illustrate the concepts and their measurement, Statistics Canada's General Social Survey Cycle 19 (2005) is used to examine the closing gender gap in educational attainment that occurred in Canada[5] (Blau et al. 2006) in section 4. One of the preoccupations of Sen's considerable body of work on social justice is the achievement of *gender justice* (See Nussbaum (2006), Sen (1990) and Sen (1995)). This could have been achieved quite swiftly by a transfer of resources from the investments in male human capital to investments in female human capital. Had that been so, an improvement in the achievements of females accompanied by deterioration in the achievements of males would have been observed. However it will be shown that, while male academic achievements did not deteriorate, the narrowing gender gap is characterized by an increased generational mobility of women relative to men. Furthermore the source of this increased mobility was the daughters of parents with lower educational attainments (which may be construed as a "good" since it implies upward mobility) rather than the daughters of parents with high educational attainments (which may be construed as a "bad" since it implies downward mobility and the attrition of inherited wellbeing). Indeed it appears that the increased mobility of women has come about as a consequence of a reduction in the dependence of their educational outcomes on those of their mothers especially at the lower end of the maternal educational attainment spectrum. However increasing immobility was observed in the lowest inheritance class. Finally, a brief discussion of the results is provided in the section 5.

2 The Constrained Equal Opportunity Imperative

2.1 Implications of Unqualified Intergenerational Mobility Policy

To illustrate matters in the transition matrix paradigm assume that parent-child characteristics have 4 discrete realizations (Though the model discussed can easily be generalized to any number of characteristic realizations and the case when the number of realizations

[5] This phenomena has also been observed in the United States, see for example Buchmann and Diprete (2006), Dynarski (2007), Goldin et al. (2006) and Jacob (2002).

for both parent and child differ in number. See Anderson and Leo (2008)). Consider a simple generational income class transition structure where the vector of parental incomes $\mathbb{x} = [1, 2, 3, 4]'$ transit to the vector of child incomes $\mathbb{y} = [1, 2, 3, 4]'$, denoting each element as x_i and y_k, $i, k \in \{1, 2, 3, 4\}$, respectively. That is x_i and y_k are realizations of random variables x (parental incomes) and y (child incomes) respectively. Let the vector of outcome probabilities for parents be $\mathbb{p}$ with elements p_k for the probability of a parent being in income class x_k. Similarly, the vector of outcome probabilities for children is $\mathbb{c}$ with elements c_i for the probability of a child being in income class y_i. In other words,

$$\mathbb{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \mathbb{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Let $\mathbb{J}$ be the matrix of joint probabilities, with element $j_{i,k}$ corresponding with the probability of a parent-child observation being in income class x_k, and y_i respectively. More precisely,

$$\mathbb{J} = \begin{bmatrix} j_{1,1} & j_{1,2} & j_{1,3} & j_{1,4} \\ j_{2,1} & j_{2,2} & j_{2,3} & j_{2,4} \\ j_{3,1} & j_{3,2} & j_{3,3} & j_{3,4} \\ j_{4,1} & j_{4,2} & j_{4,3} & j_{4,4} \end{bmatrix}$$

where $p_k = \sum_{i=1}^{4} j_{i,k}$ and $c_i = \sum_{k=1}^{4} j_{i,k}$, where $i, k = \{1, 2, 3, 4\}$. Let $\mathbb{P} = \text{dg}(\mathbb{p})$, dg being the diagonal operator, then the conventional *transition matrix* $\mathbb{T}$ can be written as $\mathbb{T} = \mathbb{J}\mathbb{P}^{-1}$ whose i^{th}, k^{th} element is $t_{i,k} = \Pr(y = y_i | x = x_k) = j_{i,k}/p_k$ and yields the child's income class vector $\mathbb{c}$ from the equation $\mathbb{c} = \mathbb{T}\mathbb{p}$ (Noting that $\mathbb{P}^{-1}\mathbb{p} = \mathbb{1}$, where $\mathbb{1}$ is vector of ones). Let $\mathbb{J}^I$ be the joint density matrix in a *pure equal opportunity* environment, where $\mathbb{J}^I = \mathbb{c}\mathbb{p}'$, i.e. independence between parent-child outcomes which yields a transition matrix, $\mathbb{T}^I$ with common columns $\mathbb{c}$ reflecting the fact that a child's life chances are the same for all parental classes. Average parent and child incomes may be written as $\mathbb{p}'\mathbb{x}$ and $\mathbb{c}'\mathbb{y}$ respectively.

In the context of zero growth in child attainment ($\mathbb{c}'\mathbb{y}$ remains unchanged), a *pure equal opportunity* program is one which moves a joint density $\mathbb{J}$ towards $\mathbb{J}^I$. Note that a move toward $\mathbb{J}^I$ that preserves the children's socioeconomic status structure will necessarily make the children of one parental income group worse off while making the children of another better off. To see this, first suppose the population's joint density matrix is such that $\mathbb{J} \neq \mathbb{J}^I$, in other words the population exhibits some dependence in mobility. Consider the socioeconomic group denoted by the index $x_1 = 1$. Let the nature of dependence be such that $j_{1,1} = \max\{j_{1,1}, j_{2,1}, j_{3,1}, j_{4,1}\}$, and $j_{1,1} \geq j_{2,1} \geq j_{3,1} \geq j_{4,1}$. In other words, child outcomes of the lowest socio-economic group is positively correlated with their parent's socioeconomic status and the relationship is monotonic. Suppose the shift towards independence or mobility shifts the attainment of children towards higher attainment. Then by definition of raising mobility, it must necessarily be true that $j_{1,1} > j^I_{1,1} = c_1 p_1$. Then for socioeconomic group x_1, the following must be true,

$$\begin{aligned} \sum_{i=1}^{m} j^I_{i,1} &\leq \sum_{i=1}^{m} j_{i,1} \\ \Rightarrow \sum_{i=1}^{m} \left(j^I_{i,1} - j_{i,1}\right) &\leq 0 \end{aligned}$$

where $m \in \{1, 2, 3, 4\}$, which means that a shift towards independence leads to a stochastic dominant shift for socioeconomic group x_1. However, since it is assumed that $\mathbb{c}'\mathbb{y}$ remains unchanged, this implies that, $j_{1,q} < j^I_{1,q} = c_1 p_q$, for some $q \in \{2, 3, 4\}$, which in turn means that,

$$\begin{aligned} \sum_{i=1}^{m} j^I_{i,q} &\geq \sum_{i=1}^{m} j_{i,q} \\ \Rightarrow \sum_{i=1}^{m} \left(j^I_{i,q} - j_{i,q}\right) &\geq 0 \end{aligned}$$

that is, consequent to the shift towards independence without any qualifying conditions on the policy, child outcomes of higher socioeconomic status families are necessarily diminished. Put another way, the children's outcome distribution in the status quo state, for higher socioeconomic status families, first order dominates that of the equal opportunity distribution.

In addition, when child outcomes are positively correlated with adult outcomes the conditional distribution of the outcomes of children with low income parents will be stochastically dominated by that of higher income parents so that, in its strongest form:

$$\sum_{i=1}^{m} \left(\frac{j_{i,l}}{p_l} - \frac{j_{i,k}}{p_k} \right) \geq 0$$

for $l < k$; l, k, $m = \{1, ..., 4\}$.

2.2 A Simple Model of the Qualified Mobility Problem Confronting the Social Planner

The social planner's problem is modelled as one of minimizing the "distance" between the targeted joint density matrix and that under perfect independence, namely:

$$\min_{j^*_{i,k}\in\mathbb{J}^*}\sum_{i=1}^{4}\sum_{k=1}^{4}\left(j^*_{i,k}-c^I_i p_k\right)^2$$

where $j^*_{i,k}$ is an element of the joint density matrix $\mathbb{J}^*$ the social planner is choosing, while $c^I_i p_k = j^I_{i,k}$ is the joint density matrix under *perfect independence.* c^I_i is an element of $\mathbb{c}^I$, the desired marginal density vector of child income, determined by the social planner. Another way to think about the choice of the social planner is that she is implicitly choosing the level of funding or assistance towards differing socioeconomic groups to achieve the desired parent-child joint density. It is clear that if there are no constraints, the optimal choice of the social planner is to simply set $\mathbb{J}^* = \mathbb{J}^I$, which is contrary to observed practice.

Consider augmenting the social planner's problem such that she faces two constraints, (A.) a growth rate constraint, g ($g \geq 0$), (which constrains $\mathbb{c}^I$) and (B.) the outcomes for children in any parental income class are not allowed to deteriorate. Note that the existing parental income distribution, $\mathbb{p}$, is fixed and immutable. With respect to the first constraint, let $\mathbb{J}$ correspond to the existing (i.e. pre-policy) transition matrix $\mathbb{T}$ which yields $\mathbb{c}$ with an average child outcome of $\mathbb{c}'\mathbb{y}$, and suppose that $\mathbb{J} \neq \mathbb{c}\mathbb{p}'$. Let $\mathbb{J}^*$ correspond to the post policy transition matrix $\mathbb{T}^*$ which yields an average child outcome of as much as $\mathbb{c}'\mathbb{y} + g$. In effect $(\mathbb{T}^*\mathbb{p})'\,\mathbb{y} = (\mathbb{J}^*\mathbb{1})'\,\mathbb{y} \leq \mathbb{c}'\mathbb{y} + g$ is a constraint on the possible choices of $\mathbb{J}^*$ since, as demonstrated above, when g is 0 no move of the elements of $\mathbb{J}$ toward an equal opportunity structure is possible without making the children of at least one parental income group worse off. Note again that *pure equal opportunity* corresponds to $\mathbb{J}^* = \mathbb{c}^I\mathbb{p}'$. The growth constraint may be rewritten as,

$$\begin{aligned}\mathbb{c}^{*\prime}\mathbb{y} - \mathbb{c}'\mathbb{y} &\leq g\\ \Rightarrow \sum_{i=1}^{4} y_i\left(\sum_{k=1}^{4} j^*_{i,k} - c_i\right) &\leq g\end{aligned}$$

where $\mathbb{c}^*$ is the corresponding vector of marginals from the constrained mobility policy chosen by the social planner.

