

CONCAVE DIFFRACTION GRATING SPECTROGRAPH

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Diffraction gratings are widely used for high resolution spectral studies. The grating available in the lab is a holographically-produced, concave grating of 600 lines/mm and radius $R = 1075$ mm. It is placed in a *Rowland* mounting. In such a mount, the grating is constrained to be on a circle of diameter equal to the radius of curvature of the grating (the Rowland circle) on which the entrance slit (the slit through which light is incident on the grating) is also located. If the entrance slit and the grating are on the circle, the slit image will be in focus on the same circle. Concave gratings are often used in the ultraviolet portion of the spectrum where their focusing properties eliminate the need for UV-transmitting lenses. In this experiment, strong lines from the visible part of the spectrum of the elements mercury and cadmium are used to calibrate the spectrograph. The spectrograph is then used to measure the wavelengths of the lines in the visible spectrum of hydrogen (Balmer series) from which a value of the Rydberg constant for hydrogen can be obtained. The lamp used also contains deuterium so the spectrum of deuterium can be obtained as well. The wavelengths of the deuterium lines are very close to the hydrogen lines so the two spectra can be used to estimate the resolving power of the spectrograph. As an option, the spectrograph can be used to identify the element(s) responsible for the impurity lines in the hydrogen/deuterium lamp spectrum.

REFERENCES

1. Hecht, *Optics* (4th ed.), pp. 476–481; general properties.
2. Anne P. Thorne, *Spectrophysics*, 2nd ed. Chap. 6, pp. 144–170; (types of diffraction gratings and mounts, focusing property of concave gratings).
3. Sumner P. Davis, *Diffraction Grating Spectrographs*; practical information on the alignment and auxiliary optics for diffraction gratings.
4. Tipler, *Modern Physics*, pp. 144-151 (reduced mass correction).

APPARATUS

The apparatus is pictured in the figure below with the Rowland Circle shown as a dashed line. The radius R of the grating is 1075 mm (± 2 mm), the width of the grating is 25 mm, and the grating constant is 600 lines/mm (error unknown). The grating is blazed to give maximum intensity in the first order. **The grating mounting has been carefully adjusted and should not be disturbed.** A movable carriage holds a viewing microscope, which can be focused on the slit image. A set of cross-hairs in the microscope allows for precise location of the spectral line under observation.

The distance x from the slit to the slit image can be measured on a wooden meter stick + associated vernier to a precision of about 0.1 mm. For a Rowland mounting with the angle of reflection $r = 0^\circ$, the grating formula becomes $d \sin i = d \frac{x}{R} = n\lambda$, where d is the line-to-line separation on the grating, i is the angle of incidence of the light on the grating, x is the slit to image distance, R is the grating-to-image distance, n is the order of the spectrum ($n = 1, 2$, etc.), and λ is the wavelength of the spectral line.

CALIBRATION OF THE SPECTROGRAPH

The wavelength of a spectral line can be determined from a measurement of x and use of the grating equation, but this method has significant limitations. The grating equation is only as precise as the values of d and R , and it depends on positioning the entrance slit exactly above the zero of the meter stick scale, a result difficult to achieve in practice.

