Alternate Final Exam: Modern Physics 201
April 24, 2009

Duration: 2.5 hours. Formula sheet provided. Point values are given with each question. Focus on key words in your written answers. Total exam is worth 100 points. Notice that question 4 covers a variety of topics and has the most total points available.

1. (a) What is a "blackbody"? How is it characterised? (2)
   (b) Draw a sketch of the blackbody spectrum [either $R(\lambda)$ or $u(\lambda)$]. Now use a dashed line to show what the "ultraviolet catastrophe" case would look like. (3)
   (c) What would the temperature be of a blackbody whose spectrum peaked at $\lambda = 7.35$ cm? What example of a blackbody did I mention in class that has roughly this temperature? (3)
   (d) The derivation of the blackbody radiation formula involves a calculation of the number of modes per unit volume per unit wavelength and a calculation of the average energy per mode. What was Planck's hypothesis and which of these calculations was affected? When was this effect largest? (3)
   (e) You don't need to do the calculation but show what steps you would need to do to obtain Wein's displacement law from Planck's formula. (3)

2. (a) Consider a Bragg scattering experiment. What is the central assumption in the derivation of Bragg's Law concerning the extra path length and the wavelength in order to observe interference maxima? Suppose that we have a neutron beam with wavelength $\lambda = 1.8$ Å that is Bragg scattered from a set of crystal planes with spacing 3.5 Å. What is the 2θ scattering angle for first order scattering? (4)
   (b) What is the k of these neutrons? What is the kinetic energy? (Use non-relativistic formulas. The mass of a neutron is 940 MeV/c^2 or 1.67×10^{-27} kg.) (3)
   (c) Suppose that we are trying to represent an individual neutron by a superposition of waves. We know the position of the neutron in x-direction to an uncertainty of 25 Å. What is the implied momentum uncertainty? (I find a value of 10's of eV/c for Δp.) (2)
   (d) The neutron wavefunction will look like an envelope surrounding a rapidly oscillating wave. Which part represents the neutron? Do the envelope and the rapidly oscillationg wave move at the same speed? Explain. (4)

3. (a) Give the expressions for the position and momentum operators. (2)
   (b) Show that $\Psi(x,t) = A\exp(i(kx - \omega t))$ is an eigenfunction of the momentum operator and give the eigenvalue. (2)
   (c) Write the Hamiltonian and time-independent Schrödinger equation for the case of a one-dimensional harmonic oscillator. (2)
   (d) If we give the potential energy as

$$V(x) = \frac{1}{2}m\omega^2 x^2 \tag{1}$$

   show that

$$\psi(x) = A\frac{x}{L}\exp\left(-\frac{x^2}{2L^2}\right) \tag{2}$$

is an eigenfunction of the Hamiltonian with

$$L = \sqrt{\frac{\hbar}{m\omega}} \tag{3}$$

and give E the energy eigenvalue. (You don't need to necessarily have the right answer here to do the rest of the question) (8)

(e) If $\psi(x)$ is given above give the appropriate $\Psi(x,t)$. (2)

(f) What is the likelihood that you will find the particle at $x = 0$ for this $\psi(x)$? (2)

4. (a) What are characteristic X-rays and how are they produced? Give a formula used to describe them. How does this formula support the Bohr model of the atom? (5)

   (b) In the Bohr model what quantity was assumed to be quantized in order to generate stationary orbits? State the quantization condition. Calculate the approximate energy of the $n = 2$ state and the $n = 4$ of the hydrogen atom (you may assume 13.6 eV to be the ionization energy of hydrogen). Show how this difference in energy states agrees with the prediction of the wavelength of the emitted photon from the Rydberg-Ritz formula. (8)

   (c) What property of electrons did Thomson measure to conclude that electrons were a universal constituent of matter? What was the name of the basic apparatus that he used? (3)

   (d) Rutherford scattering involves measuring alpha particles scattering from nuclei. But you know in the lab that alpha particles are very quickly absorbed. What modification was made to the target to observe the scattering? Why are alpha particles scattered to high angles from nuclei but not by electrons or Thomson plum-pudding atoms (a different answer for each). (4)

   (e) What is tunneling and what is the key formula? Give an example of an instrument where electron tunneling is a key feature. What is the probability of an electron with energy 4 eV tunneling through a rectangular barrier of height 8 eV and width 3 Å? (7)

   (f) You perform a photoelectric effect measurement and find the stopping voltage to be 0.4 V for light with $\lambda = 628$ nm. Calculate the energy per photon if the light intensity is doubled. How is the stopping voltage affected by the intensity change? Calculate the work function of the material using the accepted value for Planck's constant. (7)

   (g) What is the typical energy scale for ionizing radiation? What is the easiest way to experimentally distinguish the 3 classic types of radiation and what are those types of radiation? (4)

   (h) Describe Compton scattering (no formula necessary but describe what changes). (4)

5. (a) What is the law relating total energy, momentum, and rest mass of a particle? (2)

   (b) Suppose a particle was originally at rest in the lab frame and then decays into two particles. The first travels along the positive x-direction and the second along the negative x-direction. You determine that the total energy of the first particle is 80 MeV and its momentum is 30 MeV/c. The second particle has zero rest mass. Determine the rest mass of the first particle, total energy and the momentum of the second particle, and the rest mass of the original particle. (8)

   (c) How fast is the massless particle travelling? How would this speed change if we observed it from another intertial reference frame? (3)