Final Exam: Modern Physics 301
December 11, 2001

Open book. Point values are given with each question. Total exam is worth 180 points.

1. Value: 30 Points.

   (a) According to the postulates of special relativity what do inertial observers conclude about measurements of the speed of light in different reference frame? Does this answer depend on the motion of the source? (5)

   (b) An inertial frame S' moves with velocity v relative to frame S. The x-axes of the frames and the direction of motion are all parallel. At $t = t' = 0$ the origins of the reference frames coincide. State the Lorentz transformations that relate (x, y, z, t) to (x', y', z', t'). What are the inverse transformations? (10)

   (c) A metre stick is at rest in S' and lies along the x'-axis. At the moment $t = 0$ the observer in S makes a measurement of the length of the metre stick. What is the x-coordinate at the end of the metre stick at this time? According to the observer in S' when does the observer in S make her measurement? (i.e. what is t' at this location?) Your answers should agree with what you know about Lorentz contraction and the synchronization of moving clocks. (10)

   (d) Calculate the Lorentz contracted length of the metre stick if $v = 0.8\,c$. (5)

2. Value: 35 points.

   (a) According to the kinetic theory of gases, what is the average thermal energy per molecule of a monoatomic gas in three dimension at temperature T. How does the answer change if we consider a rigid diatomtic molecule? (10)

   (b) So what is the RMS velocity (m/s) and average kinetic energy (in eV) per molecule of xenon (a monoatomic gas) at 800 K if the kinetic energy equals the average thermal energy. The molecular weight of xenon is 0.131 kg/mol. (10)

   (c) You determine that 1.5% of the xenon molecules are moving with a speed between 495 and 505 m/s but only 4.7×10^{-7} of the total are moving with v_x between 495 and 505 m/s, v_y between -5 and 5 m/s, and v_z between -5 and 5 m/s. Demonstrate how I determined the ratio between these two numbers. Even if you can't determine the answer, explain why the second value is so much smaller. A diagram may be useful. (15)

3. Value: 15 points.

   (a) What was Planck's hypothesis concerning the energy levels in a blackbody radiator? (5)

   (b) Monochromatic light with $\lambda = 500$ nm is incident on a photocathode. The work function of the cathode is 1.5 eV. (This is the energy required to liberate electrodes from the cathode.) What reverse bias is necessary to cut off the photocurrent? (10)

4. Value: 45 points

   (a) Give the energies of a photon, electron, and neutron such that all have a wavelength of 1.2 Å. (15)

   (b) State Bragg's law of diffraction. (5)

(c) You measure the total scattering angle 2θ to be 90 deg with $\lambda = 1.2$ Å. What is d, the spacing of the crystal planes, that causes this Bragg reflection? (10)

(d) Suppose that you want to localize the electrons so you use a crystal that is quite small, only 20 Å in size. With your knowledge of the uncertainty principle make an estimate (in eV/c units) of the minimum uncertainty of the momentum of the scattered electrons. I am not worried about factors of 2 here and there. (10)

(e) Suppose you calculate a momentum uncertainty of 100 eV/c. What would be the spread in angle of 1.2 Å electrons? This does actually happen. "Bragg" scattering from small objects gives wide diffraction peaks. (5)

5. Point value: 55 points

(a) Write the time-dependent and time-independent Schrödinger equations. (10)

(b) Show that $\Psi(x,t) = Ae^{i(kx-\omega t)}$ is a solution of the time-dependent Schrödinger equation for a free particle [$V(x) = 0$]. Give expressions for the energy of the particle in terms of k and ω. (15)

(c) I say *particle* but considering the probabilistic interpretation of $\Psi(x,t)$ do you know where the particle *is* if $\Psi(x,t) = Ae^{i(kx-\omega t)}$? Explain. (5)

(d) The time-independent Schrödinger equation for a simple harmonic oscillator potential is

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2 x^2\psi = E\psi. \tag{1}$$

$$\psi_0(x) = A\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \tag{2}$$

is the ground state solution. Show that $\psi_0(x)$ is an eigenfunction of the Schrödinger equation and give the eigenvalue E_0 (the ground state energy). (15)

(e) The constant A is determined via normalization. Setup the normalization condition for ψ_0. Calculate an appropriate value of A assuming that A is real and positive. The following definite integral

$$\int_{-\infty}^{\infty} e^{-\sigma x^2}dx = \sqrt{\frac{\pi}{\sigma}} \tag{3}$$

will be useful. (10)

6. BONUS: What were Einstein's 3 significant contributions to the field of modern physics around 1905? (10)