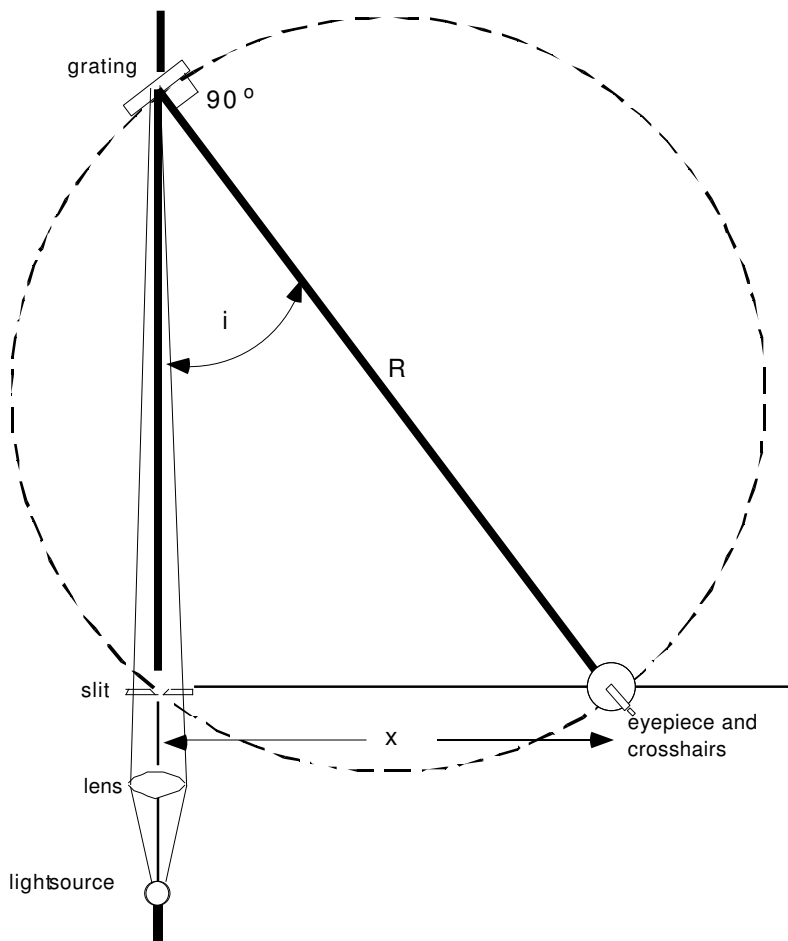


Figure 1: The concave grating spectrograph.

To avoid these limitations, the spectrograph is calibrated by measuring x for a number of spectral lines the wavelengths of which are known precisely. An empirical relation is then established from which the wavelength of an unknown line as a function of x can be obtained more precisely than it can from the grating equation. The wavelengths for the spectral lines of mercury and cadmium that may be used to calibrate the spectrograph are listed below:

λ (nm)	Element	Color
643.85	Cd	deep red
579.07, 576.96	Hg	bright yellow doublet
546.08	Hg	bright green
508.58	Cd	turquoise
479.99	Cd	light blue
467.81	Cd	dark blue
435.83	Hg	bright violet-blue
404.66	Hg	dark violet

You do not need to measure *all* of these lines, but pick at least 5 widely-spaced lines to give a good lever-arm to your calibration graph. **The calibration obtained from the measurements of these lines is to be used for all subsequent wavelength determinations.**

PROCEDURE

Position the large carriage supporting the mercury lamp assembly so that the small piece of white tape on the left edge of the carriage is above the 570 mm mark on the metal rail, and clamp the carriage securely to the rail with the silver clamping knob. If the hydrogen lamp is in the way, slide it back and clamp it to the rail before positioning the mercury lamp. **The hydrogen lamp assembly is delicate – please handle it carefully.**

Position the smaller carriage directly beneath the mercury lamp housing so that the small piece of yellow tape on the near edge of the carriage is above the 179 mm mark on the small metal rail, and clamp this carriage in position. Turn on the power supply for the mercury lamp.

Check that the lens is positioned so that the small piece of white tape on the left edge of its carriage is above the 835 mm mark on the metal rail. With the lamp and lens in these positions, the lamp image is focused on the slits with 1:1 magnification (to see the image hold a white card next to the slits).

Adjust the slit width (using the small lever marked with red tape) to approximately 0.2 mm. The width scale is marked on the slit housing. For $x \approx 35$ cm, the green line should be clearly visible.

If the observed line is not in focus, adjust the focus with the large focusing knob on the right side of the microscope assembly. Maximize the brightness of the line by adjusting the transverse position of the lens using the knob marked with yellow tape. Also move the small carriage beneath the mercury lamp housing back and forth to maximize the brightness of the line and again clamp the carriage in position. The slit width can now be decreased to allow for more precise location of the center of the line.

1. *Make five or more independent measurements of the position of the green line.*

Move the carriage with the microscope well away from the line, then reset it so the cross-hairs are centered on the line. From these measurements determine the precision with which you can locate a line. Each student in the group should make this set of measurements.

2. *Compute the positions of the calibration lines in the first order from the grating equation.* These computed positions will be help in identifying the correct calibration lines.

