Review and homework from Jan. 24 class

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We decided we wanted to measure the amount of time that passed on a single clock that was at rest in the S frame (should be a proper time) and compare it to a time interval measured with two clocks in *S'*. We elected on the clock at the Antigonish bus station which has a vertical world-line that intersects the x-axis at 60 km. Then we needed to make a choice about the two events bewteen which we would measure the time-interval. We decided to keep event #2 our bus arriving in Antigonish as an event with $x = 60$ km, $t = 333$ μs ($ct = 100$ km), $x' = 0$, $t' = 267$ μs ($ct' = 80$ km). At this point in space-time the Antigonish station clock and our bus coincide in spatial coordinate. Notice that we don't really make a big deal about $x \neq x'$ but $t \neq t'$ makes us slightly uneasy.

In class I decided the other event (#3) should be at $ct' = 0$. So it is the intersection between the x' axis and the Antigonish clock world-line. We know that $x = 60$ km so we use the inverse Lorentz transform $x = \gamma(x' + \beta ct')$ as a starting equation and then solve for $x' = 48$ km. Now you use the other inverse LT to find $t = 120$ μs ($ct = 36$ km).

With t'_2 and t'_3 we find $\Delta t' = 267$ μs and $\Delta t = 333 - 120 = 213$ μs. **Conclusion:** S clock runs slowly. Since the interval in S was measured with one clock it is a proper time. And we find that $\Delta t'$ / (proper time) $= \gamma$ just as we expect.

How can we have both $\Delta t = \gamma \Delta t'$ and $\Delta t' = \gamma \Delta t$? Wouldn't this force $\gamma = 1$? One reason is that you are timing different types of events which you can tell since they are connected by worldlines with different slopes. Depending on which worldline you choose there is a difference between proper time and a time measured with two different clocks. To measure a time with two different clocks they need to be synchronized. The observer with just one clock says that these moving clocks are slow and unsynchronised, and the trailing clocks (imagine the "fixed" observer sees a whole parade of clocks coming towards him) are more advanced in time. You interpret this on the space-time diagram by saying the positive-x axis ($ct = 0$) has $ct' < 0$ and that becomes more negative as you move farther out on the x-axis. So if you have $ct' = -200$ μs and $ct = 0$ the *S'* observer says: S started the clock too early, hence it is advanced in time.

Do you think you understand what is going on? Try these:

1. It looks like the "buses" are only 48 km apart even though Antigonish and NG are 60 km apart. What is this effect called?

2. On the second bus when $t' = 0$ the Antigonish clock doesn't read zero; it reads 120 μs. Where (and when) were the first and second buses when the Antigonish clock read zero. What was the x' of the Antigonish clock when it read zero?

3. Treating the Antigonish clock at $x = 60$ and $t = 0$ as event #4 find the x' and t' of this event (you may have just done this) and find the proper time between events #3 and #4 and the $\Delta t'$ between these events. Again you should find a ratio of γ.

Answers:

1. This is called *Lorentz* or *Lorentz-Fitzgerald contraction.* According to the buses at rest in *S'* both New Glasgow and Antigonish are moving towards them at the same speed and the

proper length is reduced. The proper length is measured in the frame where New Glasgow and Antigonish are at rest *S*. This effect arises because trying to measure the length of a moving object involves two clocks that need to be synchronised or one clock that will time an interval, both measurements are affected by relativity. The Lorentz contraction allows observers in *S'* to explain why they can make the trip "faster," even though their speed is the same; the distance is less.

2. The first bus is right where we expect it to be at $x = ct = 0$. Just put $x' = 0$ and $ct = 0$ in the Lorentz transforms to show this. The second bus is still at $x' = 48$ km. Knowing that $ct = 0$ you can use the inverse LT $0 = \gamma(x' + \beta ct')$ to solve $t' = -96$ μs (-28.8 km). Then substitute these values into the other inverse LT to give $x = 38.4$ km. So the second bus hasn't reached Antigonish yet (or Antigonish hasn't reached the second bus); makes sense since we know that we have had to go back in time to have $t = 0$ on the Antigonish clock. Also this 38.4 km is a further Lorentz contraction of the 48 km proper length between buses.

   So where is the Antigonish clock when it reads $t = 0$ according to *S'* observers? Put it in the Lorentz transform to give 1.25 $\times$60 km $x' = 75$ km. The time at the second bus is -96 μs but that isn't the t' for Antigonish! (this is a trailing clock, see above) Find ct' for Antigonish by using LT $ct' = 1.25(0 - \beta x) = -45$ km or $t' = -150$ μs.

