Physics 475: Assignment #2

Carl Adams

Due: Feb. 11, 2011

1. Foot 2.1(a) (5 points)

2. Foot 2.3 (10 points)

3. Calculate the $E_{\mathrm{s-o}}$ from equation 2.54 and 2.55 for the $n = 2$ $\ell = 1$ levels of hydrogen. Give the answer in eV and in cm^{-1}. Please use spectroscopic notation to properly label the levels. What is the spin-orbit energy shift for the $n = 2$ $\ell = 0$ state? (don't just blindly follow formulas!) (10 points)

4. The book seems scared to mention the rest of the relativistic corrections to you but I'm not!

   The relativistic mass correction is:

$$\Delta E_{\mathrm{mass}} = \left\langle n\ell m_\ell m_s \left| -\frac{p^4}{8m^3c^2} \right| n\ell m_\ell m_s \right\rangle = |E_n| \frac{\alpha^2}{n^2}\left[\frac{3}{4} - \frac{n}{\ell + 1/2}\right] \tag{1}$$

   and the Darwin term

$$\Delta E_{\mathrm{Darwin}} = \left\langle n\ell m_\ell m_s \left| \frac{\pi\hbar^2}{2m^2c^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)\delta(\vec{r}) \right| n\ell m_\ell m_s \right\rangle \tag{2}$$

   The Darwin term is only non-zero for $\ell = 0$. Calculate the Darwin term matrix element for $n = 2$ and $\ell = 0$ and use the worked-out expression for the relativistic mass correction in equation 1 to verify the claim in the text that after all relativistic corrections the $2^2\mathrm{S}_{1/2}$ and $2^2\mathrm{P}_{1/2}$ levels are degenerate. You will need information from the previous question as well. (10 points) (FYI: to apply these formulas for $Z \neq 1$ substitute in Ze^2 or $Z\alpha$.)

   The $\delta(\vec{r})$ function is not the same as $\delta(r)$. There are various mathematical details since you can represent $\delta(\vec{r})$ as the divergence of $\hat{r}/(4\pi r^2)$ and switch to a surface integral but all you need to know is that if you have $f(\vec{r})$ then

$$\int d^3r\, f(\vec{r})\delta(\vec{r}) = f(\vec{r} = \vec{0}) \tag{3}$$

   You don't need to switch to spherical polar coordinates.