3. *Record the positions of the calibration lines in first order.* Plot each point **carefully** on the graph paper provided after you measure a line. Plot x (in cm) on the vertical and λ (in nm) on the horizontal. Choose

the scales for x and λ so that the data nearly fill the page. Visually verify that a spectral line of the right color exists at that value of x . Use a straight edge to draw a line through the data points. If you have correctly identified each line and measured its position carefully, the plotted points will lie on a straight line as closely as you can plot them. **If not, remeasure the lines that are off.**

When you have finished recording the calibration lines, look at several of them (green & yellow) in the second order. Notice how much dimmer these lines are than their first order counterparts. This is the result of the grating being blazed so that maximum intensity is reflected in the first order. After looking at the second order lines, turn off the mercury lamp.

For your lab write-up:

4. Use *EXCEL* to fit a least-squares straight line to the calibration data. Obtain the slope s , the standard deviation in the slope σ_s , the intercept x_0 , the standard deviation in the intercept σ_{x_0} , and the standard deviation in an individual measurement of x , σ_x . Comment on the value of the intercept you obtained. Is it reasonable? Why might the intercept be nonzero?

5. From the slope and the radius R calculate the lines per mm in the grating and the standard deviation in this number. Compare this to the nominal value of 600 mm^{-1} , and comment on how any difference might arise.

BALMER SERIES AND THE RYDBERG CONSTANTS

The visible spectrum of hydrogen consists of a series of lines that result from transitions terminating on the energy level with principal quantum number $n = 2$ and originating from higher-lying energy states for which $n = 3, 4, 5$, etc. The series of lines is named after Johann Balmer who, in 1855, published a numerical relation accurately describing their wavelengths. Starting with the longest (red) wavelength, the lines are labeled H_α , H_β , H_γ , etc. The Balmer series wavelengths for hydrogen are described by the Balmer formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right), n = 3, 4, 5, \dots$$

The constant is called the Rydberg constant for hydrogen. A similar expression applies for deuterium, but the Rydberg constant for deuterium, R_D , is slightly larger than that for hydrogen. The difference arises from the effect of the greater mass of the deuterium nucleus. A larger nuclear mass results in a larger value for μ_e , the electron *reduced mass*. See Tipler for the formula connecting R_H and μ_e . The Rydberg constant for an infinitely heavy nucleus is one of the best known constants in physics, $= 10\,973\,731.568\,549(83) \text{ m}^{-1}$ [CODATA recommended values of the fundamental constants 1998]. Note that this is larger (by a small amount) than the value of the Rydberg constant for hydrogen.

6. Determine the value of R_H from measurements of the Balmer lines of hydrogen.

Move the large carriage supporting the mercury lamp toward the lens carriage and clamp it a cm or so from the lens carriage. Move the small carriage and mercury lamp housing toward the wall, out of the light path, and clamp it in position. Move the carriage supporting the hydrogen/deuterium lamp assembly forward until the small piece of white tape on the left edge of the carriage is above the 515 mm mark on the metal rail, and clamp the carriage with the silver clamping knob. With the lamp and lens in these positions, the lamp image is focused on the slits with 1:1 magnification (to see the image hold a white card next to the slits).

The hydrogen/deuterium lamp gets very hot after being on for a period of time, and as its temperature increases, the intensity of the emitted light decreases. To help keep the lamp cooler and the intensity higher, a small fan is positioned at the base of the black tubing, which protects the lamp. **Turn on the power supply for the fan**, and then turn on the power supply for the lamp.

For $x \approx 42 \text{ cm}$, the red (Balmer α) line should be visible. If the line is not uniformly bright, it may be necessary to adjust the transverse position of the lens and/or that of the hydrogen lamp.

The hydrogen lamp draws 20 milliamperes (mA) of current at 5000 volts potential difference across the tube. 20 mA can be a lethal current so it is important to handle the hydrogen lamp assembly carefully and touch it only at grounded surfaces.

To change the position of the lamp, slightly loosen the knob marked with blue tape, and rotate the lamp holder by grasping the short horizontal rod marked with orange tape. **Do not touch the black tubing protecting the lamp or the white caps holding the tube in place.** When you have achieved maximum brightness by adjusting the lens and lamp positions, clamp the lamp in position by tightening the knob marked with blue tape.