The second constraint requires the rows of $\mathbb{J}^*$ to first order stochastically dominate the corresponding rows of $\mathbb{J}$ following the notion that the young generation should not be made worse off by the equal opportunity policy. Put another way, the new conditional density (conditional on the child's parental status) must first order stochastically dominate the status quo conditional density. This constraint can be construed as an exogenous constraint imposed on the social planner by the median voter (Downs 1957) or probabilistic voter (Coughlin 1992) as suggested earlier. In this simple stylized model, the social planner's objective function will be subject to three stochastic dominance criteria of $\sum_{i=1}^{q} c_i^* \leq \sum_{i=1}^{q} c_i$, $q = \{1, 2, 3\}$, and that $\sum_{i=1}^{4} c_i^* = 1$, noting that $\sum_{k=1}^{4} j_{i,k}^* = c_i^* \leq c_i^I$.

The social planner's problem can now be restated as,

$$\min_{j_{i,k}^* \in \mathbb{J}^*} \sum_{i=1}^{4} \sum_{k=1}^{4} \left(j_{i,k}^* - c_i^I p_k\right)^2 \tag{1}$$

subject to:

$$\sum_{i=1}^{l} \left(j_{i,k}^* - j_{i,k}\right) \leq 0, \forall \;\; l = \{1, 2, 3\} \;\; \& \;\; k = \{1, 2, 3, 4\} \tag{2}$$

$$\sum_{i=1}^{4} y_i \left(\sum_{k=1}^{4} j_{i,k}^* - c_i\right) \leq g \tag{3}$$

Note that $\sum_{i=1}^{4} j_{i,k}^* = p_k$, $\sum_{k=1}^{4} j_{i,k}^* = c_i^*$, and $j_{i,k}^* \in [0, 1]$ $\forall i$ and k. That is she wants to ensure that in choosing the matrix of joint densities, children of each socioeconomic group do not suffer a fall in welfare on average, and that growth in child outcomes is fully exploited at the same time. The question of equal opportunity phrased in this form highlights the competing considerations increasing mobility without causing the outcomes of children of any social class to deteriorate.

After some manipulation the Lagrangian may be written as:

$$L = \left\{ \begin{array}{l} \sum_{i=1}^{3} \sum_{k=1}^{4} 2 \left(j_{i,k}^* - c_i^I p_k\right)^2 + \sum_{k=1}^{4} \left\{ 2 \sum_{i=1}^{3} \left[\left(j_{i,k}^* - c_i^I p_k\right) \sum_{l=1, l \neq i}^{3} \left(j_{l,k}^* - c_l^I p_k\right) \right] \right\} \\ + \sum_{l=1}^{3} \sum_{k=1}^{4} \lambda_{l,k} \sum_{i=1}^{l} (j_{i,k}^* - j_{i,k}) + \gamma \left[\sum_{i=1}^{3} (4 - y_i) \left(c_i - \sum_{k=1}^{4} j_{i,k}^* \right) - g \right] \end{array} \right\} \tag{4}$$

and the resultant Kuhn Tucker conditions are:

$$\frac{\partial L}{\partial j_{r,l}^*} = 4 \left(j_{r,l}^* - c_r^I p_l\right) + 2 \sum_{q=1, q \neq l}^{4} \left(j_{q,l}^* - c_q^I p_l\right) + \sum_{i=1}^{r} \lambda_{i,l} - \gamma \left(4 - y_r\right) = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda_{r,l}} = \sum_{q=1}^{r} \left(j_{q,l}^* - j_{q,l}\right) \leq 0 \qquad \lambda_{r,l} \geq 0 \tag{6}$$

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{3} (4 - y_i) \left(c_i - \sum_{k=1}^{4} j^*_{i,k} \right) - g \leq 0 \qquad \gamma \geq 0 \tag{7}$$

where $r = \{1,2,3\}$, $l = \{1,2,3,4\}$. When the constraints do not bind ($\lambda_{r,l} = 0$ for $r = \{1,2,3\}$, $l = \{1,2,3,4\}$ and $\gamma = 0$) the solution to (5) is the equal opportunity solution $j^*_{r,l} = c^I_r p_l$, $\forall r, l$. As the constraints successively bind the equal opportunity outcome is successively compromised with the solution being a combination of the initial and equal opportunity outcomes.

First note that the solution for richer parental groups ($l = 2,3,4$ in equation (5)) contains a compounding of the stochastic dominance shadow prices of each lower socioeconomic group. This implies that not meeting the stochastic dominance constraint at the lowest socioeconomic level implies costs at all socioeconomic levels. Thus suppose the initial state is one of complete immobility and $g > 0$[6], the social planner would reallocate the $j_{1,l}$'s to the extent that (7) does not bind and (6) does not bind for $l = 1$, thereby improving the mobility of the poorest children (note that increased mobility for the richest children would involve increased downward mobility making them worse off and conflicting with the dominance condition (6)). Should there still be capacity for change, the $j_{2,l}$'s would next be reallocated and so on until the growth constraint is exhausted or complete equality of opportunity is achieved.

Insofar as a move towards independence for children of higher socioeconomic status families implies a welfare reduction for them, a social planner abiding by the above program will not implement it. On the other hand, children of lower socioeconomic status families will see a shift towards independence, such that the post policy conditional density for them will first order stochastically dominate their pre-policy joint density. Finally, note that an implicit assumption in this model is that the cost of shifting children at various socioeconomic groups are constant, and the "distance" of the characteristic realization (in the case here income groups) are equidistant apart. Although relaxing the latter has no implication for this illustrative model, the former is substantial. If the costs of improving the stead of the children differ across socioeconomic groups, then improving the lot of those "high cost" children may impede the attainment of the desired level of growth in average income.

As an illustration, consider the following example; suppose the pre and post qualified

[6] Recall that if g were 0 no move toward an equal opportunity policy could be made without making some of the children in at least one of the income classes worse off.

equal opportunity policy child-adult joint densities are $\mathbb{J}^0$ and $\mathbb{J}^1$ respectively, and were given by:

$$\mathbb{J}^0 = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix} \qquad \mathbb{J}^1 = \begin{bmatrix} 0.125 & 0 & 0 & 0 \\ 0.125 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

Pre-policy child outcomes are one to one with their parents, while post-policy there has been a "convexification" of this relationship with the children of the poorest parents now receiving an average outcome of 1.5 rather than a 1. Note that the conditional variance of low categories child outcomes will be greater than the pre-policy outcome (0.25 as opposed to 0) as well as the variance of the other socioeconomic groups. This convexification of the parent-child relationship can be construed as an increase in "unpredictability" with regards to the child's outcome given her parents'. Ben-Shahar and Sulganik (2008) provides a useful partial ordering in terms of transition matrices' under such a scenario. Under their definition, the transition matrix associated with $\mathbb{J}^1$ is *weakly more unpredictably mobile* than that associated with $\mathbb{J}^0$.[7]

In the continuous variable world of generational regressions pure and qualified mobility policy effects may be illustrated as follows. Suppose in the initial state, with parental outcome $x \in X$ distributed with density $f(x)$ and c.d.f. $F(x)$, with $\mathbf{E}(x) = \mu$, $\mathbf{V}(x) = \sigma^2$ and child outcome

$$y = (1 - \xi)x + \xi e \tag{8}$$

where $0 \leq \xi \leq 1$ and e is distributed as $g(e)$ where $g(x) = f(x)$ for all x and $h(x, e) = f(x)g(e)$ (That is to say x and e are identically but independently distributed). For exposi-

[7]However, if $\mathbb{J}^1$ were the initial distribution, and $\mathbb{J}^2$ is the post-policy distribution such that,

$$\mathbb{J}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

Here child outcomes have been further "convexified" and the lowest child outcome has been completely eliminated, noting the fall in variance across time, while variances across all socioeconomic groups remain the same. This example illustrates that the changes in heteroskedasticity are not time invariant and is dependent on the initial state of intergenerational mobility within society. Nonetheless, using the partial ordering developed by Ben-Shahar and Sulganik (2008), it is clear that the transition matrix associated with $\mathbb{J}^2$ would still be more unpredictably mobile than that for $\mathbb{J}^1$.

tional convenience suppose $f(.)$ to be normal. In this set up, *Complete Immobility* implies $\xi = 0$ and *Equal Opportunity* implies $\xi = 1$. Then $\mathbf{E}(y) = \mu$ and $\mathbf{V}(y) = (1 + 2\xi(\xi - 1))\sigma^2$ for all ξ and

$$f(y|x) \sim N\left((1-\xi)x + \xi\mu, \xi^2\sigma^2\right)$$

for $\xi > 0$, according with the constraint that average child outcomes cannot increase. That is under this initial state,

$$\begin{aligned}\frac{\partial\,\mathbf{E}(y|x)}{\partial x} &= (1-\xi) &\quad (9)\\ \frac{\partial\,\mathbf{V}(y|x)}{\partial x} &= 0 &\quad (10)\end{aligned}$$

so that the intergenerational relationship is linear and constant across socioeconomic groups, and the relationship is homoskedastic.

