Fabry-Perot Interferometer Instructions and Formulae

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1 Introduction

The Fabry-Perot interferometer is an optical device that uses multiple beam interference to provide very high wavelength resolution.

2 References

1. Hecht
2. Jenkins and White

3 Operation

3.1 What do you look at?

The source should not be a point i.e. it is an *extended* source. In this way it is quite distinct from a spectrometer with a *slit* which is purposely made small and the resolution partially depends on the size of the slit. The key feature of the extended source is that it provides roughly uniform source strength over a modest angular range (i.e. it is not directed) and over a modest lateral size that is roughly comparable to the lateral size of the Fabry-Perot mirrors. A lens can be a great help since you can change the image size versus the object size.

3.2 What do you see?

When the spectrometer is properly set up with both mirrors exactly parallel what you see is a set of rings that correspond to *fringes of equal inclination* arising from *multiple beam interference*. The fringes themselves are localised at infinity so in order to see them you need to use your own eye, a telescope, or possibly a converging lens to focus them on a screen.

4 Formulae

4.1 Ring Pattern

The key formula refers to the phase difference δ between any two beams that exit from the space between the mirrors (the transmitted beams) at some angle θ. This formula comes from thin films but is also applicable here.

$$\delta = \frac{4\pi n_f}{\lambda_0} d \cos\theta_t + 2\phi \tag{1}$$

n_f is index of refraction of the film or the medium between the mirrors (air in our case so $n_f = 1$). λ_0 is the wavelength of the incident light. d is the spacing between the mirrors. θ_t is the angle of the transmitted beam measured from the normal (i.e. it is the deflection angle). ϕ is a phase difference that occurs when light reflects from a metallic surface. For the present case with a d on the order of millimetres the ϕ term is comparitively small.

The set of transmitted beams arising from multiple reflections all exit at the same angle if the mirrors are parallel to each other. In addition a ray that emenates from a different location on the source will produce a set of transmitted rays at the same angle if the beams are parallel as they leave the source. The transmitted parallel beams will then be seen as if they come from infinity as opposed to a nearby object.

When will we see a maximum? When $\delta = 2\pi m$ where m is some natural number.

$$m = \frac{2d}{\lambda_0} \cos\theta_{max} + \frac{\phi}{\pi} \tag{2}$$

Since $d \gg \lambda_0$, m will be quite large. As long as d remains fixed m decreases for the bright rings going out from the centre. This is in contrast to a grating spectrometer where m *decreases* as you go to higher deflection angles.

Suppose that we see a maximum in the centre at $\theta_t = 0$. We can estimate m for the $\lambda_0 \approx 590nm$ yellow line of sodium with $d = 2$ mm.

$$m_{centre} = 2\frac{d}{\lambda_0} + \frac{\phi}{\pi} \approx 2\frac{0.002}{5.9 \times 10^{-7}} = 6780 \tag{3}$$

Obviously you will need some coherence in the incoming wave so it can interfere with itself over this many wavelengths.

Now where are the other rings? If the centre corresponds to $m = 6780$ then the first ring corresponds to $m - 1 = 6779$. Returning to equation 2 and decreasing m by 1 we find if the maximum is at the centre then the angle of the first ring is

$$\theta = \sqrt{\frac{\lambda_0}{d}}. \tag{4}$$

We have used the approximation $\cos\theta \approx 1 - \frac{\theta^2}{2}$ for small angles in radians. With our previous example then $\theta = 17$ milliradians=0.98^o. Using a telescope is the easiest way to resolve this small angle (and also is easily set to focus at infinity).

If you wanted to plot the angles for a series of rings n at small angles, calling $n = 0$ the inner most ring and get a linear relationship you would modify equation 2

$$\begin{aligned} m_{centre} - n &= \frac{2d}{\lambda_0}\left(1 - \frac{\theta^2}{2}\right) + \frac{\phi}{\pi} && (5) \\ \theta^2 &= \frac{\lambda_0}{d} n + \frac{\lambda_0}{d}\left(2\frac{d}{\lambda_0} + \frac{\phi}{\pi} - m_{centre}\right) && (6) \end{aligned}$$

Note that these rings are separated in *angle* not in *position* so that makes them distinct from *fringes of equal thickness* which you saw with Newton's rings.

