# Physics 475: $\boldsymbol{e} / \boldsymbol{m}$ - Helmholtz Coils 

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## Additional Things for PHYS 475

1. Do the Prelab questions.
2. You will need to wire up the apparatus.
3. Show how you combine the constants for the field versus current expression.
4. Can you look up the details about the Helmholtz coils and calculate just how much the field does change as you move off-axis? I think a dipole approx for each coil can help with this.
5. Try using a handheld spectroscope to see if you can identify the glow in the tube.

## Purpose

To control electrons as they move along circular orbits by using a magnetic field applied by using Helmholtz coils. Use your results to obtain a value for $e / m$, the charge to mass ratio for electrons.

## References

Tipler, Modern Physics, pp. 92-96, Griffiths, Introduction to Electrodynamics, pp. 243, Taylor, Classical Mechanics, pp. 70-71.

## Prelab

1. Calculate the value of $e / m$ for an electron based on accepted values.
2. Suppose that electrons have been accelerated through a potential of 30 Volts. What is the kinetic energy per electron in Joules? In electron-Volts? How fast are the electrons traveling in metres per second?
3. If an electron is traveling at $10^{4} \mathrm{~m} / \mathrm{s}$ perpendicular to a magnetic field of 0.01 Tesla what force does it feel? What acceleration?

## Theory

This experiment is a variation of Thomson's classic experiment whereby the electron was discovered.

Consider a charge, $q$, moving with a velocity, $v$, in a magnetic field of known flux density, $B$. The angle between $B$ and $v$ is $\vartheta$. The charged particle experiences a force, $F$,
perpendicular to both $v$ and $B$, the direction being determined by the right hand rule, and the magnitude by the equation:

$$
\begin{equation*}
F=B q v \sin \vartheta \tag{1}
\end{equation*}
$$

i.e. it is a cross product. If the motion of the particle carrying the charge is such that $v$ is perpendicular to $B$, then $\vartheta=90^{\circ}$ and $\sin \vartheta=1$. If the particle is an electron, then the value of $q$ is usually denoted by $e$. The equation for the force then becomes:

$$
\begin{equation*}
F=B e v \tag{2}
\end{equation*}
$$

Since this force is perpendicular to $v$, the resulting motion of the particle must be along the arc of a circle, in which case the centripetal force Bev must be equal to the centrifugal force $m v^{2} / r$ where $r$ is the radius of the circular path followed by the electron.

$$
\begin{equation*}
B e v=\frac{m v^{2}}{r} \tag{3}
\end{equation*}
$$

If the velocity of the electron is the result of accelerating it through a potential difference of $V$ volts, and if the applied voltage is small enough that the electron does not approach speeds which are a significant fraction of the speed of light, then the classical kinetic energy is equal to the decrease in potential energy:

$$
\begin{equation*}
\frac{1}{2} m v^{2}=e V \tag{4}
\end{equation*}
$$

Rearranging equation (3)
yields:

$$
\begin{equation*}
v^{2}=\frac{B^{2} e^{2} r^{2}}{m^{2}} \tag{5}
\end{equation*}
$$

And rearranging (4) and substituting in (5) gives:

$$
\begin{equation*}
\frac{e}{m}=\frac{2 V}{B^{2} r^{2}} \tag{6}
\end{equation*}
$$

Since $V$ and $r$ are measured directly, the problem is largely that of producing and measuring the uniform field $B$. This is accomplished using a pair of Helmholtz coils. (I have omitted the mathematical derivation of the field produced by the coils. The interested students can look for the information in the Griffiths reference above.) The Helmholtz coils are arranged so that the distance between the two coils is equal to the diameter of the coils. The magnetic field (in the direction along the axis of the Helmholtz coils) is:

$$
\begin{equation*}
B=2 \mu_{o}\left(\frac{4}{5}\right)^{3 / 2} \frac{N I}{D} \tag{7}
\end{equation*}
$$

where: $\mu_{o}=$ permeability of free space $=4 \pi \times 10^{-7} \mathrm{~T} \mathrm{~m} / \mathrm{A}, N=$ number of turns ( 72 for this apparatus), $I=$ coil current (A), $D=$ coil diameter ( 0.66 m for this apparatus). This expression is strictly true only at the centre of the coils but the special thing about the Helmholtz coils is that the second derivative of the field variation is zero giving a very uniform field.