3. I have all the numbers I need now. Looking at the time differences between events 3 and 4: $\Delta t'$=0-(-150)=150 μs and Δt=120-0=120 μs. They are in the ratio of γ. The observers in *S'* say that more time has elapsed than the observers with the single clock do. Just what we expect.

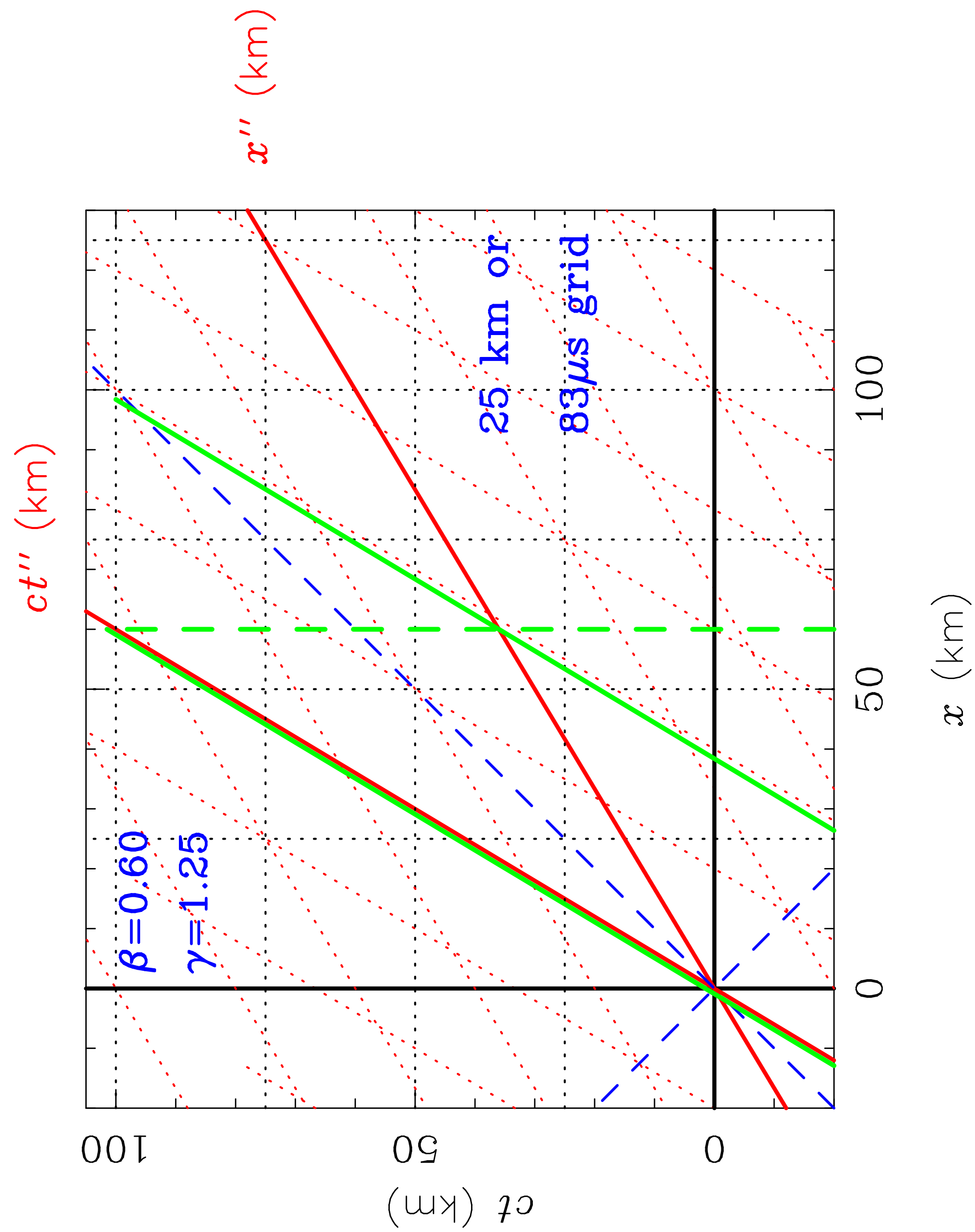

Figure 1: Space-time diagram for bus discussion. Green lines are world-lines of buses and the Antigonish bus station. No special meaning for "double prime".