Lecture Summaries: Physics 322: Electromagnetic Theory I
Carl Adams, Fall 2009

1. Sept. 10
   (a) Review of course outline
   (b) Basic goal of EM theory
   (c) Topics and technologies that involve EM theory
   (d) Concepts of electric and magnetic vector fields and the charges and currents.
   (e) Gauss' Law: spreading of electric field, ϵ_0 as a physical constant
   (f) Gauss' Law for magnetism: absence of magnetic "charges"
2. Sept. 14
   (a) Ampéere's Law with Maxwell's displacement current
   (b) Electric currents (and current density) as a source of "curling" magnetic fields
   (c) The permeability of free space as a defined quantity
   (d) Faraday's Law and a reminder from circuits
   (e) Possibility of wave-solutions where the time-dependent fields are self-generating.
   (f) The continuity equation and local charge conservation
   (g) The force equation including the Lorentz force; relationship to experiments in PHYS 201.
   (h) Decoupling of **E** and **B**
   (i) The electric potential and the Laplacian operator (very brief)
   (j) Considering how the position vector and scalar quantities are seen differently in rotated coordinate systems.
3. Sept. 16
   (a) Working out R_{ij} the rotation matrix for our simple case
   (b) Einstein summation convention and repeated indicies
   (c) The Levi-Civita alternating tensor ϵ_{ijk} and the cross product
   (d) The Knoenecker delta δ_{ij}
   (e) Definition of a vector $\bar{A}_i = R_{ij} A_j$
   (f) Generalisation to tensors (in 3-dimensional flat space)
   (g) Extra: generalisation to tensors with general transforms
4. Sept. 17
   (a) Motivation for vectors and tensors: physical laws applicable to all reference frames.
   (b) Concept of a field and sketching vector fields.
   (c) Example of converting a Cartesian vector field to a rotated frame.
   (d) The necessity of the inverse rotation transform and ways to determine it.
   (e) Using different coordinate systems

(f) Necessity of changing unit vectors, line elements, area elements, volume elements, and forms of gradient, divergence, curl, and the Laplacian (scalar and vector)

(g) Not quite complete example of this for $\mathbf{F}(\mathbf{r}) = x\hat{x}$ and its divergence in Cartesian and spherical polar coordinates.

5. Sept. 21

   (a) Response to student question: what *is* the electric field?

   (b) The Laplacian, a 2nd derivative.

   (c) Scalar and vector forms of the Laplacian

   (d) Vector identities on the front and back cover.

   (e) Vanishing 2nd derivatives

   (f) The line integral

6. Sept. 23

   (a) An example of a line integral.

   (b) Parametrizing the line

   (c) The differential line element.

   (d) Complicated integral, simple result

7. Sept. 24

   (a) A variation: line integral over a source

   (b) Warning concerning moving non-Cartesian unit vectors outside of integrals.

   (c) Surface integrals

   (d) Volume integrals

   (e) Fundamental theorem of calculus

   (f) Physical picture of adding up small steps being equivalent to looking at boundaries.

   (g) Gradient, Gauss', and Stokes' theorems

8. Sept. 28

   (a) The gradient and Laplacian of $1/r$.

   (b) Using Gauss theorem to understand the discontinuity at the origin.

   (c) The differential area element $d\mathbf{a}$

   (d) Concept of a delta function, a sharp spike.

   (e) The finite integral of a delta function.

   (f) Definition of $\delta^3(\mathbf{r} - \mathbf{r}')$

   (g) Dirac delta function in 1-dimension and its properties under integration.

