Final Exam: Modern Physics 201
April 19, 2006

Duration: 3 hours. Text book and one double sided formula sheet. Point values are given with each question. Focus on key words in your written answers. Total exam is worth 100 points plus a 6 point bonus question. Maximum mark is still 100%.

$mc^2 = 5.11 \times 10^5$ eV for an electron, $\hbar c$=1973 eV Å, $\frac{e^2}{4\pi\epsilon_0} = 14.4$ eV Å

1. (a) Consider a Young's double slit experiment. Is the observation of an interference pattern a wave or particle phenomena? Suppose that the slits are 1.6 μm apart and illuminated by a monochromatic plane wave. You observe the first intensity maximum at an angle of 20 degrees relative to the undeflected beam. What is the wavelength of the incident radiation? (should be in the visible range) (4)

   (b) Now consider the question of "what slit does the radiation go through?". How do particles and waves act differently when you ask this question? (4)

   (c) I want to answer this question experimentally. In class I mentioned I could put snow outside the two doors of the class and look for footprints. What we will do is put some test particles in a box in front of the top slit. What is the spread in the y-momenta of the particles in the box? (Make an estimate of the size of the box in the y-direction and use the uncertainty principle. It works equally well in the y and x directions.) How does this compare to the x-momentum? Please continue to use the parameters given above. What do you think will happen to the interference pattern? (8)

2. (a) What phenomenon suggests that nuclei (like atoms) also have different energy levels and structures? What are the typical energy levels in nuclei? What is a typical size for a nucleus (6)

   (b) Suppose that an alpha particle (mass roughly 4000 MeV/c^2) is trapped inside of a nucleus. The alpha particle has an energy of 5 MeV and moves freely inside the nucleus. If we approximate the nuclear barrier as rectangular with a height of 25 MeV and width of 10 fm what is the probability that an alpha particle will escape any given time it hits the barrier. (This will be a very small probability.) In quantum mechanics what is this phenomena called that allows the alpha particle to escape? You may find the combination $\hbar c = 197.3$ MeV·fm useful. (8)

3. (a) Use the Rydberg-Ritz formula to calculate the wavelength of the light that comes from the $n = 3$ to $n = 2$ transition in hydrogen. (4)

   (b) What are the two radiative processes for the transition from a higher energy state to a lower energy state? Which one is involved in lasers? (4)

   (c) What was the importance and the result of the Franck-Hertz experiment? (4)

4. (a) Give the time-independent Schrödinger equation for one dimension. (2)

   (b) Write the time-independent Schrödinger equation for the case of a one-dimensional infinite square well for $0 \le x \le L$. What values must $\psi(x)$ have at the walls? Why? (4)

   (c) What part of the above equation is the Hamiltonian operator? (2)

   (d) Show that

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \quad (1)$$

   is an eigenfunction of the Hamiltonian and give the eigenvalue E. (4)

(e) Sketch this wavefunction. (2)

(f) Calculate E if the particle is an electron and the well is 1.5 Å wide. Is $E = 0$ possible for a trapped particle ? (4)

5. (a) Consider a "light clock" in the frame S. The top mirror is on the y-axis at a position of $L = 3$ m. The source and detector are at both at the origin. Suppose that a pulse is sent at $t = 0$. According to observers in S at what time does the pulse return to the origin? (give both a symbolic and numerical answer, you may find $c = 0.3$ m/ns useful). Is this a proper time interval? Why or why not? (4)

   (b) What is the name of the transforms that relate time and position coordinates in different reference frames? What is the special property the frames must posess (aside from details like axis-orientation and synchronization at the origin). (3)

   (c) The transforms are

$$x' = \gamma(x - vt) \quad (2)$$
$$y' = y \quad (3)$$
$$z' = z \quad (4)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad (5)$$

   Give this in 4-vector form. (2)

   (d) Now suppose that frame S' is moving with $v = -0.8c$ (note the negative sign) with the usual conventions. Just sketching, what is the appearance of the light clock in this frame as it moves. (We have drawn similar diagrams in class.) (3)

   (e) Observers make measurements of quantities in the different frames. Do they agree or disagree on values for

      i. the speed of light
      ii. the distance the pulse has travelled
      iii. the time elapsed
      iv. the distance of the mirror from the x-axis

   (4 points)

   (f) Use the transforms to obtain a symbolic and numeric solution for the time elapsed (it will just be t' if the pulse left from the origin at $t = t' = 0$) in the S' frame. What is this result called? (4)

6. I have repeatedly mentioned times when the predictions of classical physics disagree with certain phenomena. These disagreements led to the development of modern physics. Pick two of these disagreements and briefly describe them highlighting experimental evidence, names of scientists involved, classical predictions and disagreement, and the assumption or development of modern physics with an appropriate formula (if it is fairly straight forward to write down) leading to the proper physical description. (Franck-Hertz is not an option!) (20)

7. BONUS: What is a typically yearly background dose of ionizing radiation in either mrem or millisieverts? (6)