Final Exam: Modern Physics 201
April 23, 2013

Duration: 2.5 hours. (CPA: I actually gave the students 3 hours.) Formula sheet provided. Point values are given with each question. Focus on key words in your written answers. Total exam is worth 79 points but I will mark it out of 77.

1. A one-dimensional infinite square well potential is given by

$$V(x) = \begin{cases} \infty & \text{for } x < 0 \\ 0 & \text{for } 0 \le x \le L \\ \infty & \text{for } x > L \end{cases} \tag{1}$$

   (a) Make a sketch of this potential. If a classical particle was in this potential where would the particle experience forces? Where would it feel no force? (3)

   (b) Write the time-independent Schrödinger equation for the region from $0 \le x \le L$. What are the boundary conditions for $\psi(x)$ at $x = 0$ and $x = L$? (2)

   (c) The general solution to the Schrödinger equation inside the well is

$$\psi(x) = A \sin kx + B \cos kx. \tag{2}$$

   Use the boundary condtions, as far as possible, to determine values for A, B, and k. (If you know how to do this it should be straightforward but if you need clues check the formula sheet.) (4)

   (d) Consider the following function (defined inside the well, $\psi = 0$ outside the well).

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right). \tag{3}$$

   Show that this is an energy eigenfunction and give the expression for the energy eigenvalue. (4)

   (e) Suppose that the transition for the $n = 2$ to $n = 1$ state of an infinte square well gives a photon with a wavelength of 694.3 nm. What is the L of this infinite square well? (4)

   (f) What happens to the allowed energies and wavefunction as you change from an infinite square well to a finite square well? I am looking for two features as we discussed in class. The answers can be qualitative. (3)

2. (a) Write the formula $\Psi(x, t)$ for a harmonic wave moving to the right as we did in class. Comment on Δx and Δp for this state. (4)

   (b) Further suppose that this wave represents an electron with a kinetic energy of 35 eV. Using the deBrogile relations and the relationship between p and E for a classical particle give λ, k and ω. (4)

   (c) The electron encounters a potential barrier of height $V_0 = 45$ eV and width 5.0 nm. What is the transmission probability? (3)

3. (a) What was the key feature of Rutherford scattering that led to the conclusion that the mass of the atom is concentrated in a nucleus? What was the probe used for Rutherford scattering? (3)

(b) You want to design a Rutherford scattering experiment so that you get one count per 60 s (ΔN) at an angle of 30 degrees. The detector has an area of 0.5 cm^2 and is 0.30 m from the target. The target is a gold foil that is 1.0 μm thick, atomic number 79, and has an atomic density $n = 5.9 \times 10^{28}$ atoms/m^3. The incident particles have an energy of 1.6 MeV. What must the incident flux I_0 be? (I found something of the order of $10^3\ \text{s}^{-1}$.) (6)

4. Calculate the wavelength of the $n_i = 4$ and $n_f = 2$ transition in hydrogen. Suppose that you had a diffraction grating with $d = 1.7 \times 10^{-6}$ m. What is the scattering angle for the $m = 1$ reflection? (use Young's diffraction formula). You should get an answer that compares to your own laboratory measurements. (4)

5. What was the big problem with the Rayleigh-Jeans description of blackbody radiation (demonstrate graphically and give the name of the prediction)? Why do we call it a "black" body? If we assume *you* are a blackbody with a temperature of 310 K use the Stefan-Boltzmann law to determine the total intensity of radiation (Watts per square metre). Use Wein's displacement law to predict the wavelength of the radiation maximum at this temperature. (and set your "night vision" goggles.) (10)

6. (a) What is the equivalence principle in general relativity? (2)

   (b) Calculate $\Delta\Phi(r)$ using formula 20 if you assume $g = 9.8$ m/s^2 and $h = 300$ km. If $\Delta\Phi(r)/c^2$ represents the fractional slow down in clocks at the surface of the earth estimate how slow an earth clock is after 3.15×10^7 s (i.e. 1 year). (4)

7. Give the Bragg scattering angle 2θ for $d = 2.5$ Å for electrons with an energy of 50 eV. (Assume $n = 1$.) (4 points)

8. Suppose a particle is moving through the lab frame and we discover it has a total energy of 1.50 MeV and a momentum of 1.41 MeV/c.

   (a) What is the rest mass and the velocity u of the particle? (3)

   (b) The particle travels between point A and point B in the lab frame in 0.35 μs. In the particle's rest frame how much time has elapsed? What is this effect called? (3)

9. Consider two frames S and S' with the usual convention. Suppose that $\beta = 0.8$ and $\gamma = 1.67$. Suppose that a pole with proper length 4.0 m is at rest in the S' frame.

   (a) What is the Lorentz contracted length of the pole? Who measures this length: observers in S or S'? (2)

   (b) Sketch a space-time diagram showing the worldlines of the ends of the pole. Now suppose that in the S frame there were fixed signposts at $x = 0$ and $x = 2.40$ m. Draw their world lines. (In this question try and show the world lines that lie along axes by making them "bushy" or using different pen/pencil colours.) (4)

   (c) Use Lorentz transforms to show that the event of "locating" the front end of the pole in S at $t = 0$ occurs at a "negative time" in S' and calculate that time. (3)