Let $\Phi(.)$ denote the standard normal c.d.f, and $\phi(.)$ denote the standard normal density function. Then notice that for all $\xi < 1$, children with parental outcome x^* have a distribution of outcomes that first order dominate those of children with parental outcome x^{**} when $x^* > x^{**}$. This is so since for all y,

$$\begin{aligned}F(y|x^*) &= \Phi\left(\frac{Y - ((1-\xi)x^* + \xi\mu)}{\xi\sigma}\right)\\ &\leq \Phi\left(\frac{Y - ((1-\xi)x^{**} + \xi\mu)}{\xi\sigma}\right)\\ &= F(y|x^{**})\end{aligned}$$

with strict inequality holding for some Y. Essentially well endowed children are better off than poorly endowed children except under perfect mobility as mentioned in the previous section.

Pure EO policies increase ξ uniformly across x. Consider the marginal effect of an increase in ξ on the probability that a child's outcome is less than Y given parental outcome x^*:

$$\begin{aligned}
\frac{\partial \Pr(y < Y|x^*)}{\partial \xi} &= \frac{\partial F(Y|x^*)}{\partial \xi} \\
&= \frac{\partial \Phi\left(\frac{Y - \mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}\right)}{\partial \xi} \\
&= \phi\left(\frac{Y - \mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}\right)\frac{\partial \frac{Y - \mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}}{\partial \xi} \\
&= \phi\left(\frac{Y - \mathbf{E}(y|x^*)}{\sqrt{\mathbf{V}(y|x^*)}}\right)\left(\frac{x^* - Y}{\lambda^2 \sigma}\right)
\end{aligned}$$

That is the marginal effect on child outcome of this policy is positive for $x^* > Y$ and negative for $x^* < Y$. Thus, $\frac{\partial \Pr(y<Y|x^*)}{\partial \xi} \leq 0$ as $x^* \leq Y$ and $\frac{\partial \Pr(y<Y|x^*)}{\partial \xi} > 0$ as $x^* > Y$. Hence, $F_{post}(y|x^*) - F_{pre}(y|x^*) \geq 0$ for $y \leq x^*$ and $F_{post}(y|x^*) - F_{pre}(y|x^*) < 0$ for $y > x^*$, so that the Pre− and Post−Policy Change cumulative densities, F_{pre} and F_{post} cross just once at x^*. Here note that both Pre− and Post−Policy outcome distributions are normally distributed so:

$$\int_{\infty}^{x^*} \left(F_{pre}(y|x^*) - F_{post}(y|x^*)\right) dy \gtreqqless \int_{x^*}^{\infty} \left(F_{pre}(y|x^*) - F_{post}(y|x^*)\right) dy \quad \text{as} \quad x^* \gtreqqless \mathbf{E}(Y)$$

which means that Pre−Policy outcomes second order dominate post policy outcomes for high (above average) parental groups so that average child outcomes diminish for these groups whereas Post−Policy outcomes second order dominate Pre−Policy outcomes in the counter cumulative density[8] sense for low (below average) parental groups so that average child outcomes increase for these groups. If in this very stylized symmetric world parents vote selfishly in the interests of their children, probabilistic or median voter models would predict a tie between pro EO policy (below median parents) and status quo (above median parents) voters (the median voter would be indifferent between the two states) and the policy would face considerable electoral uncertainty.

Consider now a Qualified Equal Opportunity policy where the policy maker is inclined to increase ξ more for children from lower socioeconomic status families and less for those from

[8]Second order dominance of the counter cumulative density

$$\int_{x}^{\infty} \left(F_{pre}(z) - F_{post}(z)\right) dz \quad \geq \quad 0 \;\; \forall x$$

with strict inequality holding somewhere, is a sufficient condition for $\mathbf{E}_{pre}(Y) = \mathbf{E}_{post}(Y)$ (Anderson (2004); Levy and Wiener (1998)).

higher socioeconomic status families (and has the growth capacity to do so), so that ξ now becomes a linear decreasing function of x with $\xi'(x) < 0$, $0 < \xi(x) \leq 1$ (assume $\xi''(x) = 0$). Denote the density of the child's outcome as $f^q(.)$, and the distribution as $F^q(.)$. It follows that:

$$f^q(y|x) \sim N\left((1-\xi(x))x + \xi(x)\mu, \xi(x)^2\sigma^2\right)$$

Here in this new state, among families affected by the Qualified Equal Opportunity policy

$$\frac{\partial\,\mathbf{E}(y|x)}{\partial x} = 1 - \xi(x) + \xi'(x)(\mu - x) \tag{11}$$

$$\frac{\partial^2\,\mathbf{E}(y|x)}{\partial x^2} = -2\xi'(x) + \xi''(x)(\mu - x) = -2\xi'(x) > 0 \tag{12}$$

First note that the parent-child relationship is no longer constant across socioeconomic groups and that $\mathbf{E}(y|x)$ is convex in x compared to the linear relationship of equation (9). In addition,

$$\frac{\partial\,\mathbf{V}(y|x)}{\partial x} = 2\xi(x)\xi'(x)\sigma^2 < 0 \tag{13}$$

implying heteroskedasticity that diminishes with x instead of homoskedasticity of equation (10). In terms of voting behaviour the impact on the status quo group has been lessened and that on the pro policy group increased so that, within the context of a probabilistic voting model the electoral uncertainty regarding the policy has diminished.

To restate the key insights from this simple model: Firstly mobility for the children of a particular socioeconomic group is only improved if it can be achieved without the status of other groups deteriorating on average. Secondly under a qualified mobility policy the parent-child outcome relationship is convexified over the cohorts. Thirdly the variance of the parent-child relationship becomes increasingly negatively related to parental status over successive cohorts.

3 Measuring Conditional or Qualified Mobility

3.1 Examining Intergenerational Mobility via the Transition Matrix and Parent - Child Joint Distribution

The *transition matrix*, $\mathbb{T}$, between the common quantiles of the marginal density vectors $\mathbb{p}$ and $\mathbb{c}$ can be more informative than a regression coefficient as to the nature of the dependence when it is non-linear. This has given rise to the application of techniques derived from Markov

Chain processes and the development of mobility indices, some based upon the nature of the transition matrix directly, some based upon other concepts[9], but all of them reflecting to varying degrees the extent to which the underlying variables, x and y, are independent. With complete mobility the columns of the *transition matrix* would be identical (corresponding to independence between parent and child outcomes) while with complete immobility the leading diagonal would have as its elements 1.

For the present discussion, assume that the realizations are continuous and let $x \in X = [\underline{x}, \overline{x}] \subset \{0\} + \mathbb{R}^+$ and $y \in Y = [\underline{y}, \overline{y}] \subset \{0\} + \mathbb{R}^+$. Let $j(x, y)$ be the joint density function of the parent-child realization, and let $p(x)$ and $c(y)$ be the marginal density functions of the realizations for parent and child respectively. Following Anderson et al. (2010), the degree of mobility is assessed via the joint distribution of x and y (namely $j(x, y)$) since such an approach is amenable to evaluating mobility conditional on particular ranges of parental outcome x (in other words socioeconomic group(s) of interest). The approach is based on the notion that if x and y are independent for a particular range of x and y, say $a_x < x < b_x$ and $a_y < y < b_y$ then:

$$\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy - \int_{a_x}^{b_x} p(x)dx \int_{a_y}^{b_y} c(y)dy = 0 \tag{14}$$

This relation provides the basis of the contingency table test which examines whether or not $\Pr(a_x < x < b_x, a_y < y < b_y) = \Pr(a_x < x < b_x)\Pr(a_y < y < b_y)$ for the set of intervals $\{(\mathbb{a}_x, \mathbb{b}_x) \in X\}$ and $\{(\mathbb{a}_y, \mathbb{b}_y) \in Y\}$, where $\mathbb{a}_x$ and $\mathbb{a}_y$ are vectors of lower integral limits, and $\mathbb{b}_x$ and $\mathbb{b}_y$ are vectors of upper integral limits for x and y respectively, and they delineate mutually exclusive and exhaustive intervals in X and Y respectively.

An overall mobility index (Anderson et al. 2010) may be constructed from a sum of the terms

$$\min\left\{\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy, \int_{a_x}^{b_x} p(x)dx \int_{a_y}^{b_y} c(y)dy\right\} \tag{15}$$

over the collections of intervals. This index is a measure of the extent to which the empirical joint density and the joint density implied by independence, overlap or coincide. The index has a support of $[0, 1]$, where 1 indicates complete independence (mobility), with lower

[9]Bartholemew (1982), Blanden et al. (2004), Chakravarty (1995), Dearden et al. (1997), Hart (1983), Maasoumi (1986), Maasoumi (1986), Prais (1955), Shorrocks (1978), have all produced mobility indices many of which are discussed in Maasoumi (1996).

values indicating relative dependence (immobility). Further, the value of the statistic is asymptotically normally distributed (Anderson et al. 2010), consequently permitting simple statistical comparison of mobility states.

