Assignment #2: PHYS 322: Electromagnetic Theory I
Due: October 5, 2009

Questions 1, 3, and 4 are worth 15 points and question 2 is worth 10.

1. (a) If $\mathbf{F}$ is a vector field show that

$$(\nabla \times (\nabla \times \mathbf{F}))_i = (\nabla (\nabla \cdot \mathbf{F}))_i - \nabla^2 F_i \tag{1}$$

where i is referring to the i-th Cartesian component of the vector field. The Laplacian takes its conventional Cartesian form $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})$. F_i enters the formula like it is a scalar (although it isn't!)
Use the identity

$$\epsilon_{ijk}\epsilon_{k\ell m} = \delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell} \tag{2}$$

and "contract" using the Kroenecker deltas. Please include enough steps so I can follow your logic.

(b) You might be saying *This is crazy! What kind of vector field has a double curl!* Try taking the curl curl of

$$\mathbf{E}(\mathbf{r}, t) = E_0 \exp\left(i(kz - \omega t)\right) \hat{\mathbf{x}} \tag{3}$$

where $i = \sqrt{-1}$. For the purpose of this discussion t is a constant.

(c) Calculate $\nabla^2 E_x$ and $\nabla(\nabla \cdot \mathbf{E})$ and verify the vector identity in equation 1.

(d) Make a sketch of the real part of $\mathbf{E}$ with $t = 0$. Try and make an orthographic projection sketch. What do you think it is?

2. Consider the scalar field

$$V(\mathbf{r}) = C\frac{\cos\theta}{r^2} \tag{4}$$

(a) Calculate $\mathbf{E} = -\nabla V$.

(b) Sketch $\mathbf{E}$.

(c) Calculate $\nabla \cdot \mathbf{E}$. This is another candidate for an electric field that solves Laplace's equation where $r \neq 0$.

3. Problem 2.3 in Griffiths except that the linear charge density is now $\lambda(x') = Cx'$ where C is some constant. Also don't worry about the $L \to \infty$ form of the expression. The line integral involved is not the dot product of a vector field and $d\boldsymbol{\ell}$ but instead an integral over source charges where those charges are along a wire. Your answer should not include any source (primed) coordinates. You may tackle the integrals with Maple, a table, or see my notes on-line.

4. An example of the divergence or Gauss theorem.

Consider the following vector field in cylindrical polar coordinates

$$\mathbf{E}(s, \phi, z) = \frac{Qs}{2\pi R^2 \ell \epsilon_0}\,\hat{\mathbf{s}} \tag{5}$$

where Q, R, and ℓ are constants. Consider a Gaussian surface defined by a cylinder of length ℓ and radius R that is coaxial to the z-axis. (Since there is no z-dependence in the problem the z-position of the cylinder is unimportant; just pick something.)

(a) Calculate the flux integral over this closed surface $\oint \mathbf{E} \cdot d\mathbf{a}$. You will need to break up the cylinder into end caps and the curved surface to easily parametrize it. Some parts will contribute nothing to the total.

(b) According to Gauss' Law $\rho = \epsilon_0 \nabla \cdot \mathbf{E}$ if $\mathbf{E}$ is the electric field and ρ is the local charge density. Calculate $\rho(s, \phi, z)$ with these assumption and then perform the volume integral to calculate the charge enclosed by cylinder. (A short cut on the *volume* integral is fine.)

(c) Using these two results confirm that Gauss' (or divergence) theorem (not law) is verified (just move the ϵ_0 around on the right hand side).