From "Teach Spin" Notes

$$\mu = 0.40 \pm .01 \text{ A-m}^2$$

Our value for the magnetic moment from this experiment is different from the value obtained in the static experiment because a different ball was used in each experiment. The magnetization of the disks in each of the balls may be slightly different.

C. Precessional motion of a spinning sphere

1. -- Objectives

The objective again is to measure the dipole moment of a permanent magnet inside the cue ball. A secondary objective is to observe and quantify the motion of a spinning sphere subject to an external torque.

2. -- Equipment

Magnet, power supply, air bearing, cue ball, strobe light, stopwatch, calipers, and balance.

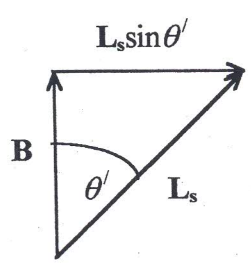
3. -- Theory

When the magnetic moment is displaced at some angle from the direction of the magnetic field, the magnetic dipole (and subsequently the ball) experiences a torque that causes a change in the ball's angular momentum in the direction of the torque. This is the central principle of this experiment. The ball is displaced from the vertical position and spun, with its spin-axis running through the handle of the ball. This creates a large spin angular momentum. The spin axis will remain in a fixed position until the magnetic field is turned on. When the magnetic field is turned on, the magnetic dipole will experience a torque. This torque will cause a change of angular momentum in the direction of the torque, but because the ball already has a large spin angular momentum, it will precess. This motion is similar to that of the spinning gyroscope in the earth's gravitational field. Students may be familiar with gyroscopic motion from their mechanics course.

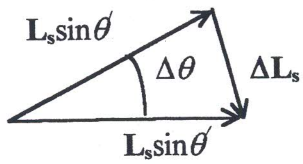
The differential equation for the motion of the ball is:

$$\boldsymbol{\mu} \times \mathbf{B} = \frac{d\mathbf{L}}{dt}$$

A side view of the angular momentum vectors is shown below:



If we look at the change of the angular momentum for a short time Δt from above, the picture looks like the one below:



Since $s = r\theta$, we can show that:

$$\Delta L_s = \Delta\theta\, L_s \sin\theta'$$

$$\frac{\Delta L}{\Delta t} = \frac{\Delta\theta}{\Delta t} L \sin\theta'$$

as $\Delta t \to 0$,

$$\frac{dL}{dt} = \Omega_p L \sin\theta'$$

where Ω_p is the precessional angular velocity. Since μ is in the same direction as $\mathbf{L}$,

$$\frac{dL}{dt} = \mu B \sin\theta'$$

So $$\mu B \sin\theta' = \Omega_p L \sin\theta'$$

and $$\mu B = \Omega_p L$$

or $$\Omega_p = \frac{\mu}{L} B$$

The precessional frequency (in radians/second) is the dependent variable. It can be determined by measuring the time needed for the handle of the ball to precess through 2π radians, and then dividing that time by 2π. The magnetic field is again the independent variable. The magnitude of the angular momentum L is a constant that can be measured using the strobe light. The handle of the ball has a white dot on its top. As the ball spins the strobe light reflects off of this white dot. When the strobe light is flashing at the same frequency at which the white dot is spinning, the dot will appear stationary. Thus, from the displayed strobe frequency and the measurement of the moment of inertia, the spin angular momentum of the ball can be calculated. The graph of Ω_p vs. B will yield a straight line if L is held constant. From the slope of this line, μ can be determined.

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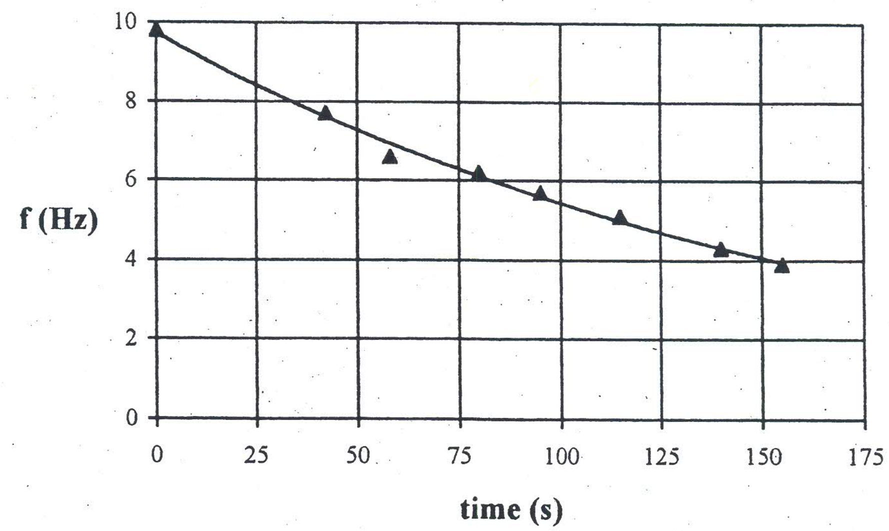
4. -- Procedure