9. Sept. 30

   (a) The 3-D delta function and its ability to simplify integrals

   (b) Question concerning the realness and the usefulness of the Dirac delta function: generalised dot product and orthogonality and Green's functions

   (c) Helmholtz Theorem: how to get a vector function from its divergence and curl when function goes to zero at the boundary
   (d) Construction of the scalar and vector potentials
   (e) Proof of Helmholtz theorem using Dirac delta functions

10. Oct. 1
    (a) A pictorial example of the $\psi(\mathbf{r})$ function for a unit cube "source" of divergence.
    (b) Geometric/qualitative picture of the relationship between $\nabla \cdot \mathbf{F}(\mathbf{r})$, $\psi(\mathbf{r})$, and $\mathbf{F}(\mathbf{r})$.
    (c) Coulomb's Law
    (d) Superposition, positive/ negative charges, other properties
    (e) The field concept
    (f) Source points versus field points

11. Oct. 5
    (a) The electric field of a charged line
    (b) Conversion to convenient, symmetric coordinates
    (c) Conversion back to Cartesian coordinates
    (d) Working out all the parts of the integrand
    (e) Checking the answer against a known limit

12. Oct. 7
    (a) The electric field of charged shell; two possible regions
    (b) Exploiting symmetry to remove θ and ϕ dependence.
    (c) Eliminating E_θ and E_ϕ components to give a purely radial field.
    (d) Focussing on $E_z(0,0,z)$ "without loss of generality".
    (e) Determining the differential field element
    (f) The geometric approach to determining $\mathscr{r}$ and $\hat{\mathscr{r}} \cdot \hat{z}$
    (g) Calculating the double integral.

13. Oct. 8
    (a) Explaining the difference between inside and outside results
    (b) The surprisingly simple result for **E** of a spherical shell (**last topic on 1st midterm**)
    (c) Differential and integral forms of Gauss' Law
    (d) Using the Dirac delta function and **E** of a point charge as a bridge from Coulomb's Law to Gauss' Law
    (e) Demonstrating that **E** is the gradient of some function and therefore is irrotational

14. Oct. 14
    (a) The forms of **E** for spheres, lines, and planes.
    (b) Using Gauss' Law and symmetry to determine **E** of a solid, uniformly charged sphere of radius R.

   (c) A Gaussian surface
   (d) Details of calculation for a solid charged wire of radius a and linear charge density λ

15. Oct. 15 Midterm

16. Oct. 19

   (a) Continuation, field of a charged wire
   (b) The constant field of an infinite charged plane
   (c) Definition of scalar potential in terms of a path-independent path integral

17. Oct. 21

   (a) Using the gradient theorem to show that $\mathbf{E} = -\nabla V$
   (b) Determining V from $\mathbf{E}$ for a charged wire
   (c) Options for choosing $\mathcal{O}$
   (d) Getting the correct sign when calculating path integrals

18. Oct. 22

   (a) Poisson's equation
   (b) Green's function for Poisson's equation
   (c) Solution to Poisson's equation for $\mathbf{E} \rightarrow 0$ as $r \rightarrow \infty$
   (d) Boundary conditions for $\mathbf{E}$ at charged surfaces
   (e) Using Gaussian pillboxes and Stokes' loops

19. Oct. 26

   (a) Further discussion on the boundary conditions
   (b) The example of a charge spherical shell
   (c) Which small surfaces are smaller?
   (d) Taking the proper limits
   (e) Defining the normal and tangential components
   (f) Boundary conditions for scalar potential

20. Oct. 28

   (a) Work-energy theorem
   (b) The energy of an assembled charge array
   (c) General formula for a continuous charged distribution
   (d) Warning concerning infinity for point charges
   (e) The formula for the energy density of electric fields

21. Oct. 29

   (a) Limitations of Coulomb's Law for Conductors
   (b) The equilibrium assumption for conductors in electrostatics

(c) **E**=0 and $\rho = 0$ inside of a conductor

(d) Inquiry: what is the scalar potential at different points in a conductor?

