Midterm Quiz: Modern Physics 201
February 10, 2009

Formula sheet provided. Total 40 points. Individual values follow each question. Usual conventions for frames S and S' apply.

1. The following question has several parts. The entire question is internally consistent but it is not always necessary to know the answer from a previous part to find an answer for the later part. Do everything that you can. If I ask for a specific method I want to see that method.

   Suppose that you are at the origin of the frame S. You have two identical electron guns that are on opposite sides of the origin, each 3 m away. You have arranged for the electrons to be released at $t = 0$ ns each with a speed 0.2 m/ns or two-thirds of the speed of light in vacuum. You detect both electrons arriving at the origin at the same time. Once the electrons leave the guns they move at a constant velocity.

   (a) At what time do the electrons meet at the origin.(2 points)

   (b) Draw a space-time diagram that shows the world lines of the electrons and your world line from $t = 0$ until the electrons meet. (3)

   (c) Imagine that S' is an inertial frame such that the electron released from the left is at rest in this frame. Include the space-time axes of S' and highlight any special relationship between the world line of the electron from the left and the new space-time axes. (2)

   (d) Even if you are having trouble with the space-time diagram use the Lorentz transforms to calculate t' and x' of two events: (i) the electron on the left leaving the gun and (ii) the electrons meeting at the origin of S. (note: the electron on the left is at rest in S' but is not at the origin of S') (4)

   (e) The observer in S' "sees" the electron released from the right moving towards him. By applying the Lorentz transform the observer says this event [event (iii) if you like]: the release of the electron on the right has space-time coordinates $x' = 4.02$ m at $t' = -8.93$ ns. Based on this information and the answer to the previous question, calculate u'_x for this electron directly using $\Delta x'$ and $\Delta t'$ (pay attention to the signs). Are electrons emitted at the same time according to the S' observer? (4)

   (f) Now use the Lorentz velocity transformation directly to calculate u'_x for the electron on the right (pay attention to the signs!). Should this and does this agree with the $\Delta x'/\Delta t'$ calculation? (3)

   (g) Based on a Galilean transform what should u'_x have been? Comment on how you interpret this in terms of the maximum speed of object. (2)

   (h) Consider two events: the origins of S and S' coinciding by the usual convention and the electron leaving the gun on the right. Show where these events occur on the space-time diagram with at least roughly correct scale. (3)

   (i) Demonstrate numerically that the space-time interval separating the two events in part (h) is the same in both reference frames. [You can do this with information already given even if you aren't sure about the space-diagram in part (h) or some of earlier Lorentz transforms]. Are the events separated by a space-like or time-like interval? (3)

2. (a) What was the point of the Michelson-Morley experiment? What was its result and the conclusion? (reminder: the experiment is very closely related to the assignment problem with the airplane race) (3)

(b) In the context of the twin paradox why and how did we correctly distinguish Homer and Ulysses (the answer is not simply "Homer stayed still and Ulysses moved" since this is a relative concept)? Who is younger when they meet again? (3)

(c) What are x^0, x^1, x^2, and x^3 i.e. the components of the position 4-vector in terms of c, t, x, y, z? (2)

3. A police officer is attempting to measure your speed with a radar gun. According to the police officer the radar gun emits a frequency of 2×10^9 Hertz= 2 GHz. You observe the police officer coming towards you at $\beta = 0.1$.

   (a) What is the blueshifted frequency of the radar gun? (2)

   (b) A consequence of the binomial theorem is the approximation that $(1+x)^n \approx 1 + nx$ for $x \ll 1$. Make appropriate use of this approximation to show that

$$\sqrt{\frac{1+\beta}{1-\beta}} - 1 \approx \beta \tag{1}$$

   to first order in β for $\beta \ll 1$. (2)

   (c) Make use of this approximation to calculate the frequency shift (that is the difference in frequency $\Delta f = f_{\text{blueshift}} - f_0$) that you measure for the more reasonable approach speed of the police officer of 250 km/hr. (2)