a. First measure the constants. The moment of inertia of the ball is assumed to be that of a solid sphere. Both the mass of the ball and its radius need to be measured with the balance and the calipers, respectively. The other constant is the spin angular momentum of the ball. A constant value of the spin angular momentum of the ball can be accomplished by fixing the frequency of the strobe light. We recommend choosing a frequency somewhere between 4.5 and 6 Hertz. Set the strobe light at a frequency in that range and check it throughout the experiment to make sure that it remains constant. In order for the strobe light to illuminate the white dot, the room should be darkened. It is not necessary to make the room completely dark in order to use the strobe light effectively. You can maintain ample light for writing and working with the instrument.

b. Once the strobe light has been set to some particular frequency, record that number. Turn the air on, leave the field gradient off, and set the field direction "up". Have a stopwatch ready, but don't turn on the current quite yet. Before you begin making measurements, practice spinning the ball. A good technique is to spin the ball (give it a good, hard spin!) and then use the tip of your fingernail to direct it to spin about the handle's axis with the handle in the strobe light. Notice that the frequency of the ball's spin *does change with time*. If you graph this change, it closely approximates an exponential decay. The graph of this decay is shown below:



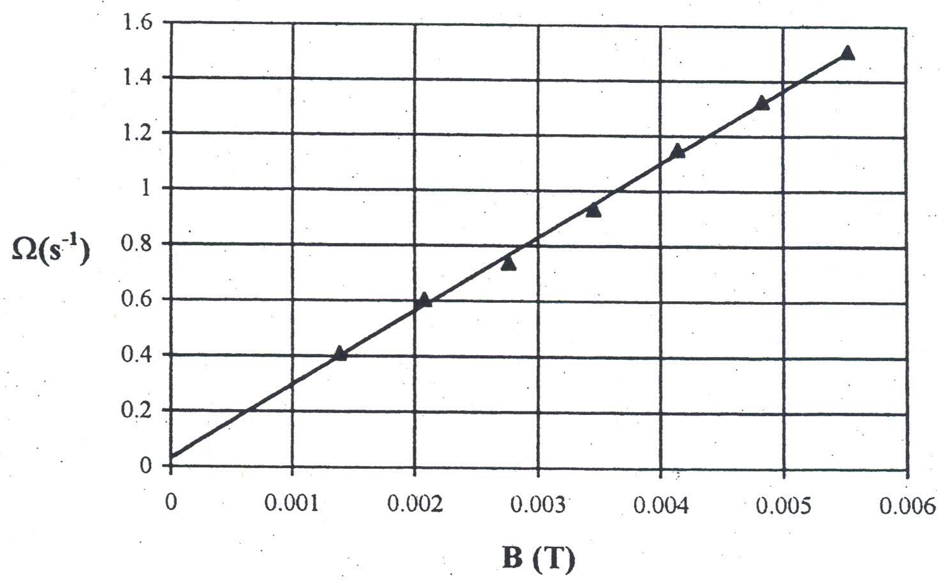
It's because of this decay that the range of frequency between 4.5 and 6 hertz is advised. In this range, the rotational frequency does not change significantly during the time it takes the ball to precess through one period.

When the students master the spin technique, they can begin the measurements. Leave the current off, spin the ball up, adjust it so that it's bathed in the strobe light, and then watch the white dot. You'll see it all around the handle at first, but as the ball slows down, the white dot will begin to have a regular rotation of its own. This rotation will slow down until the white dot stops. As soon as it stops, turn the current up to 1 amp, and time a period of the ball's precession. Record this time for that current, turn the current off, and spin the ball up again. Do the same procedure, but this time use 1.5 amps, and continue on in 0.5 amp intervals until you reach 4 amps. This should give you enough data. Record your data in a table such as the one below (again, sample data is given):

I (A)	B (T)	T_p (s)	$\Omega_p = (2\pi)/T$ (s^{-1})
1.0	0.00138	15.35	0.409
1.5	0.00207	10.40	0.604
2.0	0.00276	8.53	0.737
2.5	0.00345	6.73	0.934
3.0	0.00414	5.47	1.149
3.5	0.00483	4.74	1.326
4.0	0.00552	4.17	1.507

5. -- Report

The student should again include in the report a data table, sample calculations, the graph of Ω vs. B, and the calculation of μ. Discussions of conclusions, observations, and error are appropriate. Sample data are plotted below, along with our calculated value of the magnetic moment from this experiment:



$\mu = 0.38 \pm .01$ **A-m²**