Measure the position of the three visible Balmer lines, (α , β and γ) for hydrogen in first order. (The other lines in the Balmer series fall outside the visible range.) Note: there are a number of lines in the spectrum of the lamp that come from impurities rather than from hydrogen or deuterium. These tend to be notably dimmer than the hydrogen lines, but you can use your calibration from the mercury lamp to make sure you are observing the correct lines. Extract the Rydberg constant for hydrogen R_H from the parameters of a straight-line fit to an appropriately chosen plot of your data. Hint: The Balmer formula can be reduced to a linear equation by substituting the proper functions of λ and n .

RESOLVING POWER OF THE GRATING

The *resolving power*, \mathcal{R} of a diffraction grating is given by

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda_{\min}},$$

where $\Delta\lambda_{\min}$ is the minimum wavelength difference between two spectral lines that are just resolved (i.e., seen as separate lines). For a grating, the theoretical resolving power is $\mathcal{R} = nN$, where n is the order number and N is the total number of lines in the grating that are illuminated by the light source. In practice, the resolution will always be less than the theoretical value as it is reduced by effects of misalignments, imperfect collimation of the input light, finite slit width and limitations of the detection system.

7. Obtain estimates for the resolving power of the grating in the first order by studying the hydrogen/deuterium isotope shift.

To measure the separation between the H_α and D_α lines, a CCD array will be used. The array has 1024 equally spaced sensing elements (pixels) in the space of 7.99 mm, so it is capable of resolving very closely spaced lines.

Before switching to the array, position the microscope carriage to observe H_α and D_α lines in the microscope and adjust the slit width until the H_α and D_α lines just overlap.

Carefully lift the microscope assembly and set it aside, taking care not to move the microscope/detector carriage. Loosen the silver clamping knob on the black post holder and swing the CCD array board into position so that the plane of the board is perpendicular to the light path. Clamp the array in position. The sensing elements of the array are now on or very near the Rowland circle. Turn on the oscilloscope and the power supply for the CCD board and amplifier box. The scope sweep speed should be set to 10 milliseconds (ms) per division, and the vertical gain to 0.5 volt per division.

The peak due to the H_α and D_α lines should be visible on the CCD readout as displayed on the scope. The short “blips” at just over 2 volts amplitude define the beginning and end of the CCD readout cycle. One “blip” should be visible near each edge of the scope screen. Move the carriage back and forth and notice how the position of the peak shifts on the scope display. Some adjustment of the scope time base may be necessary to get the full array readout on the screen. To adjust the offset on the CCD readout, cover the entrance slit assembly with the piece of creased black paper. Next, switch the scope input to ground (GND position on switch just below the CH 1 VOLTS/DIV knob) and adjust the vertical position of the scope trace (POSITION knob above CH 1 VOLTS/DIV knob) so that it is 2 divisions below the center of the

scope screen. Switch the scope input to DC, and adjust the OFFSET knob on the amplifier box so that the scope trace is again 2 divisions below the center of the scope screen. Now remove the black paper covering the slit assembly.

Move the carriage to the left so that the very top of the peak is just at the left edge of the CCD readout. Note the x reading (denoted x_1) on the meter stick. Now move the carriage to the right so that the very top of the peak is just at the right edge of the CCD readout, and again note the x reading (denoted x_2) on the meter stick.

From your calibration you can derive the wavelength associated with x_1 and x_2 , and thereby the wavelength difference $\Delta\lambda = \lambda(x_2) - \lambda(x_1)$ across the 1024 pixels. The change in wavelength per pixel will be $\Delta\lambda/1024 = \Delta\lambda_{\text{pixel}}$.

Now move the carriage to the left so that the entire peak is just at the left edge of the readout cycle. Increase the scope sweep speed to 0.5 ms per division and increase the vertical sensitivity to 0.1 VOLTS/DIV. Decrease the slit width so that the H_α and D_α lines are clearly resolved. The readout from individual pixels should now be visible on the scope (each pixel shows up as a flat section with a faint spike at the edge).

