Final Exam: Modern Physics 201
April 13, 2009

Duration: 2.5 hours. Formula sheet provided. Point values are given with each question. Focus on key words in your written answers. Total exam is worth 100 points. Notice that question 4 covers a variety of topics and has the most total points available.

1. (a) What is a "blackbody"? How is it characterised? (2)

   (b) Draw a sketch of the blackbody spectrum [either $R(\lambda)$ or $u(\lambda)$]. Now use a dashed line to show what the "ultraviolet catastrophe" case would look like. (3)

   (c) What would the temperature be of a blackbody whose spectrum peaked at $\lambda = 490$ nm? What example of a blackbody did I mention in class that has roughly this temperature? (3)

   (d) The derivation of the blackbody radiation formula involves a calculation of the number of modes per unit volume per unit wavelength and a calculation of the average energy per mode. What was Planck's hypothesis and which of these calculations was affected? (2)

   (e) By using the approximation $e^x \approx 1 + x$ for $x \ll 1$ show how the Planck formula for $u(\lambda)$ and the Rayleigh-Jeans formula for $U(\lambda)$ agree for $\frac{hc}{\lambda k_B T} \ll 1$. (4)

2. (a) Consider a Bragg scattering experiment. What is the central assumption in the derivation of Bragg's Law concerning the extra path length and the wavelength in order to observe interference maxima? Suppose that we have a neutron beam with wavelength $\lambda = 1.3$ Å that is Bragg scattered from a set of crystal planes with spacing 3.5 Å. What is the 2θ scattering angle for first order scattering? (4)

   (b) What is the k of these neutrons? What is the kinetic energy? (Use non-relativistic formulas. The mass of a neutron is 940 MeV/c^2 or 1.67×10^{-27} kg.) (3)

   (c) Suppose that we are trying to represent an individual neutron by a superposition of waves. We know the position of the neutron in x-direction to an uncertainty of 50 Å. What is the necessary range of k and the implied momentum uncertainty? (I find a value of 10's of eV/c for Δp.) (4)

   (d) Make a sketch of what the neutron wavefunction might look like as a function of x at one instant of time. (2)

3. (a) Write $\Psi(x,t)$ for a monochromatic wave travelling to the right. (We used this as our prototype for discussing the Schrodinger wave equation and operators.) (2)

   (b) Give the expressions for the position and momentum operators. (2)

   (c) Write the Hamiltonian and time-independent Schrödinger equation for the case of a one-dimensional harmonic oscillator. (2)

   (d) If we give the potential energy as

$$V(x) = \frac{1}{2}m\omega^2 x^2 \tag{1}$$

   show that

$$\psi(x) = A\exp\left(-\frac{x^2}{2L^2}\right) \tag{2}$$

is an eigenfunction of the Hamiltonian with

$$L = \sqrt{\frac{\hbar}{m\omega}} \tag{3}$$

and give E the energy eigenvalue. (You don't need to necessarily have the right answer here to do the rest of the question) (8)

(e) What role does the constant A play? (2)

(f) Write (but don't calculate) the expression for the expectation value $< x^2 >$. (2)

4. (a) What was the key feature of Rutherford scattering that led to the conclusion that the mass of the atom is concentrated in a nucleus? What was the probe used for Rutherford scattering? (3)

(b) In the Bohr model what quantity was assumed to be quantized when generated stationary orbits. State the quantization condition. Calculate the approximate energy of the $n = 2$ state and the $n = 3$ of the hydrogen atom (you may assume 13.6 eV to be the ionization energy of hydrogen). Show how this difference in energy states agrees with the prediction of the wavelength of the emitted photon from the Rydberg-Ritz formula. (8)

(c) What property of electrons did Thomson measure to conclude that electrons were a universal constituent of matter? What was the name of the basic apparatus that he used? (3)

(d) Using the Moseley formula or a modification of the Rydberg-Ritz formula calculate the wavelength and energy of K_α radiation from iron ($Z = 26$ for iron but remember that it is modified if you use the RR formula). What supplies the original energy to create the X-ray and what is another way this energy can be released? (6)

(e) What is tunneling and what is the key formula? Give an example of an instrument where electron tunneling is a key feature. What is the probability of an electron with energy 4 eV tunneling through a rectangular barrier of height 6 eV and width 5 Å? (7)

(f) You perform a photoelectric effect measurement and find the stopping voltage to be 0.6 V for light with $\lambda = 546$ nm. Calculate the energy per photon if the light intensity is doubled. How is the stopping voltage affected by the intensity change? Calculate the work function of the material using the accepted value for Planck's constant. (7)

(g) What is the typical energy scale for ionizing radiation? What is the easiest way to experimentally distinguish the 3 classic types of radiation and what are those types of radiation? (4)

(h) Describe Compton scattering (no formula necessary but describe what changes). (4)

5. (a) What is the law relating total energy, momentum, and rest mass of a particle? (2)

(b) Suppose a particle was originally at rest in the lab frame and then decays into two particles. The first travels along the positive x-direction and the second along the negative x-direction. You determine that the total energy of the first particle is 70 MeV and its momentum is 20 MeV/c. The second particle has zero rest mass. Determine the rest mass of the first particle, total energy and the momentum of the second particle, and the rest mass of the original particle. (8)

(c) How fast is the massless particle travelling? How would this speed change if we observed it from another intertial reference frame? (3)