Note that condition (14) could be equally well written as

$$\frac{\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy}{\int_{a_x}^{b_x} p(x)dx} - \int_{a_y}^{b_y} c(y)dy = 0 \tag{16}$$

This relation asks if the conditional probability of a child's outcome given its parent's outcome is equal to the marginal probability of the child's outcome. Conditional or qualified mobility may be examined by considering the sum of terms of the form:

$$\min\left\{\frac{\int_{a_y}^{b_y}\int_{a_x}^{b_x} j(x,y)dxdy}{\int_{a_x}^{b_x} p(x)dx}, \int_{a_y}^{b_y} c(y)dy\right\} \tag{17}$$

In this case the sum is taken over $(\mathbb{a}_y, \mathbb{b}_y)$ that exhaust the range of y. Such a statistic measures the proximity of the conditional distribution to its corresponding marginal distribution where the conditioning region is the range of the parental characteristic of interest. It has the same numeric and statistical properties as the overall mobility statistic outlined above and is more informative in the sense that mobility conditional upon a particular inherited circumstance can be examined. Finally, these techniques can be easily generalized to examine questions involving more than 2 variables (see Anderson et al. (2010)).

3.2 Generational Regressions

Intergenerational mobility has often been examined via the regression coefficient (β) of a child's characteristic when adult (y) on the corresponding parental characteristic (x) (Solon 1992).

$$y = \alpha + \beta x + \gamma x^2 + \epsilon$$

where ϵ is the population error term (and $\gamma = 0$ for now). In effect that literature interpreted β as a mobility index, building upon Becker and Tomes (1979) to create a rich class of models highlighting the forces that determined the value of β, where it inferred *mobility* (*equal opportunity*) as $\beta \to 0$ and *immobility* (*unequal opportunity*) as $\beta \to 1$. Since Atkinson (1983) there has been interest in the nonlinearity of generational income elasticity ($\gamma < 0$) or asymmetry of mobility[10], largely stimulated by Becker and Tomes's (1986) conjecture that parent-child outcome relationships are concave due to asymmetries in borrowing constraints. Presumably theories of diminishing returns to human capital transfer and regression to the mean would also produce a similar conjecture. However here it is suggested that, whatever the initial generational regression relationship, a qualified equal opportunity program would *convexify* (reduce concavity) and *increase the extent to which conditional error heteroskedasticity* of the child outcome is negatively related to adult income. It should be noted that an unqualified equal opportunity program in our model suggests that the stochastic errors associated with child outcomes would be homoskedastic with respect to socioeconomic status.

To reiterate, the predictions of the Qualified Equal Opportunity theory have to do with changes in the curvature coefficient (that on squared parental education) in the generational regressions and the heteroskedasticity coefficient (that on parental education in the log residuals equation) over successive cohorts. Tests of these changes are reported in tables 6 and 7 respectively.

4 An Example: Narrowing the Educational Gender Gap in Canada

One profound change in the latter part of the 20$^{\text{th}}$ century was the emancipation of women and the declining significance of gender in labour and consequently educational outcome (Blau et al. 2006). The introduction of the pill, abortion rights and legislation against gender discrimination in the workplace improved the wellbeing and status of women in those years (Pezzini 2002, Goldin and Katz 2002, Siow 2002). One dimension in which this found expression is in the narrowing gap in academic achievement of men and women (Dynarski 2007). To study this phenomenon in light of the hypothesized qualified mobility mandate, the educational achievements of successive cohorts of Canadian individuals and their parents are compared. Relating to our previous discussion, the educational outcome of children here is the variable y, while that of the parent's is x. A priori under a qualified

[10]Behrman and Taubman (1990), Solon (1992), Mulligan (1999), Corak and Heisz (1999), Couch and Lillard (2004), Grawe (2004) and Bratsberg et al. (2007) all being examples.

mobility policy, we should see an improvement in mobility of children of lower socioeconomic status families regardless of gender. Further, the circumstances favouring women implies that the gains to them over the years should also be greater than it was for men.

4.1 Summary of Data

The data on academic achievements of children and their parents in Canada are drawn from Statistics Canada's *General Social Survey Cycle 19* (2005). Table 1 outlines the attainment index which associates integers 1 through 5 with the highest academic achievements of individuals aged 25 and above and their parents in 2005.

Table 1: Attainment Definition

Index/Year	2005
1	Some Secondary or Elementary or No Education
2	High School Diploma
3	Some University
4	Trade or Technical Diploma or Certificate
5	Bachelors or Masters or Doctorate Degree

Table 2 summarizes the proportion of individuals in each educational attainment category and the corresponding proportion of observations with their parents in those categories by the individual's gender and cohort (decade in which they were born). Note that for individuals born in the 1940s and earlier, the upper attainment levels are dominated by males, but this changes in favour of females in later cohorts, corresponding with the increased female labour force participation in the post World War II decades.

Table 3 presents a comparison of male and female academic attainment distributions across the cohorts, highlighting the turnaround in the academic achievements of males and females over time. Interpreting the continuous child outcome y from our previous discussion as education attainment, we can then denote $A(y)$ as the monotonically increasing educational attainment index function. Let the distribution function of attainment for males and females be $C_m(y)$ and $C_f(y)$ respectively. Then a necessary and sufficient condition for $E[A(y)]$ to be greater for males than females is $C_m(y) \leq C_f(y)$ for all y, the first order dominance criterion. For the cohorts born before 1940, we see three significantly negative differences at 5% level of significance and no significantly positive differences revealing that

Table 2: Summary Statistics by Gender and Cohort

Decade	Gender	No. of Obs.	Variable	Dropout	High School	Some College	Technical Education	University
70s	Male	895	Own	0.060335	0.15531	0.16425	0.31173	0.30838
			Father's	0.28492	0.3095	0.059218	0.1162	0.23017
			Mother's	0.20447	0.40894	0.056983	0.13184	0.19777
	Female	1187	Own	0.04802	0.11542	0.14575	0.32266	0.36816
			Father's	0.32098	0.27548	0.068239	0.14827	0.18703
			Mother's	0.27885	0.34709	0.073294	0.16428	0.13648
60s	Male	1039	Own	0.081809	0.14918	0.12801	0.35226	0.28874
			Father's	0.43503	0.30318	0.032724	0.07026	0.15881
			Mother's	0.38499	0.36959	0.029836	0.087584	0.12801
	Female	1340	Own	0.052985	0.14179	0.15149	0.34104	0.31269
			Father's	0.4791	0.24776	0.050746	0.08209	0.1403
			Mother's	0.43358	0.30522	0.049254	0.10672	0.10522
50s	Male	995	Own	0.1206	0.17286	0.15075	0.26533	0.29045
			Father's	0.60905	0.19397	0.036181	0.044221	0.11658
			Mother's	0.50151	0.33166	0.030151	0.056281	0.080402
	Female	1201	Own	0.076603	0.18068	0.14488	0.32889	0.26894
			Father's	0.58701	0.22315	0.037469	0.045795	0.10658
			Mother's	0.54621	0.26978	0.036636	0.079933	0.067444
40s	Male	659	Own	0.13505	0.1563	0.13809	0.23672	0.33384
			Father's	0.6434	0.22003	0.028832	0.028832	0.078907
			Mother's	0.58574	0.26859	0.021244	0.054628	0.069803
	Female	884	Own	0.15271	0.18439	0.1267	0.28959	0.24661
			Father's	0.67647	0.17308	0.030543	0.041855	0.078054
			Mother's	0.65271	0.20023	0.024887	0.062217	0.059955
$\leq$ 30s	Male	569	Own	0.32689	0.15641	0.11775	0.14587	0.25308
			Father's	0.73111	0.15114	0.040422	0.02109	0.056239
			Mother's	0.68366	0.19156	0.031634	0.045694	0.047452
	Female	887	Own	0.34724	0.18602	0.1195	0.20068	0.14656
			Father's	0.7283	0.14431	0.027057	0.036077	0.064262
			Mother's	0.71477	0.16234	0.020293	0.049605	0.052988

Table 3: Males vs. Females Cumulative Densities and First Order Dominance Results

Decade	Gender	Statistic	Dropout	High School	Some University	Technical Education	University
	Male	CDF	0.08452	0.24911	0.41637	0.74110	1.00000
	Female	CDF	0.07226	0.20109	0.34833	0.67962	1.00000
70s		Difference	0.01226	0.04802	0.06804	0.06148	
		σ	0.01070	0.01661	0.019260	0.01786	
		P(Z≤z)	0.87409	0.99808	0.99979	0.99971	
	Male	CDF	0.12700	0.29028	0.42598	0.75980	1.00000
	Female	CDF	0.07988	0.24310	0.39942	0.73793	1.00000
60s		Difference	0.04711	0.04717	0.02655	0.02186	
		σ	0.01108	0.01598	0.01776	0.01561	
		P(Z≤z)	0.99999	0.99843	0.93260	0.91939	
	Male	CDF	0.15589	0.35201	0.50144	0.76940	1.00000
	Female	CDF	0.13067	0.33454	0.46521	0.78523	1.00000
50s		Difference	0.02522	0.01747	0.03623	-0.01583	
		σ	0.01278	0.01728	0.01817	0.01515	
		P(Z≤z)	0.97580	0.84401	0.97693	0.14802	
	Male	CDF	0.25280	0.41698	0.53265	0.76772	1.00000
	Female	CDF	0.22996	0.42621	0.54231	0.82171	1.00000
40s		Difference	0.02284	-0.00923	-0.00966	-0.05399	
		σ	0.01757	0.02025	0.02045	0.01662	
		P(Z≤z)	0.90314	0.32425	0.31832	0.00058	
	Male	CDF	0.44424	0.58003	0.68525	0.83723	1.00000
	Female	CDF	0.44261	0.61818	0.72443	0.90454	1.00000
≤ 30s		Difference	0.00163	-0.03815	-0.03918	-0.06731	
		σ	0.01903	0.01879	0.01753	0.01310	
		P(Z≤z)	0.53413	0.02118	0.01272	0.00000	

male attainment outcomes first order stochastically dominate that of their female counterparts. However tracking upwards in table 3, it is clear that the attainment gap narrowed across the cohorts, since the difference between male and female distributions disappeared by the 1950s, and in fact the trend of the pre-1940 years were completely reversed by the 1970s (Noting that there were three significant positive differences for the 1970s cohort).

