Final Exam: Modern Physics 301
December 13, 2002

Open book. Point values are given with each question. Total exam is worth 165 points. Do all of the first four questions and any three of the last four questions.

1. (a) How did classical physics (Maxwell's equations) fail to describe the hydrogen atom with respect to the Rydberg-Ritz formula? Comment on the stability of the hydrogen atom in classical physics. (6)

   (b) What was Bohr's assumption concerning quantization of angular momentum in the hydrogen atom? If angular momentum $L = mvr$ show how this is consistent with deBroglie's hypothesis that the wavelength of the electron is $\lambda = h/p$. (6)

   (c) Derive the expressions for the quantized radii and energy levels in the hydrogen atom ($Z = 1$) based on Bohr's quantization condition. Make the following assumptions: the proton is much heavier than the electron so the reduced mass is equal to the electron mass. The orbits are circular and the angular momentum is given by $L = mvr$. The total energy E for the electron in a circular orbit is given by

$$E = -\frac{1}{2}\frac{ke^2}{r} \tag{1}$$

   and the Coulomb and centripetal forces are equal

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}. \tag{2}$$

   Once you have derived the expressions calculate the first Bohr radius and the energy of the ground state. (18)

2. (a) Consider a hydrogen molecule H_2. It is a diatomic rigid molecule at $T = 500$ K. Write the expression for its total energy if it has mass m and moment of inertia I perpendicular to the bond direction. According to the equipartition theorem what is the thermal average energy per hydrogen molecule at this temperature. (total energy, including translational and rotational). (10)

   (b) How many hydrogen molecules are there in 1 mole? What is the mass of one hydrogen molecule? What is the mass of one hydrogen molecule in MeV/c^2 if it is composed of two protons (ignore any contribution from binding energy). (6)

   (c) Much like the hydrogen atom the hydrogen molecule has only certain allowed values of angular momentum. This implies that the first rotational energy level is 15 meV above the ground state. (This is $\hbar/I$ if you are curious.)

      i. What is the wavelength of the photon that would correspond to this transition? (4)

      ii. What is the fractional occupation of this level compared to the ground state at 500 K and at 30 K? How would you expect this to affect the number of degrees of freedom in the equipartition theorem? (10)

3. A train car acts as an inertial frame S' moving with velocity $v = 0.866c$ relative to frame S fixed to the platform. The x-axes of the frames and the direction of motion are all parallel. At $t = t' = 0$ the origins of the reference frames coincide. In addition to the coincidence of origins, lightning simultaneously strikes both ends of the platform and the ends of the train car at $t = 0$. I call the lightning strike at $x = 0$ and $t = 0$ lightning strike 1 and the other strike 2.

(a) The train car has a rest length of 10 m. How far apart are the lightning strikes in S ? (4)

(b) When and where do the two lightning strikes occur in S' ? What is the order of the strikes? (1 before 2, 2 before 1, or 1 and 2 at the same time) (6)

(c) Observers in S and S' are at the location of lightning strike 2. Use the constancy of the speed of light to conclude how much time passes in each reference frame before the observers "see" lightning strike 1? At what location x in S does the observer in S' (at $x' = 10$ m) see lightning strike 1 (or where is the end of the train car when the light from strike 1 catches up to it)? (10)

4. If your one-dimensional wavefunction has the form $\psi(x)$ the position $\hat{x}$ and $\hat{p}_x$ momentum operators are

$$\hat{x} = x \tag{3}$$

$$\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}. \tag{4}$$

(a) Show that the electron plane wave $\psi(x) = Ae^{ikx}$ is an eigenfunction of the momentum operator and give the eigenvalue. (10)

(b) Show that the commutator $[\hat{x}, \hat{p}_x] = i\hbar$ (hint: expand the commutator and let it act on a wavefunction $\psi(x)$). (10)

(c) In our dealings with the simple harmonic oscillator we found eigenfunctions

$$\psi_0(x) = A\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \tag{5}$$

$$\psi_1(x) = A\sqrt{\frac{2m\omega}{\hbar}}x\exp\left(-\frac{m\omega}{2\hbar}x^2\right) \tag{6}$$

for the ground state and first excited state. A is some normalization constant. Show that the lowering operator

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}_x\right) \tag{7}$$

acting on ψ_1 gives ψ_0, and $\hat{a}$ acting on ψ_0 gives zero. (20)

Do any 3 of these final 4 questions. Each one is worth 15 points in total.

5. Give a sketch of the spectral distribution $R(\lambda)$ for a blackbody radiator at three distinct temperatures. In what region does the equipartition theorem fail? (No reasoning required, just state your answer). How many modes (counting both polarizations) are there with λ between 495 and 505 nm in a volume of 0.01 m^3? (15)

6. State the position-momentum uncertainty principle. Suppose that an alpha particle is confined to a space the size of a nucleus ($\Delta x = 5$ fm). What is its minimum kinetic energy according to the uncertainty principle? Notice that this is quite a bit smaller than typical energies for radiated α-particles implying that the wave-like properties of the alphas are hidden at this length scale. What is the wavelength of an alpha particle with $\frac{p^2}{2m} = 1$ MeV? (15)

7. What was the feature of Rutherford scattering that implied that the nucleus was much smaller than the size of the atom? What fraction of a beam of incident 6 MeV α-particles are scattered to an angle greater than 50 degrees on a silver foil ($Z = 47$) with $nt = 5.86 \times 10^{18}$ cm^{-2} (atomic density times the thickness). Give the impact parameter b for particles that scatter at 50 degrees. (15)

8. (a) A muon ($m_\mu c^2 = 106$ MeV) is created in the upper atmosphere and moves towards the earth. If the time dilation factor is 5 what is the total energy (MeV), speed (in terms of c) and the momentum (MeV/c) in the reference frame of the earth. (8)

   (b) The muon comes to rest in the earth frame and decays into an electron ($m_e c^2 = 0.511$ MeV) and a neutrino ($m_\nu \approx 0$). The energy of the electron is so high that $E \approx pc$ and the energies and momenta are equally split between the particles. What is the total energy of the electron and its γ-factor? (7)