4.2 Lines that are separated by roughly 1 Å

The example that I give in the lab for this is the yellow line of sodium. If you look at it with a conventional spectrometer with a grating the sodium yellow line looks single because of the resolving power of the spectrometer. In principle you could use equation 6 to find the slopes of the two graphs for the slightly different λ components but you might not be able to tell which ring pattern is which. A better method involves changing d and noting the positions of *maximum discordance* and *maximum concordance*. If λ_1 and λ_2 are quite different from each other then you observe two distinct ring patterns of different colour. However, if λ_1 and λ_2 are fairly close together the ring spacing (related to the slope in equation 6) is almost the same. What is different for the two wavelengths is the value of m_{centre} (i.e. the number of wavelengths that fit betweent the mirrors) so the intercept in equation 6 is different. If the intercepts happen to be the same then the two ring patterns almost overlap, which you would call maximum concordance. If d is such that the bright spot is in the centre (at $\theta = 0$) then

$$\begin{aligned} m_{1,centre} &= \frac{2d}{\lambda_1} + \frac{\phi}{\pi} && (7) \\ m_{2,centre} &= \frac{2d}{\lambda_2} + \frac{\phi}{\pi} && (8) \end{aligned}$$

If you change d slightly by Δx then the rings you see have different m_{centre} values. However, the intercept changes at different rates for the different wavelengths. It is easiest to see this with an example. Suppose that $\lambda_1 = 590.0$ nm and $\lambda_2 = 590.1$ nm. For our earlier example the nominal values of m_{centre} are $m_{1,centre} = 6779.7$ and $m_{2,centre} = 6778.5$. So roughly one more λ_1 fits between the mirrors than λ_2. As we increase In fact let's calculate d if we have $m_{1,centre} - m_{2,centre} = 1$. Substitute this into equation 8 and take the difference between the two equations

$$1 = 2d\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \qquad (9)$$

$$d = \frac{1}{2}\frac{\lambda_1\lambda_2}{\Delta\lambda} \tag{10}$$

which gives $d = 1.74$ mm for my example numbers. I have choosen the difference between the m values to be 1. How much do I increase the d for the m values to be different by j?

$$d = \frac{j}{2}\frac{\lambda_1\lambda_2}{\Delta\lambda} \tag{11}$$

Each time this happens the rings will be lined up for maximum concordance so a plot of d versus j will give a straight line with slope $\lambda^2/2\Delta\lambda$.

4.3 Very fine splitting

Suppose that you keep d fixed and look at a ring that is some angle $\theta 0$ away from the centre. We introduce a magnetic field that causes the wavenumber $\sigma = \lambda^{-1}$ to change by $\Delta\sigma$

$$\Delta\sigma = \sigma\left(\frac{\cos\theta_0}{\cos\theta} - 1\right) \tag{12}$$

θ is the angle of a new line. You can expand for a small difference in θ_0 and θ (angular splitting) and for small angles. If $\theta = \theta_0 + \Delta\theta$

$$\Delta\sigma \approx \sigma\left(\frac{1-\frac{\theta_0^2}{2}}{1-\frac{(\theta_0+\Delta\theta)^2}{2}} - 1\right) \tag{13}$$

$$\Delta\sigma \approx \sigma\left(\left(1-\frac{\theta_0^2}{2}\right)\left(1+\frac{\theta_0^2+2\theta_0\Delta\theta+\Delta\theta^2}{2}\right) - 1\right) \tag{14}$$

$$\Delta\sigma \approx \sigma\left(\left(1+\frac{\theta_0^2}{2}+\theta_0\Delta\theta+\frac{\Delta\theta^2}{2}-\frac{\theta_0^2}{2}-\frac{\theta_0^4}{4}-\frac{\Delta\theta\theta_0^3}{2}-\frac{\Delta\theta^2\theta_0^2}{4}\right) - 1\right) \tag{15}$$

$$\Delta\sigma \approx \sigma\theta_0\Delta\theta \tag{16}$$

You could also use a Taylor expansion around θ_0 to say that

$$\cos(\theta_0 + \Delta\theta) \approx \cos\theta_0 - \cos\theta_0 \sin\theta_0\,\Delta\theta. \tag{17}$$

You can also use your angle information to find angular splitting versus magnetic field. You can find the formula in your text for how much frequency changes in the normal Zeeman effect and relate it to e/m.