The constants may be combined to yield the following equation for the determination of $e / m$ :

$$
\begin{equation*}
B=1.96 \times 10^{-4} I \tag{8}
\end{equation*}
$$

with field in Tesla.

## Procedure

The apparatus is composed of two distinct sections: the Helmholtz coils and the tube containing the cathode and the bars where you observe the electron orbits.

The coils are connected to a high current power supply (RSR DC Power Supply HY3005). The connections to the coils are made at the topmost and bottommost plugs on the right hand side of the main hinged rib of the coil frame (right as defined by having the rib closest to you). Given the polarity of the coil connections the magnetic field produced by the coils should be "up". (Follow the connections and use the right hand rule to verify this.)

The tube circuit connections are made on the left hand side of the main rib. The heated filament (black connectors at the top and bottom) which comprises the cathode is connected to another RSR HY3005 power supply. The accelerating voltage between the cathode and anode is provided by a Vizatek power supply. The connection to the anode is made on the central red pin on the left hand side. The accelerating voltage is read from the front of the Vizatek.

All of the power supplies can operate in either constant current or constant voltage modes. In this experiment the resistances of the loads (coils, cathode heater, cathodeanode voltage) don't really change much so the distinction isn't really that important. However, if you want to really do things properly the coils and the cathode heater should run in constant current mode and the cathode-anode voltage should run in constant voltage mode. Each supply defaults to the mode where it produces up to the lower of the two limits set by the knobs on the front panel. This makes operation a little confusing. Suppose you want a constant current of 1 Amp . You might think "turn up the current". Nope. As a first step keep the current knob turned down, turn up the voltage. The supply thinks (?) the operator is telling me to supply a small current and a large voltage. I will pick the lower so I will operate in constant current mode. I think there is an orange light on the front to tell you it is operating in constant current mode. Now turn up the current knob. If at any time the supply need to increase its voltage beyond the limit you set when
you turned up the voltage it reverts to constant voltage mode and ignores you (a green CV light will come on). THESE SUPPLIES CAN PROVIDE ENOUGH VOLTAGE TO GIVE YOU A SHOCK! Keep this in mind and turn down the voltage control before you change any wiring. You should always turn the knobs down rather than just turning the supplies off.

Figures 1a and 1 b show the inside of the tube. Fig. 1a is from the top and Fig. 1 b is from the right hand side. Fig. 1a also shows a typical orbit the electrons might follow if a magnetic field is applied out of the page.

Figure la.


A: Five crossbars attached to wire staff
B: Typical path of a beam of electrons
C: Cylindrical anode
D: Distance from filament to far side of crossbars
E: Lead wire and support for anode
F: Filament (straight wire type)
G: Lead wires and support for filament
L: Insulating plugs
The cross bars are used as reference points so you can determine the radius of the orbits $r$ as described in equation 6. Consult the table at the end of the lab for these values.

1) The magnetic field of the earth is strong enough to influence the electron orbits. Our analysis is only correct if the magnetic field is perpendicular to the velocity vector of the electrons. This will be impossible to achieve unless the magnetic field of the earth is exactly opposite the field from the Helmholtz coils and the initial electron velocity is perpendicular to the field direction. Use the normal compass provided to find the direction of the magnetic north (level the compass
and make sure there are no stray fields from electric currents). This is not the direction of the earth's magnetic field. It is the direction of magnetic field of the earth projected into the plane of the compass. All you really know if that the $B$ vector is in the plane that contains the compass needle and is perpendicular to the plane of the compass. (imagine you and a friend are playing catch with the sun directly overhead; the shadow of the ball on the ground tells you about the relative amounts of $V_{x}$ and $V_{y}$ but gives you no information about $V_{z}$.) Now adjust the dip compass so that it is in this plane and it will tell you what the direction is of the magnetic field. Setup the coils by rotating the frame on the table and adjusting orientated in the Earth's