4.2 The Overlap Measure

The "Qualified Equal Opportunity" hypothesis suggests that the conditional density of child attainment for lower socioeconomic groups should be a closer match to the marginal density of child attainment relative to the children from higher socioeconomic status groups since a qualified mobility policy would leave the latter group largely untouched. Section 3.1 provides a test that could easily be performed, which intuitively measures the degree of overlap between two densities. Specifically, the discrete realization analog of the measures in (17) is

$$\min\left\{\frac{j_{i,k}}{p_k}, c_i\right\} \tag{18}$$

The Overlap measure between the conditional density and the marginal density for each parental attainment (socioeconomic group) is then

$$\sum_{i=1}^{m}\min\left\{\frac{j_{i,k}}{p_k}, c_i\right\} \tag{19}$$

for each $k \in \{1, 2, ..., n\}$. If child outcomes and parental circumstances are independent, the Overlap measure will record values close to 1. To the extent that they are not independent the statistic will record a value less than 1. The results of this measure for each parental attainment outcome by gender of the children are reported in table 4. Since the measure is asymptotically normal (Anderson et al. 2010), we can examine how the measure differs across each cohort (reported in table 5), parental attainment groups which we use as a proxy for socioeconomic group status (reported in table 6), and across gender of the children (reported in table 7). Tables 5 to 7 then essentially detail the direction and evolution, and the statistical significance of the changes.

Table 4: Qualified Mobility Indices by Parental Attainment Class, Cohort and Gender

	Mother's Attainment					Father's Attainment				
	Female Child									
	Drop Out	High School	Some College	Technical Education	University	Drop Out	High School	Some College	Technical Education	University
1970s Cohort	0.8437	0.9871	0.8183	0.8933	0.7393	0.8364	0.9680	0.8793	0.9021	0.7859
	(0.0189)	(0.0053)	(0.0400)	(0.0212)	(0.0338)	(0.0188)	(0.0096)	(0.0349)	(0.0221)	(0.0270)
1960s Cohort	0.9112	0.9733	0.8294	0.8703	0.7606	0.9141	0.9380	0.8246	0.8073	0.7128
	(0.0112)	(0.0077)	(0.0437)	(0.0272)	(0.0350)	(0.0108)	(0.0130)	(0.0451)	(0.0363)	(0.0324)
1950s Cohort	0.9223	0.9271	0.6800	0.8411	0.7888	0.9267	0.9232	0.8615	0.8670	0.6419
	(0.0099)	(0.0138)	(0.0695)	(0.0352)	(0.0440)	(0.0095)	(0.0159)	(0.0509)	(0.0435)	(0.0419)
1940s Cohort	0.9200	0.8855	0.8562	0.7690	0.7390	0.9310	0.8579	0.8362	0.7283	0.6447
	(0.0105)	(0.0229)	(0.0675)	(0.0523)	(0.0572)	(0.0101)	(0.0276)	(0.0644)	(0.0695)	(0.0568)
$\leq$ 1930s Cohort	0.924	0.7779	0.7282	0.7176	0.6872	0.9219	0.7658	0.8620	0.7679	0.7337
	(0.0098)	(0.0331)	(0.0928)	(0.0596)	(0.0609)	(0.0101)	(0.0365)	(0.0610)	(0.0651)	(0.0532)
	Male Child									
	Drop Out	High School	Some College	Technical Education	University	Drop Out	High School	Some College	Technical Education	University
1970s Cohort	0.8820	0.9417	0.8568	0.9366	0.7827	0.84353	0.9389	0.8730	0.8537	0.7347
	(0.0232)	(0.0119)	(0.0460)	(0.0215)	(0.0303)	(0.02219)	(0.0140)	(0.04336)	(0.0332)	(0.0303)
1960s Cohort	0.9027	0.9578	0.8914	0.8341	0.7611	0.8675	0.9222	0.8524	0.7882	0.7126
	(0.0143)	(0.0100)	(0.0519)	(0.0382)	(0.0364)	(0.0152)	(0.0148)	(0.0575)	(0.0460)	(0.0344)
1950s Cohort	0.9168	0.9611	0.8318	0.7933	0.7441	0.9191	0.9400	0.8881	0.8859	0.6700
	(0.0120)	(0.0103)	(0.0661)	(0.0527)	(0.0485)	(0.0106)	(0.0166)	(0.0492)	(0.0425)	(0.0427)
1940s Cohort	0.9267	0.8973	0.8470	0.8255	0.8429	0.9116	0.8344	0.8213	0.8754	0.7127
	(0.0128)	(0.0220)	(0.0873)	(0.0624)	(0.0525)	(0.0128)	(0.0297)	(0.0817)	(0.0721)	(0.0594)
$\leq$ 1930s Cohort	0.9227	0.8319	0.7096	0.6476	0.7245	0.9129	0.7965	0.7295	0.6192	0.6204
	(0.0131)	(0.0350)	(0.107)	(0.0858)	(0.07777)	(0.0130)	(0.0407)	(0.0825)	(0.1178)	(0.0798)

Note: Standard Errors are in Parenthesis.

Table 5: Standard Normal Tests of Qualified Mobility Differences Across Cohorts (a negative value denotes an increase in mobility over successive cohorts)

	Mother's Attainment (Female Child)					Father's Attainment (Male Child)				
	Drop Out	High School	Some College	Technical Education	University	Drop Out	High School	Some College	Technical Education	University
60s-70s	3.07346	-1.46651	0.186821	-0.66926	0.43799	0.892696	-0.81934	-0.28536	-1.15459	-0.48282
	(0.99894)	(0.071254)	(0.574099)	(0.25166)	(0.66930)	(0.81399)	(0.20629)	(0.38768)	(0.12413)	(0.3146)
50s-70s	3.6876	-4.0440	-1.7245	-1.27288	0.89222	3.0724	0.051138	0.23078	0.59709	-1.2357063
	(0.99989)	(0.000026)	(0.042312)	(0.10153)	(0.81386)	(0.99894)	(0.52039)	(0.591262)	(0.72478)	(0.10828)
50s-60s	0.74861	-2.9152	-1.8187	-0.65721	0.50186	2.7852	0.80071	0.47141	1.5610	-0.77615
	(0.77295)	(0.0017774)	(0.034479)	(0.25552)	(0.69212)	(0.99733)	(0.78835)	(0.68132)	(0.94074)	(0.21883)
40s-70s	3.5244	-4.3165	0.48225	-2.2051	-0.0033131	2.6565	-3.1857	-0.55874	0.27315	-0.33071
	(0.99979)	(0.0000079)	(0.68518)	(0.013722)	(0.49868)	(0.99605)	(0.00072)	(0.28817)	(0.60763)	(0.37043)
40s-60s	0.57025	-3.631	0.33284	-1.7198	-0.32097	2.219	-2.6478	-0.31136	1.0198	0.0010195
	(0.71574)	(0.000141)	(0.63037)	(0.042735)	(0.37412)	(0.98676)	(0.0040511)	(0.37776)	(0.84610)	(0.50041)
40s-50s	-0.16745	-1.5551	1.8174	-1.1444	-0.68908	-0.45141	-3.1071	-0.70052	-0.12586	0.58282
	(0.43351)	(0.059965)	(0.96542)	(0.12624)	(0.24539)	(0.32585)	(0.00094469)	(0.24180)	(0.44992)	(0.71999)
30s-70s	3.7794	-6.2454	-0.89167	-2.7772	-0.74821	2.6969	-3.3092	-1.5397	-1.9165	-1.3396
	(0.99992)	(0.000000)	(0.18629)	(0.0027412)	(0.22717)	(0.99650)	(0.00047)	(0.061815)	(0.027649)	(0.090184)
30s-60s	0.86495	-5.7553	-0.98622	-2.33000	-1.0452	2.2702	-2.9024	-1.2222	-1.3370	-1.0610
	(0.80647)	(0.000000)	(0.16201)	(0.00990)	(0.14796)	(0.98840)	(0.0018514)	(0.11081)	(0.090609)	(0.14435)
30s-50s	0.11757	-4.1637	0.41627	-1.7832	-1.3523	-0.37354	-3.2657	-1.6512	-2.1305	-0.54789
	(0.5468)	(0.00002)	(0.66139)	(0.037276)	(0.088143)	(0.35437)	(0.00054593)	(0.049346)	(0.016563)	(0.2919)
30s-40s	0.28275	-2.6749	-1.1148	-0.64743	-0.62106	0.06857	-0.75110	-0.79093	-1.8556	-0.92736
	(0.61132)	(0.0037381)	(0.13246)	(0.25868)	(0.26728)	(0.52733)	(0.22629)	(0.21449)	(0.031754)	(0.17687)

Note: $\Pr(Z \leq z)$ in Parenthesis.

