Solutions for Midterm Quiz: Modern Physics 301
November 5, 2002

Question 1: 12 points. Question 2: 9 points. Question 3: 9 points. Total 30 points. Individual values follow each question.

1. (a) State the Lorentz transforms. (3)
   Answer:

$$
\begin{aligned}
x' &= \gamma(x - vt) & (1)\\
y' &= y & (2)\\
z' &= z & (3)\\
t' &= \gamma(t - \frac{vx}{c^2}) & (4)\\
\text{with } \gamma &\equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} & (5)
\end{aligned}
$$

   (b) A pulse of light leaves the origin of the lab reference frame S at $t = 0$ and travels along the x-axis. At what time does it reach a detector at $x = 1$ m? (You may find that $c = 0.3$ metres per nanosecond is useful.) (2)
   Answer:

$$t = \frac{x}{c} = \frac{1}{0.3} = 3.33\,\text{ns} \qquad (6)$$

Our usual "moving" inertial reference frame S'is moving along the x-axis with $v = \frac{\sqrt{3}}{2}\,c$. The origins coincide at $t = t' = 0$.

   (c) Calculate γ. (2)
   Answer:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{3}{4}}} = 2 \qquad (7)$$

   (d) Use the Lorentz transforms to give x' and t' for the light pulse reaching the detector. (numerical values in metres and nanoseconds) (3)
   Answer:

$$x' = \gamma(x - vt) = 2(1 - (0.866)(0.3)(3.33)) = 2(0.135) = 0.27\,\text{m} \qquad (8)$$

$$t' = \gamma(t - \frac{vx}{c^2}) = 2(3.33 - \frac{(0.866\,c)(1)}{0.3c}) = 2(0.443) = 0.887\,\text{ns} \qquad (9)$$

   (e) Based on these results show that these values of x' and t' are consistent with a constant speed of light in the two reference frames. (Numerical demonstration is fine). (2) Answer: The speed of light in S' is x'/t'

$$c_{S'} = \frac{x'}{t'} = \frac{0.27}{0.887} = 0.3\,m/ns = c. \qquad (10)$$

   So the speed of light is the same in both reference frames.

2. Consider a rigid O_2 molecule with mass m and moments of inertia $I_{x'}$ and $I_{y'}$ (the z'-axis is parallel to the bond between the two oxygen atoms).

   (a) Give an expression for the energy of the molecule (translational plus rotational) in terms of $x, y, z, \omega_{x'}$, and $\omega_{y'}$. (ω is the rotational velocity in radians per unit time) (2)

   Answer: On page 59 of *Tipler*

$$E_k = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_{x'}\omega_{x'}^2 + \frac{1}{2}I_{y'}\omega_{y'}^2 \tag{11}$$

   (b) In the language of the kinetic theory of matter what is the general name for these variables that make up the energy expression? (2)

   Answer: Variables that enter into the energy expression as squared terms are known as *degrees of freedom.*

   (c) According to the equipartition theorem what is the thermal average value of the energy (in eV) for each molecule at $T = 1000$ K. (2)

   Answer: There is $\frac{1}{2}k_BT$ associated with each degree of freedom. There are 5 degrees of freedom in this case. So

$$\bar{E_k} = \frac{5}{2}k_bT = (2.5)(8.62 \times 10^{-5})(1000) = 0.215\, eV \tag{12}$$

   (d) Suppose that the average translational kinetic energy of the molecule is 0.13 eV or 2×10^{-20} J and the mass is 5.33×10^{-26} kg. What is the rms speed? (3)

   Answer: The average translational kinetic energy expression is

$$\bar{E}_{trans} = \frac{1}{2}m\bar{v_x^2} + \frac{1}{2}m\bar{v_y^2} + \frac{1}{2}m\bar{v_z^2}. \tag{13}$$

   But the equipartion theorem states that each of these average velocities are equal to the rms speed.

$$\bar{E}_{trans} = 3(\frac{1}{2}mv_rms^2) \tag{14}$$

   Giving

$$v_rms = \sqrt{\frac{2\bar{E}_{trans}}{3m}} = \sqrt{\frac{(2)(2 \times 10^{-20})}{(3)(5.33 \times 10^{-26})}} = 500 \text{ m/s}. \tag{15}$$

3. (a) What is the energy of a photon with $\lambda = 620$ nm? (2)

   Answer: $E = \frac{hc}{\lambda} = \frac{1240}{620} = 2$ eV

   (b) For $T = 4640$ K calculate $hc/(\lambda k_BT)$ and $\exp(-\frac{hc}{\lambda k_BT})$. (3)

   Answer:

$$\frac{hc}{\lambda k_BT} = \frac{2}{(8.617 \times 10^{-5})(4640)} = 5 \tag{16}$$

$$\exp(-\frac{hc}{\lambda k_BT}) = e^{-5} = 6.7 \times 10^{-3} \tag{17}$$

(c) According to Planck's hypothesis what is $\bar{E}$, the thermal average energy for this mode? (consult your assignment or formula 3-28). (4)

Answer:

$$\bar{E} = \frac{hc}{\lambda}\frac{1}{\exp(\frac{hc}{\lambda k_B T}) - 1} \approx \frac{hc}{\lambda}e^{-\frac{hc}{\lambda k_B T}} = (2)(6.7 \times 10^{-3}) = 13\, meV \qquad (18)$$