(e) Poisson's equation and boundary conditions

(f) The uniqueness theorem

(g) **E** inside of a cavity

(h) **E** outside of a spherical conductor

22. Nov. 2

(a) The FISHTANK method

(b) General solution to Laplace's equation in spherical polar coords only r dependence

(c) Fitting to the boundary conditions

(d) Determining the surface charge

(e) Capacitance $C = Q/V$ a geometric quantity

23. Nov. 4 (quite a few students missing)

(a) Parallel plate capacitor

(b) Dirichlet (voltage specified) and Neumann (charge specified) boundary conditions

(c) Finding $V(\mathbf{r})$, C, and U with Dirichlet boundary conditions

(d) Finding $V(\mathbf{r})$ and C with Neumann boundary conditions but maintaining the FISHTANK approach

(e) Finding $V(\mathbf{r})$ with Neumann boundary conditions using the "**E** is known through Gauss Law" approach

24. Nov. 5

(a) (you are not responsible for this) Laplace's equation doesn't allow local minima and maxima

(b) Great freedom allowed in Poisson's equation by including virtual charges

(c) Solving for $V(\mathbf{r})$ of a point-charge above an infinite grounded conducting plane

(d) Solution by imagining an image charge to fit the boundary conditions

(e) Determining the induced charge on the plane by calculating the gradient of V

(f) Gauss Law shows that total induced charge is equal to $-q$

25. Nov. 9

(a) Using the image charge method to solve for a grounded sphere with an external point charge

(b) Checking that boundary conditions are satisfied.

(c) Separating Laplace's equation in Cartesian coordiates with z symmetry

(d) The role of the k constants and how they originate

(e) How to choose the functions? Role of grounded boundaries.

26. Nov. 12

(a) The use of "Fourier's trick" to solve for the coefficients to fit the last boundary condition

(b) Sketches of the terms that solve the problem

27. Nov. 16

    (a) Separating Laplace's equation in spherical polar coordinates.

    (b) The R(r) solutions as r^{ℓ} and $r^{-1-\ell}$

    (c) Generating $P_{\ell=0}(u)$ and $P_{\ell=1}(u)$ using Rodrigues's formula

    (d) Notice how familiar the solutions are

28. Nov. 18

    (a) Generating $P_{\ell=2}(u)$ using Rodrigues's formula

    (b) Looking at the separable solutions of Laplace's equation as *Tetris* pieces

    (c) Using separable solutions to Laplace's equation to solve the problem of a conducting sphere in a uniform electric field

29. Nov. 19: Midterm II

30. Nov. 23

    (a) Mini-plan for the next few classes

    (b) Multipole expansion

    (c) Expansion of $1/\mathscr{r}$ in terms of r_k/r

    (d) Use and an example of Taylor series

    (e) Definition of monopole, dipole, and quadrupole terms

    (f) What is a 2nd rank tensor and how do we represent it?

31. Nov. 25

    (a) Numeric example of a charge displaced along the z-axis

    (b) Calculating Q_T, $\mathbf{p}$, and $\mathcal{Q}_2$

    (c) Different approaches to handling $\mathcal{Q}_2$

    (d) Convergence of monopole, dipole, and quadrupole terms to the exact answer.

32. Nov. 26

    (a) The relationship between Legendre polynomials and the mutipole expansion when there is no ϕ dependence

    (b) A form for the octupole term in our previous expansion and how it helps our approximation

    (c) The effect of choosing a different origin

    (d) The field of an ideal dipole

33. Nov. 30

    (a) Dipole moments in materials: induced, thermal, ferroelectrics

(b) Definition of **P** the polarisation
(c) The potential of a polarised object
(d) Bound charges
(e) Definition of **D** the displacement field (or electric displacement)
(f) Gauss Law for **D**

34. Dec. 2

    (a) Boundary conditions for **E** and **D**
    (b) The constitutive relation and the various dielectric constants
    (c) A parallel-plate capacitor filled with dielectric
    (d) Locations of bound and free charge

35. Dec. 3

    (a) Half-filled capacitor with left-half filled
    (b) **E** and **D** in the different regions and the boundary conditions
    (c) Locations of charges
    (d) Half-filled capacitor with one conductor covered