Table 6: Standard Normal Tests of Qualified Mobility Differences Across Parental Attainments

	Mother's Attainment (Female Child)					Father's Attainment (Male Child)				
	1970s Cohort	1960s Cohort	1950s Cohort	1940s Cohort	$\leq$ 1930s Cohort	1970s Cohort	1960s Cohort	1950s Cohort	1940s Cohort	$\leq$ 1930s Cohort
High School	7.30220	4.57230	0.28040	-1.36540	-4.23950	3.63340	2.57580	1.05920	-2.39030	-2.72490
-Drop Out	(1.00000)	(1.00000)	(0.61040)	(0.08610)	(0.00000)	(0.99990)	(0.99500)	(0.85520)	(0.00840)	(0.00320)
Some College	-0.57380	-1.81270	-3.45100	-0.93340	-2.09900	0.60400	-0.25440	-0.61590	-1.09240	-2.19610
-Drop Out	(0.28300)	(0.03490)	(0.00030)	(0.17530)	(0.01790)	(0.72710)	(0.39960)	(0.26900)	(0.13730)	(0.01400)
Some College	-4.18370	-3.24170	-3.48600	-0.41170	-0.50430	-1.44670	-1.17420	-0.99860	-0.15040	-0.72940
-High School	(0.00000)	(0.00060)	(0.00020)	(0.34030)	(0.30700)	(0.07400)	(0.12020)	(0.15900)	(0.44020)	(0.23290)
Technical Edu.	1.75020	-1.39240	-2.22430	-2.83110	-3.41620	0.25470	-1.63860	-0.75800	-0.49470	-2.47860
-Drop Out	(0.96000)	(0.08190)	(0.01310)	(0.00230)	(0.00030)	(0.60050)	(0.05070)	(0.22420)	(0.31040)	(0.00660)
Technical Edu.	-4.29810	-3.64870	-2.27660	-2.04160	-0.88380	-2.36060	-2.77350	-1.18510	0.52660	-1.42350
-High School	(0.00000)	(0.00010)	(0.01140)	(0.02060)	(0.18840)	(0.00910)	(0.00280)	(0.11800)	(0.70070)	(0.07730)
Technical Edu.	1.65870	0.79490	2.06730	-1.02080	-0.09610	-0.35220	-0.87140	-0.03340	0.49670	-0.76700
-Some College	(0.95140)	(0.78670)	(0.98060)	(0.15370)	(0.46170)	(0.36240)	(0.19180)	(0.48670)	(0.69030)	(0.22160)
University	-2.69800	-4.10400	-2.96150	-3.11190	-3.84160	-2.90030	-4.11990	-5.65570	-3.27280	-3.61800
-Drop Out	(0.00350)	(0.00000)	(0.00150)	(0.00090)	(0.00010)	(0.00190)	(0.00000)	(0.00000)	(0.00050)	(0.00010)
University	-7.24840	-5.94300	-2.99920	-2.37790	-1.30940	-6.12460	-5.59400	-5.88760	-1.83210	-1.96680
-High School	(0.00000)	(0.00000)	(0.00140)	(0.00870)	(0.09520)	(0.00000)	(0.00000)	(0.00000)	(0.03350)	(0.02460)
University	-1.51020	-1.22920	1.32190	-1.32340	-0.37010	-2.61470	-2.08500	-3.34470	-1.07530	-0.95020
-Some College	(0.06550)	(0.10950)	(0.90690)	(0.09280)	(0.35570)	(0.00450)	(0.01850)	(0.00040)	(0.14110)	(0.17100)
University	-3.86640	-2.47880	-0.92850	-0.38630	-0.35750	-2.64730	-1.31680	-3.58260	-1.74210	0.00870
-Technical Edu.	(0.00010)	(0.00660)	(0.17660)	(0.34960)	(0.36040)	(0.00410)	(0.09390)	(0.00020)	(0.04070)	(0.50350)

Note: $\Pr(Z \leq z)$ in Parenthesis.

¿From table 4, according with expectations, note the strong tendency of the Overlap measure to move towards 1 among children of parents with High School education to Technical training for both genders. This pattern is strongest when comparisons are made between the cohorts born in the 1960s and 1970s against the earlier cohorts and it is stronger (in terms of the change in the Overlap measure) among females. This pattern is not mimicked by children with parents with University education and particularly parents who did not complete their education. The former accords with our Qualified Mobility Policy conjecture, since a high dependence between parent-child outcomes in the status quo would render these children outside the sphere of influence of this policy. All measures are significantly different from 1 suggesting that a pure equal opportunity imperative has not been pursued or achieved. Finally, note that maternal effects were greater than paternal for both genders.

The drive toward higher mobility can be examined by comparing cohorts *within a particular parental attainment class*, with successful policies rendering statistically significantly higher mobility measures with successively younger cohorts. However, from the perspective of the qualified equal opportunity program, the comparison should be between particular parental attainment groups *within a particular cohort* where such programs would result in statistically significantly lower mobility coefficients in higher attainment groups. These comparisons are reported in Tables 5 and 6 respectively, which look specifically at daughters of mothers and sons of fathers comparisons[11].

¿From Table 5, observe that excepting "Drop Out" parents, all of the significant changes across cohorts are increasing mobility changes, predominantly among children with "High School" parents (and then more so with females than males as adjudged from table 6). There are a few significant increases among the daughters of "Technical Education" parents but no significant mobility changes across cohorts in the children (of either gender) of University Graduates, all of which is consistent with a Qualified Mobility program. What is at odds with the Qualified Mobility scenario is the significant reductions in mobility experienced by the younger cohorts in the "Drop Out" parent category. This suggests a forgotten segment of the populace that public policy has neglected. In the stylized model, it has implicitly been assumed that the cost of advancing children across the distribution is the same but in all probability this is not the case. A more appropriate model would explicitly include the cost to the social planner of affecting the different cells of the density vector. Intuitively, if the cost of improving the mobility of the lowest socioeconomic group is relatively the highest, then

[11] The other comparisons did not differ in substance from these and have been omitted for space reasons

it is those children that might be left behind. The results of Table 6, reporting the within cohort across parental attainment category comparisons, are equally supportive of a Qualified Mobility paradigm. Again excluding the “Drop Out” category, mobility is significantly higher in the lower attainment categories and is more so in the recent as compared to the older cohorts.

Table 7: Mobility Differences Daughters of Mothers - Sons of Fathers

	Parental Attainment				
	Drop Out	High School	Some College	Technical Education	University
1970s Cohort	0.0051	3.2198	-0.9266	1.0055	0.1001
	(0.5021)	(0.9994)	(0.1771)	(0.8427)	(0.5399)
1960s Cohort	2.3160	3.0624	-0.3185	1.5373	0.9775
	(0.9897)	(0.9989)	(0.3750)	(0.9379)	(0.8359)
1950s Cohort	0.2246	-0.5940	-2.4426	-0.8129	1.9354
	(0.5889)	(0.2763)	(0.0073)	(0.2081)	(0.9735)
1940s Cohort	0.5035	1.3646	0.3290	-1.1952	0.3198
	(0.6927)	(0.9138)	(0.6289)	(0.1160)	(0.6254)
$\leq$ 1930s Cohort	0.6883	-0.3558	-0.0099	0.7458	0.6651
	(0.7544)	(0.3610)	(0.4960)	(0.7721)	(0.7470)

Note: $\Pr(Z \leq z)$ are in parenthesis

Finally a comparison of the qualified mobility of daughters of mothers with that of sons of fathers reported in Table 7 reveals that with one exception (among children in the 1950s cohort, with parents with some college education), all of the significant differences relate to higher mobility of daughters in more recent cohorts. Furthermore the advances have taken place among children of parents with high school education. No significant differences were identified in the $\leq$ 1930s cohort and only one significant difference was observed in the 1940s cohort at 10% level of significance. This signals the advances that females have made over males in the last half century.

4.3 The Generational Regression Approach

In analyzing educational mobility in the context of generational regressions, the model considered is of the form:

$$y_{i,k} = \alpha_k + \beta_{1,k} x_{i,k} + \beta_{2,k} x_{i,k}^2 + \epsilon_{i,k} \tag{20}$$

where $\mathbf{E}(\epsilon_{i,k}) = 0$ and $\mathbf{E}(\ln \epsilon^2_{i,k}) = \gamma + \phi x_{i,k}$ where $i = \{1, 2, ..., n_k\}$, $k = \{male, female\}$. As before y corresponds to the child and x the parent's outcome (in terms of educational attainment) and heteroskedasticity is modeled in terms of the log squared error being a linear function of parental attainment. Note that parent and child variables here are both discrete integer variables requiring some sort of multinomial technique for analysis since the residuals from regressions which employ them will have heteroskedastic errors. However the hypotheses considered here are that the regression relationship will become increasingly convexified over successive cohort regressions and the heteroskedasticity becomes increasingly negatively related to parental status both of which can readily be examined via simple regression techniques with these albeit discrete variables. The results are reported in tables 8 and 9. At the outset it should be noted that the generational transfer technology appears to be concave i.e. it appears to exhibit diminishing returns to parental ability.

Examining the coefficients of the regression for all the female cohorts from table 8, note first that for females both maternal and paternal effects are highest for cohorts born in the 1940s, but gradually declining with each cohort. Table 9 reports the same results for males and exhibits a similar pattern of falling effect due to parental educational attainment, all of which are evidence of increased educational mobility within both genders. Further, examining the coefficient for heteroskedasticity for each gender in turn, note that all the coefficients are all negative and statistically significant, affirming the prediction of the model that variances should be decreasing across socioeconomic groups (in terms of parental education attainment). In addition, for both child genders, the maternal effect was stronger, and heteroskedasticity seem to be greatest among the 1950s, post World War II cohorts, reflecting the dependence of changes in heteroskedasticity on prior levels of mobility or dependence.