the tilt until the axis of the coils matches the direction of the earth's magnetic field.
2) The apparatus should be essentially wired up but make a quick check through the connections to see if it makes sense with the explanation above. Turn on the accelerating voltage supply and set it to 20 V. Dim the room lights. The HY3005 that powers the filament can operate as either a constant voltage (or voltage limited) source or as a current limited source. You want to operate is as a current limited source as explained earlier. Before turning it on turn all adjustment knobs to their lowest setting (fully counter clockwise). Now turn on the power supply. Turn up the coarse voltage adjustment roughly half-way. Nothing should happen except that the orange CC light should come on if it wasn't before. The supply would like to give you the voltage you dialed up but it can't because you have the current limit set to zero (and hence it is in CC or current controlled mode). Now slowly turn up the current adjustment. Once you reach 3.8 A as given on the meter on the front you should see a glow coming from the filament through the slot in the anode. Turn it up higher. Once you reach 4.3 A the glow should be bright and you should see an electron beam coming out of the anode, slightly curving away from the centre of the tube and then bouncing off the glass tube. The glow you see is coming from some residual gas purposely left inside of the tube as it is being excited by the electrons. (How would you figure out what this gas is?) If you have reached 4.3 A and you don't see a nice beam you can go up to
4.5 A but don't go beyond this or you will burn out the filament. If the supply seems "stuck" check and see if it is in constant voltage mode. You may need to turn up the voltage control knob. Try to operate with as low a filament current as possible. The actual current in the electron beam is_between 5 and 15 mA . Note: neither of these is the current that goes into equations 7 or 8 . As long as you have an electron beam you don't need to adjust the filament current for the rest of the lab.
3) Turn on the coil power supply with adjustments at minimum. Turn the current limit up to 6 A and then adjust the voltage so that a small current is sent through the Helmholtz coils. Verify that the beam describes a circular path which is dependent on the current through the Helmholtz coils. The orbit of the electrons should be parallel to the plane of the coils. If not make sure your original line-up was okay. If the beam is moving up or down you can also twist the tube slightly in its mount so that the velocity vector starts in the correct plane.
4) Increase the field current until the electron beam describes a circle with the sharp outer edge of the beam just striking the outer edge of the crossbar furthest from the filament. The outer edge of the crossbar is used since it is to this point that the manufacturer has specified the distance. Also, the outer edge of the beam is used since it is determined by the electrons with the greatest velocity, those which left the negative end of the filament with the greatest potential difference, as recorded by the voltmeter. Record the field current necessary to make the electron beam strike each of the crossbars in turn. Repeat this procedure for five different accelerating voltage up to 50 volts. At the highest accelerating voltage it may not be possible to reach the innermost crossbar.
5) Before you begin to analyse your data try and reproduce a couple of your points and see how close you come to the same current for a given crossbar and voltage. This reproducibility represents your current uncertainty.
6) Analyse your data by putting all of it on one graph. I find a computer is quite useful since there are a lot of points and you can use the computer to calculate the magnetic field values using equation 8 . The problem is that equation 8 does not account for the earth's magnetic field. To avoid this problem plot

$$
B \quad \text { verus } \frac{\sqrt{2 V}}{r}
$$

You will have a y-intercept that will correspond to the magnetic field of the earth. From the slope of your graph calculate $e / m$.

| Crossbar number | Distance <br> $(\mathrm{m})$ | Radius, r <br> $(\mathrm{m})$ |
| :---: | :---: | :---: |
| 1 | 0.065 | 0.0325 |
| 2 | 0.078 | 0.0390 |
| 3 | 0.090 | 0.0450 |
| 4 | 0.103 | 0.0515 |
| 5 | 0.115 | 0.0575 |

