Assignment #1: PHYS 322: Electromagnetic Theory I
Due: September 21, 2009

Questions 1 and 3 are worth 10 points and question 2 is worth 20.

1. (a) Describe in words how we formalised the definition of a vector in this class. What was the prototype vector? Why is it important to state the laws of physics in terms of vectors (or tensors in the more general sense)?

   (b) Suppose I give you the following scalar function:

$$f(x, y, z) = x \tag{1}$$

   Calculate its gradient and let this be the vector field $\mathbf{F}(\mathbf{r})$

   (c) Now imagine that we change to a new set of rotated Cartesian coordinates denoted by $\bar{\mathbf{r}}$, $\bar{x}$, $\bar{y}$, and $\bar{z}$ where we have rotated by 30 degrees counter-clockwise around the z-axis. Give the rotation matrix that describes the linear transformation from non-rotated to rotated coordinates. (It is also common to use "primes" to indicate the new set of coordinates.)

   (d) Give $f(\bar{x}.\bar{y}, \bar{z})$. You will need to find the inverse transform. (R_{ij} is *orthogonal* if that is useful.)

   (e) Now calculate the gradient of $f(\bar{x}.\bar{y}, \bar{z})$ using the $\bar{\nabla}$ operator i.e. take derivatives with respect to the rotated coordinates. Call this function $\bar{\mathbf{F}}(\bar{\mathbf{r}})$.

   (f) Now demonstrate that $\mathbf{F}(\mathbf{r})$ is a *vector* field by explicitly showing that $\bar{F}_i = R_{ij}F_j$. (At the final step you would usually compare the expressions all in rotated or non-rotated coordinates to confirm equality.)

2. One difficulty that you can often run into in converting vector fields between Cartesian, spherical polar, and cylindrical polar is that the unit vectors change and you need to get used to the idea that non-Cartesian unit vectors *are not constant*. The unit vector and coordinate transforms are on the back cover and the differential forms of gradient, divergence, and curl are on the front cover. There are also times that the "physical picture" of the differential operators can fool you. Here are two examples that will explain what I mean.

   (a) Consider the vector field $\mathbf{F}(\mathbf{r}) = \hat{r}$. Make a sketch of this vector field. Using the formula for spherical polar coordinates on the front cover calculate $\nabla \cdot \mathbf{F}$. Obviously $\hat{r}$ is not "constant" if it has non-vanishing divergence.

   (b) Now re-write $\mathbf{F}(\mathbf{r})$ in Cartesian and cylindrical polar coordinates using information on the front and back covers. Does the field still look "constant"? Calculate the divergence in both coordinate systems and confirm that all coordinate systems give the same answer (go back and check if they don't!).

   (c) Now consider the following modification $\mathbf{E}(\mathbf{r}) = \mathbf{F}/r^2$ and calculate the divergence in spherical polar coordinates. Despite its "radial" appearance we have a vector field that has no divergence except at the origin.

   (d) Now the same sequence for $\mathbf{F}(\mathbf{r}) = \hat{\phi}$ (cylindrical polars). Make a sketch, calculate the curl, transform to the other two coordinate systems, and recalculate the curl.

   (e) Consider the modification $\mathbf{B}(\mathbf{r}) = \mathbf{F}/s$ and calculate the curl (you can pick the coordinate system). We now have a field that "circulates" but has no curl except along the z-axis.

3. You have probably done this before but it is a technique you must be comfortable with. Consider a semi-circle that has its centre at the origin, lies in the $x - y$ plane, radius a and is entirely in the $y \geq 0$ section of space. (Often I would use "primed" variable to designate these as "source" coordinates but since I won't be discussing field coordinates I will not use primes.) The semi-circle has a surface charge density that varies linearly as a function of y $\sigma = Cy$.

   (a) What would the units of C be to give a surface charge density in conventional units (Coulombs per square metre)?

   (b) Calculating the total charge on the semi-circle involves a double integral. What is the differential charge on a small area dx by dy (it will be a function and it will be proportional to both dx and dy)?

   (c) How much charge is there on a small slab of the semi-circle of width dx? (Use your previous answer and perform a single integral. You should get an answer that is a function of x but y has been "integrated out".)

   (d) What is the total charge on the semi-circle?

   (e) Write the actual double integral you performed with the appropriate limits.