Table 8: Mobility OLS and an Examination of Heteroskedasticty by Cohort, Female Children

	Father	Mother	Father	Mother	Father	Mother	Father	Mother	Father	Mother
	1970s Cohort		1960s Cohort		1950s Cohort		1940s Cohort		$\leq$ 1930s Cohort	
Intercept	3.0850	3.0323	3.0178	2.9444	2.7769	2.5677	2.4334	2.3333	2.0438	2.0559
	(36.3055)	(32.0526)	(38.4946)	(35.4313)	(31.9278)	(29.0950)	(26.0298)	(23.7804)	(26.5908)	(26.3233)
Parent's	0.4909	0.4864	0.4273	0.5022	0.5426	0.7881	0.8104	0.8630	0.5386	0.5127
Education	(7.3014)	(6.7228)	(6.6026)	(7.5242)	(7.6256)	(10.9567)	(9.7828)	(10.0323)	(7.4610)	(7.0241)
(Parent's	-0.0479	-0.0421	-0.0337	-0.0500	-0.0491	-0.0953	-0.0954	-0.1037	-0.0468	-0.0458
Education)^2	(-3.7857)	(-3.1192)	(-2.7056)	(-3.8448)	(-3.4767)	(-6.4503)	(-5.4720)	(-5.7141)	(-2.9196)	(-2.7774)
	Heteroskedasticity									
Parent's	-0.1397	-0.1480	-0.1656	-0.2663	-0.2346	-0.3613	-0.1692	-0.2036	0.1112	0.1329
Education	(-4.8040)	(-4.3470)	(-5.6973)	(-7.5670)	(-8.7535)	(-11.0595)	(-4.1489)	(-4.8055)	(3.4174)	(3.8232)
R^2	0.1382	0.1338	0.1114	0.102	0.1263	0.1537	0.1491	0.1539	0.1081	0.0964
$\overline{R}^2$	0.1317	0.1273	0.1057	0.0963	0.1204	0.1481	0.142	0.1469	0.1025	0.0907
σ^2	1.3402	1.347	1.4123	1.4271	1.606	1.5556	1.8101	1.7999	1.8248	1.8489
No. of Obs.s	1467	1467	1740	1740	1653	1653	1335	1335	1760	1760

t-statistics are in parentheses.

Nine Provincial Indicators were included in each main regression.

Table 9: Mobility OLS and an Examination of Heteroskedasticty by Cohort, Male Children

	Father	Mother	Father	Mother	Father	Mother	Father	Mother	Father	Mother
	1970s Cohort		1960s Cohort		1950s Cohort		1940s Cohort		$\leq$ 1930s Cohort	
Intercept	2.9950	3.0173	2.8551	2.8272	2.7213	2.6538	2.3847	2.5445	2.0318	2.1087
	(29.0205)	(27.8419)	(30.5550)	(30.5137)	(27.7411)	(27.9917)	(20.7241)	(22.0155)	(19.7722)	(20.5475)
Parent's	0.2941	0.3470	0.5143	0.5847	0.5266	0.6409	1.0118	0.8339	0.6733	0.7182
Education	(3.7405)	(4.3857)	(6.8010)	(7.9760)	(6.3012)	(8.3503)	(10.5623)	(8.8184)	(6.9507)	(7.3131)
(Parent's	-0.0057	-0.0217	-0.0415	-0.0577	-0.0505	-0.0720	-0.1360	-0.1125	-0.0491	-0.0815
Education)^2	(-0.3932)	(-1.4983)	(-2.8964)	(-4.0500)	(-3.0845)	(-4.5121)	(-6.7923)	(-5.4738)	(-2.2273)	(-3.5751)
	Heteroskedasticity									
Parent's	-0.1505	-0.1907	-0.2409	-0.2562	-0.1764	-0.2045	-0.0376	-0.1068	0.0979	0.1676
Education	(-3.9561)	(-4.9216)	(-8.6583)	(-8.2600)	(-4.5017)	(-5.8717)	(-0.8183)	(-2.6851)	(2.0506)	(3.6398)
R^2	0.1446	0.109	0.1479	0.1407	0.0959	0.1097	0.1676	0.1324	0.1342	0.1096
$\overline{R}^2$	0.1361	0.1002	0.141	0.1338	0.0887	0.1026	0.1589	0.1234	0.1255	0.1007
σ^2	1.3869	1.4446	1.5544	1.5676	1.7885	1.7611	1.9688	2.0519	2.1154	2.1755
No. of Obs.s	1124	1124	1378	1378	1392	1392	1072	1072	1112	1112

t-statistics are in parentheses.

Nine Provincial Indicators were included in each main regression.

Table 10: Standard Normal Tests of the Reduction in the Degree of Concavity in Successive Cohorts (a negative value denoting a reduction)

	Male		Female	
	Father	Mother	Father	Mother
70s-60s	-1.7596	-1.7690	0.8019	-0.4194
	(0.0392)	(0.0384)	(0.7887)	(0.3375)
70s-50s	-2.0526	-2.3302	-0.0629	-2.6567
	(0.0201)	(0.0099)	(0.4749)	(0.0039)
70s-40s	-5.277	-3.6077	-2.2034	-2.7229
	(0.0000)	(0.0002)	(0.0138)	(0.0032)
70s-30s	-1.6470	-2.2113	0.0533	-0.1705
	(0.0498)	(0.0135)	(0.5213)	(0.4323)
60s-50s	-0.4150	-0.6674	-0.8194	-2.303
	(0.3391)	(0.2523)	(0.2063)	(0.0107)
60s-40s	-3.8384	-2.1906	-2.8805	-2.4070
	(0.0000)	(0.0142)	(0.0020)	(0.0080)
60s-30s	-0.2893	-0.8844	-0.6475	0.2015
	(0.3862)	(0.1883)	(0.2586)	(0.5796)
50s-40s	-3.3039	-1.5570	-2.0624	-0.3601
	(0.0005)	(0.0597)	(0.0196)	(0.3593)
50s-30s	0.0520	-0.3413	0.1068	2.2389
	(0.5207)	(0.3664)	(0.5425)	(0.9874)
40s-30s	2.9179	1.0106	2.0498	2.3649
	(0.9982)	(0.8439)	(0.9798)	(0.9910)

p-values in parenthesis.

Tables 10 and 11 tests the "convexification" and heteroskedasticity comparisons across the five cohorts respectively. Through the five cohorts, there seem to have been a significant decline in concavity of the "production function", somewhat more pronounced for males than females. For males born in the 1970s, the quadratic term was in fact not significant in terms of transmission from both fathers and mothers. Concerning the heteroskedasticity parameter, it appears to have become substantially more negative when the comparison is made between the female cohort born in the 1950s against earlier cohorts. The patterns of increasingly negative heteroskedasticity is likewise noted for males throughout the cohorts from the earliest year to the cohort born in the 1960s. Taken together, the above findings highlight the increase in mobility across the decades for both genders, emphasizing the primary point made by the qualified mobility program hypothesis that it will not impinge

Table 11: Standard Normal Tests of the Increase in the Degree of Negative Heteroskedasticity in Successive Cohorts (a positive value denoting an increase)

	Male		Female	
	Father	Mother	Father	Mother
60s-70s	-1.9192	-1.3191	-0.6309	-2.4152
	(0.0275)	(0.0936)	(0.2641)	(0.0079)
50s-70s	-0.4737	-0.2638	-2.3998	-4.5190
	(0.3179)	(0.3960)	(0.0082)	(0.0000)
50s-60s	1.3441	1.1094	-1.7440	-1.9779
	(0.9105)	(0.8664)	(0.0406)	(0.0240)
40s-70s	1.8913	1.5123	-0.5892	-1.0226
	(0.9707)	(0.9348)	(0.2778)	(0.1532)
40s-60s	3.7831	2.9635	-0.0713	1.1380
	(0.9999)	(0.9985)	(0.4716)	(0.8724)
40s-50s	2.2967	1.8487	1.3398	2.9467
	(0.9892)	(0.9677)	(0.9099)	(0.9984)
30s-70s	4.0688	5.9538	5.7493	5.7732
	(1.000)	(1.0000)	(1.0000)	(1.0000)
30s-60s	6.1310	7.6332	6.3443	8.0706
	(1.0000)	(1.0000)	(1.0000)	(1.0000)
30s-50s	4.4406	6.4447	8.2027	10.3603
	(1.0000)	(1.0000)	(1.0000)	(1.0000)
30s-40s	2.0448	4.5098	5.3745	6.1401
	(0.9796)	(1.0000)	(1.0000)	(1.0000)

p-values are in parenthesis

on the progress or lack of mobility for the well endowed (here the well endowed being male children).

5 Conclusions

It has been demonstrated that in the absence of sufficient flexibility or capacity in a society, the unqualified pursuit of an equal opportunity goal results in some of the inheriting generation being made worse off while others are made better off relative to the status quo. If some sort of Pareto-Utilitarian goal is also an objective of the policy maker (in effect that no inheriting class should be made worse off) in a constant cost world, a qualified equal opportunity outcome emerges in which the most disadvantaged are addressed first. With such a program, complete independence of outcome from circumstance will not be observed across all socioeconomic groups and conventional measures of mobility will not record complete mobility. However such policies have predictable consequences for generational regressions and suggest ways that mobility measures could be re-interpreted. Evaluating conditional mobility policies via the transition matrix or joint distribution of outcomes and circumstance requires indices which identify changes in mobility by subgroup or conditional mobility measurement. In the context of Generational Regressions Qualified Equal Opportunity policies induce a reduction in concavity in the prevailing regression relationship as well as inducing heteroskedasticity in the corresponding error process which is negatively related to the conditioning variable.

