Physics 323 Experiment # 2 Pre-lab Hints

I. RC Circuit - I think you can do all of these.

The basic starting point is to have a voltage loop such that

$$V_{in} - V_R = V_{out} = V_C$$

and assume V_{out} doesn't draw any current

also $V_c = \frac{Q}{C}$ ← charge on capacitor

and $i_R = \dot{Q}$.

Then solve the resulting first order linear differential equation with an exponential function.

I am sure this will be explained in a circuits text.

(In the "Lab" notice $37\% = \frac{1}{e} = e^{-1}$

and $63\% = 1 - e^{-1}$.)

2. <u>Differentiator</u> - again assume V_{OUT} draws no current.

$$V_{IN} - V_C = V_{OUT}$$

but V_{OUT} is also equal to iR

and $V_C = \frac{Q}{C} = \frac{1}{C}\int i\,dt + k$

some integration constant

$$V_{IN} - \frac{1}{C}\int i\,dt + k = iR$$

Take the time derivative $\frac{dV_{IN}}{dt} - \frac{1}{C}i = R\frac{di}{dt}$

$$\frac{dV_{IN}}{dt} - \frac{1}{C}\frac{V_{OUT}}{R} = R\frac{di}{dt}$$

$$V_{OUT} = RC\left(\frac{dV_{IN}}{dt} - R\frac{di}{dt}\right)$$

$$= RC\left(\frac{dV_{IN}}{dt} - \frac{dV_{OUT}}{dt}\right)$$

if $\frac{dV_{OUT}}{dt} << \frac{dV_{IN}}{dt}$ then $V_{OUT} = RC\frac{dV_{IN}}{dt}$

Essentially you are running a high pass filter at low frequency. Not much signal gets through but what does is proportional to the time derivative of V_{IN}.

(3) Integrator (The circuit shown in (1))

$$\underbrace{V_{IN} - iR = V_{OUT}}_{\text{integrate}} = V_C = \frac{Q}{C}$$

$$\int V_{IN}\,dt - R\underbrace{\int i\,dt}_{\substack{Q \\ = CV_{OUT}}} = \int V_{OUT}\,dt$$

$$RC\ V_{OUT} = \int (V_{IN} - V_{OUT})\,dt$$

$$V_{OUT} = \frac{1}{RC}\int (V_{IN} - V_{OUT})\,dt$$

$$\approx \frac{1}{RC}\int V_{IN}\,dt \qquad \text{if} \quad V_{OUT} \ll V_{IN}$$

essentially running a low pass filter at high frequency charges (integral of current) are building up on the capacitor to give some V_{OUT} but the capacitor largely "shorts" the circuit at high frequency forcing V_{OUT} small in magnitude

(4) <u>Low-Pass Filter</u>

we will see in class

$$dB = 20 \log_{10} \frac{V_{out}}{V_{in}}$$

(so 3 dB actually talks about power (V^2) amplification or attenuation)

So solve this formula to find V_{out}/V_{in} for 3dB (or -3dB) (because for this passive filter $|V_{out}| < |V_{IN}|$)

We now have sinusoidal-in and sinusoidal-out so you can use the expressions for complex impedance.

$$V_{OUT} = \frac{Z_c}{Z_c + R} V_{IN}$$

Sub in $Z_c = \frac{1}{j\omega C}$

and find the magnitude of $\frac{Z_c}{Z_c + R}$ to give you the correct $|V_{OUT}| / |V_{IN}|$ for -3dB

And also find phase. $x + jy = Re^{i\phi}$ $R = \sqrt{x^2+y^2}$, $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

The sketch will be Amplitude $\left(\frac{|V_{OUT}|}{|V_{IN}|}\right)$ and ϕ vs f

(5) now have $V_{out} = \frac{R}{R + Z_C} V_{IN}$

(6) V_{AC} is quoted as root mean squared (RMS)

and that is also what a voltmeter on AC measures

(if $V(t) = V_0 \sin \omega t$

then $V_{RMS} = \left(\frac{1}{T} \int_0^T (V_0 \sin \omega t)^2 \, dt \right)^{1/2}$

$= \frac{1}{\sqrt{2}} V_0$)

The high-pass filter is shown in (5). Low-pass in (4)

(7) Maybe use nodal voltages for this one... (A)