A very nice hard copy of the CCD readout can be obtained with the Tektronix TDS 3012 digital scope. If there are waveforms displayed other than Channel 1 (yellow), you can remove them from the display by pushing the appropriately colored button (white for REF, red for MATH), and then pushing the OFF button. Adjust the vertical sensitivity for Channel 1 to 100 millivolts (mV) per division and the sweep speed to 1 millisecond (ms) per division. These parameters are displayed at the bottom of the scope screen. Check that the scope is in the Sample mode by pressing the MENU button in the ACQUIRE column, and then pressing the button to the right of the screen and next to the Sample selection. If the scope is not running, press the RUN/STOP button in the ACQUIRE column to start acquiring traces. (If the green indicator next to the DELAY button is on, push the DELAY button to turn the DELAY feature off.) If the H_α and D_α lines are not visible, adjust the POSITION knob in the HORIZONTAL column until they appear on the scope screen.

To see more detail of the H_α and D_α lines, increase the sweep speed to 200 microseconds (μs) per division. Turn on the DELAY feature by pressing the DELAY button, and adjust the POSITION knob in the HORIZONTAL column until the H_α and D_α lines appear on the scope screen. The DELAY time interval is indicated at the bottom of the screen, and it will typically be several ms.

To get rid of the bounce in the display the averaging feature of the scope is useful. Press the MENU button in the ACQUIRE column, then press the Average button to the right of the scope screen next to the Average selection. An average of 32 sweeps will make for a steady display. To change the number of sweeps in the average, rotate the knob with the \oplus on it at the upper right of the screen. To get a hard copy of the scope display, press the button at the lower left of the screen with the printer icon on it, and the printer below the scope will make a copy of the scope display. Sample copies of the scope display for both the H_α/D_α and H_β/D_β lines are available in the lab.

Estimate to one-tenth of a pixel the number of pixels separating the two lines and note this number for later use.

Estimate the width of both lines to one-tenth of a pixel. From these measurements and the separation of the H_α and D_α lines in pixels estimate how many pixels apart the two peaks could be and still be resolved. Multiply this number by $\Delta\lambda_{\text{pixel}}$ to obtain a minimum resolvable $\Delta\lambda$, and a value for \mathcal{R} . How does value compare to the theoretical value of \mathcal{R} for this grating? Why is this value for \mathcal{R} likely to be a lower limit?

Repeat the measurements above for H_β and D_β . Follow the same procedure as when looking for the H_α and D_α lines, but start with a vertical sensitivity of 20 mV (and sweep speed of 1 ms) on the digital scope. Turn off the Averaging and DELAY features when first locating the H_β and D_β lines. After locating the lines, increase the sweep speed and vertical sensitivity (don't forget to turn the DELAY feature back on) to see the lines in more detail. Turn on the Averaging feature to get rid of the bounce in the display.

The H_β and D_β lines are much less intense than the H_α and D_α lines, and the background pixel-to-pixel variation of the CCD readout will visibly affect the line shapes. To eliminate this background variation,

the REFERENCE and MATH features of the scope can be utilized. After finding the H_β and D_β lines and getting them displayed on the digital scope, proceed as follows. Cover the slit assembly with the black paper. Adjust the vertical sensitivity of the digital scope to 5 millivolts per division. Press the REF button, turn Ref 1 on (press the button below the Ref 1 selection on the screen), and store Channel 1 to Reference 1. Remove the black paper covering the slit. Now press the MATH button, and select the subtraction operation (-), with the first source set to Channel 1, and the second source set to Reference 1. The MATH trace (red one) will now display the difference between the signal and background, with the pixel-to-pixel variation largely eliminated and the line shapes displayed much more cleanly.

8. Determine R_D from R_H and the measured wavelength difference between H_α and D_α . Hint: Use the Balmer formula to express λ_H/λ_D and the fact that $\Delta\lambda \equiv \lambda_D - \lambda_H \ll \lambda_H$.

9. Determine the nuclear mass ratio m_D/m_H from the ratio R_D/R_H .

You will need to use the electron/proton mass ratio $m_e/m_p = 1/1836.15$. If the nuclear mass ratio you calculate is not close to 2, try again. The reduced mass μ is given by

$$\mu = \frac{m_e}{1 + m_e/m_{\text{nucleus}}} .$$

SHUTDOWN

When you are finished please move the CCD array out of the way, put the viewing microscope back in place, and put the covers back over the optics.

Shut off all lamp power supplies and electronics.