To illustrate the concept and the associated indices, the successes of various equal opportunity policies pursued either implicitly or explicitly in the emancipation of women was evaluated in terms of how they narrowed the gender gap in educational attainment in Canada. Hypotheses relating to generational regressions that are consistent with a qualified equal opportunity program are not rejected for daughters whereas they are for sons. From the conditional mobility indices comparisons, the gender gap appears to have been narrowed by an increase in the mobility of the daughters of parents of lower educational status, without any change in the mobility of daughters or sons in the highest parental educational attainment category. All of which is what would have been expected from a Qualified Equal Opportunity or Conditional Mobility Policies.

It also appears that there is a segment of children, both males and females, of dropout parents whom society has neglected in that their mobility has diminished. It is conjectured that, contrary to what is implicitly assumed in the model here presented, the cost of improving the stead of the deprived are not the same as those associated with other better endowed segments of the populace. If those cost are significantly higher, the social planner may be less inclined to improve their mobility in the first instance.

References

Anderson, Gordon, "Toward an Empirical Analysis of Polarization," *Journal of Econometrics*, 2004, *122* (1), 1–26.

—— **and Teng Wah Leo**, "Indices and Tests for Matching and Mobility Theories," 2008. Mimeo Department of Economics University of Toronto.

——**, Oliver Linton, and Yoon-Jae Whang**, "Nonparametric Estimation of a Polarization Measure," 2009. Mimeo University of Toronto, Department of Economics.

——**, Ying Ge, and Teng Wah Leo**, "Distributional Overlap: Simple, Multivariate, Parametric and Non-Parametric Tests for Alienation, Convergence and General Distributional Difference Issues," *Econometric Reviews*, 2010, *29* (3), 247–275.

Arneson, Richard, "Equality and Equality of Opportunity for Welfare," *Philosophical Studies*, 1989, *56*, 77–93.

Atkinson, Anthony, *The Measurement of Economic Mobility in Social Justice and Public Policy* 1983. Cambridge, MA.

Bartholemew, David, *Stochastic Models for Social Processes* 1982. Wiley New York.

Becker, Gary and Nigel Tomes, "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," *Journal of Political Economy*, 1979, *87*, 1153–1189.

—— **and** ——, "Human Capital and the Rise and Fall of Families," *Journal of Labor Economics*, 1986, *4*, 1–39.

Behrman, Jere and Paul Taubman, "The Intergenerational Correlation Between Children's Adult Earnings and their Parents Income: Results from the Michigan Panel Survey of Income Dynamics," *Review of Income and Wealth*, 1990, *36*, 115–127.

Ben-Shahar, Danny and Eyal Sulganik, "Partial Ordering of Unpredictable Mobility with Welfare Implications," *Economica*, 2008, *75*, 592–604.

Blanden, Jo, Alissa Goodman, Paul Gregg, and Stephen Machin, "Changes in Intergenerational Mobility in Britain," 2004. In *Generational Income Mobility in North America and Europe*, edited by Miles Corak. Cambridge University Press.

Blau, Francine, Mary Brinton, and David Grusky, *The Declining Significance of Gender?* 2006. Russell Sage Foundation.

Bratsberg, Bernt, Knut Roed, Oddbjorn Raaum, Robin Naylor, Markus Jantti, Tor Viking Eriksson, and Eva Osterbacka, "Nonlinearities in Intergenerational Earnings Mobility: Consequences for Cross Country Comparisons," *Economic Journal*, 2007, *117*, 72–92.

Buchmann, Claudia and Thomas Diprete, "The growing Female Advantage in College Completion: The Role of Family Background and Academic Achievement," *American Sociology Review*, 2006, *71*, 515–541.

Cavanagh, Matt, *Against Equality of Opportunity* 2002. Clarendon Press.

Chakravarty, Satya, "A Note on the Measurement of Mobility," *Economics Letters*, 1995, *48*, 33–36.

Cohen, Gerald Allan, "On the Currency of Egalitarian Justice," *Ethics*, 1989, *99*, 906–944.

Corak, Miles, *Generational Income Mobility in North America and Europe* 2004. Cambridge University Press.

___ **and Andrew Heisz**, "The Intergenerational Earnings and Income Mobility of Canadian Men: Evidence from Longitudinal Income Tax Data," *Journal of Human Resources*, 1999, *34*, 504–533.

Couch, Kenneth and Dean Lillard, "Linear Patterns of Intergenerational Mobility in Germany and the United States," 2004. In *Generational Income Mobility in North America and Europe*, edited by Miles Corak. Cambridge University Press.

Coughlin, Peter J., *Probabilistic Voting Theory*, New York: Cambridge University Press, 1992.

Dardanoni, Valentino, Gary Fields, John Roemer, and Maria Laura Sanchez Puerta, "How Demanding Should Equality of Opportunity Be, and How Much Have We Achieved?," 2006. In *Mobility and Inequality:Frontiers of Research in Sociology and Economics*, edited by S. Morgan, D. Grusky and G. Fields. Stanford University Press.

Dearden, Lorraine, Stephen Machin, and Howard Reed, "Intergenerational Mobility in Britain," *Economic Journal*, 1997, *107*, 47–66.

Downs, Anthony, *An Economic Theory of Democracy*, New York: Harper Collins, 1957.

Dworkin, Ronald, "What is Equality? Part 1: Equality of Welfare," *Philosophy and Public Affairs*, 1981, *10* (3), 185–246.

—— , "What is Equality? Part 2: Equality of Resources," *Philosophy and Public Affairs*, 1981, *10* (4), 283–345.

—— , *Sovereign Virtue: The Theory and Practice of Equality* 2000. Harvard University Press.

Dynarski, Susan, "Cradle to College: The Puzzle of Gender Differences in Educational Outcomes," 2007. Mimeo Harvard University Kennedy School of Government.

Friedman, Milton, *The Moral Consequences of Economic Growth* 2005. Alfred Knopf New York.

Goldin, Claudia and Lawrence Katz, "The Power of the Pill: Oral contraceptives and Women's Career and Marriage Decisions," *Journal of Political Economy*, 2002, *110*, 730–770.

—— , —— , and Ilyana Kuziemko, "The Homecoming of American College Women: The Reversal of the College Gender Gap," *Journal of Economic Perspectives*, 2006.

Grawe, Nathan, "Reconsidering the Use of Nonlinearities in Generational Earnings Mobility as a Test for Credit Constraints," *Journal of Human Resources*, 2004, *39*, 813–827.

Hart, Peter, "The Size Mobility of Earnings," *Oxford Bulletin Of Economics and Statistics*, 1983, *45*, 181–193.

Jacob, Brian, "Where the boys aren't: Non-Cognitive Skills, Returns to School and the Gender Gap in Higher Education," *Economics of Education Review*, 2002, *21*, 589–598.

Jencks, Christopher and Laura Tach, "Would Equal Opportunity Mean More Mobility?," 2006. In *Mobility and Inequality: Frontiers of Research in Sociology and Economics*, edited by S. Morgan, D. Grusky and G. Fields. Stanford University Press.

Kanbur, Ravi and Joseph Stiglitz, "Intergenerational Mobility and Dynastic Inequality.," 1986. Woodrow Wilson Discussion Paper No 111 Princeton University.

Levy, Haim and Zvi Wiener, "Stochastic Dominance and Prospect Dominance with Subjective Weighting Functions," *Journal of Risk and Uncertainty*, 1998, *16* (2), 147–163.

Maasoumi, Esfandiar, "The Measurement and Decomposition of Earnings Mobility," *Econometrica*, 1986, *54*, 991–997.

——, "On Mobility," 1996. In *Handbook of Applied Economic Statistics*, edited by D. Giles and A. Ullah. Marcel Dekker.

Mulligan, Casey B., "Galton versus the Human Capital Approach," *Journal of Political Economy*, 1999, *107*, 184–224.

Nussbaum, Martha, "Poverty and Human Functioning: Capabilities as Fundamental Freedoms," 2006. In *Poverty and Inequality*, edited by D. Grusky and R. Kanbur. Stanford University Press.

Pezzini, Silvia, "The Effect of Women's Rights on Women's Welfare: The Results of a Natural Experiment," *Economic Journal*, 2002, pp. 208–227.

Piketty, Thomas, "Theories of Persistent Inequality and Intergenerational Mobility," 2000. In *Handbook of Income Distribution, Volume 1*, edited by A. Atkinson and F. Bourguignon. North Holland.

Prais, Sig, "Measuring Social Mobility," *Journal of the Royal Statistical Society Series A*, 1955, pp. 56–66.

Roemer, John, *Democracy, Education and Equality: Graz Schumpeter Lectures*, Vol. 40 of *Econometric Society Monographs* 2006. Cambridge University Press.

Sen, Amartya, "Gender and Cooperative Conflicts," 1990. In *Persistent Inequalities*, edited by I. Tinker. Oxford University Press.

——, "Gender Inequality and Theories of Justice.," 1995. In *Women, Culture and Development* edited by M. Nussbaum and J. Glover. Clarendon Oxford.

Shorrocks, Anthony, "The Measurement of Mobility," *Econometrica*, 1978, *46*, 1013–1024.

Siow, Aloysius, "Do Innovations in Birth Control Technology Improve the Welfare of Women," 2002. Mimeo University of Toronto.

Solon, Gary, "Intergenerational Income Mobility in the United States," *American Economic Review*, 1992, pp. 393–408.

van de Gaer, Dirk, Erik Schokkaert, and Michel Martinez, "Three Meanings of Intergenerational Mobility," *Economica*, 2001, *68* (272), 519–537.