Observing the normal Zeeman effect in transverse and longitudinal configuration
Spectroscopy with a Fabry-Perot etalon

Principle

The Objective of this experiment is to observe the normal Zeeman effect in the light from a cadmium lamp and to perform quantitative measurements to determine value of the Bohr magnetron.

There are two parts to this experiment:

1. To perform qualitative observations of the Zeeman effect including
   - Observing the line triplet for the normal transverse Zeeman effect.
   - Determining the polarization state of the triplet components.
   - Observing the line doublet for the normal longitudinal Zeeman effect.
   - Determining the polarization state of the doublet components.
2. To perform quantitative measurements on the normal transverse Zeeman effect and to calculate the value of the Bohr magnetron

Theory

Normal Zeeman effect

The Zeeman effect is the name for the splitting of atomic energy levels or spectral lines due to the action of an external magnetic field. The effect was first predicted by H. A. Lorenz in 1895 as part of his classic theory of the electron, and experimentally confirmed some years later by P. Zeeman. Zeeman observed a line triplet instead of a single spectral line at right angles to a magnetic field, and a line doublet parallel to the magnetic field. Later, more complex splitting of spectral lines were observed, which became known as the anomalous Zeeman effect. To explain this phenomenon, Goudsmit and Uhlenbeck first introduced the hypothesis of electron spin in 1925. Ultimately, it became apparent that the anomalous Zeeman effect was actually the rule and the "normal" Zeeman effect the exception.

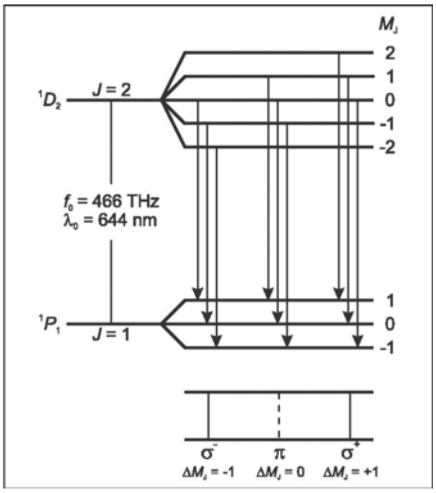

Figure 1 Level splitting and transitions of the normal Zeeman effect in Cadmium

The normal Zeeman effect only occurs at the transitions between atomic states with the total spin $S = 0$. The total angular momentum $J = L + S$ of a state is then a pure orbital angular momentum ($J = L$). For the corresponding magnetic moment, we can simply say that:

$$\mu = \frac{\mu_B}{\hbar} J \qquad (1)$$

where

$$\mu_B = \frac{\hbar e}{-2m_e} \qquad (2)$$

(μ_B = Bohr's magneton, m_e = mass of electron, e = elementary charge, $\hbar$ = $h/2\pi$, h = Planck's constant).

In an external magnetic field B, the magnetic moment has the energy

$$E = -\mu B \qquad (3)$$

The angular-momentum component in the direction of the magnetic field can have the values

$$J_z = M_J \hbar \quad with \quad M_J = J, J-1, ..., -J \qquad (4)$$

Therefore, the term with the angular momentum *J* is split into *2J + 1* equidistant Zeeman components which differ by the value of M_J. The energy interval of the adjacent components M_J, M_{J+1} is

$$\Delta E = \mu_B B \qquad (5)$$

We can observe the normal Zeeman effect e.g. in the red spectral line of cadmium (λ_0 = 643.8 nm, f_0 = 465.7 THz). It corresponds to the transition $^1D_2$ ($J = 2, S = 0$) → $^1P_1$ ($J = 1, S = 0$) of an electron of the fifth shell (see Fig. 1). In the magnetic field, the $^1D_2$ level splits into five Zeeman components, and the level $^1P_1$ splits into three Zeeman components having the spacing calculated using equation 5.

Optical transitions between these levels are only possible in the form of electrical dipole radiation. The following selection rules apply for the magnetic quantum numbers M_J of the states involved:

$$\Delta M_J \begin{cases} = \pm 1 & for\ \sigma\ components \\ = 0 & for\ \pi\ components \end{cases} \qquad (6)$$

Thus, we observe a total of three spectral lines (see Fig. 1);the π component is not shifted and the two σ components are shifted by

$$\Delta f = \pm \frac{\Delta E}{h} \qquad (7)$$

with respect to the original frequency. In this equation, ΔE is the equidistant energy split calculated in 5.

Angular distribution and polarization

Depending on the angular momentum component ΔM_J in the direction of the magnetic field. the emitted photons exhibit different angular distributions. Fig. 2 shows the angular distributions in the form of two-dimensional polar diagrams. They can be observed experimentally, as the magnetic field is characterized by a common axis for all cadmium atoms.
In classical terms, the case $\Delta M_J = 0$ corresponds to an infinitesimal dipole oscillating parallel to the magnetic field. No quanta are emitted in the direction of the magnetic field, i.e. the π-component cannot be observed parallel to the magnetic field. The light emitted perpendicular to the magnetic field is linearly polarized, whereby the E-vector oscillates in the direction of the dipole and parallel to the magnetic field (see Fig. 3)
Conversely, in the case $\Delta M_J = \pm1$ most of the quanta travel in the direction of the magnetic field. In classical terms, this case corresponds to two parallel dipoles oscillating with a phase difference of 90°. The superposition of the two dipoles produces a circulating current. Thus, in the direction of the magnetic field, circularly polarized light is emitted; in the positive direction, it is clockwise-circular for $\Delta M_J = +1$ and anticlock wise-circular for $\Delta M_J = -1$ (see Fig. 3).

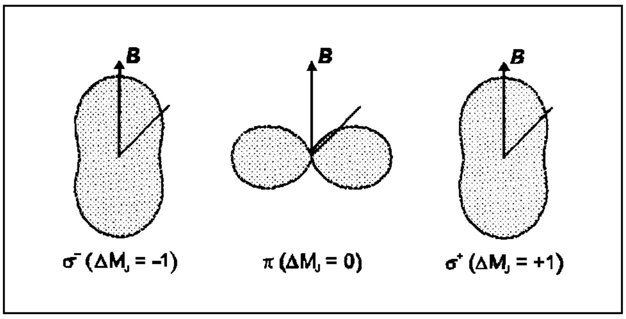

Fig. 2 Angular distributions of the electrical dipole radiation (ΔM_J: angular-momentum components of the emitted photons in the direction of the magnetic field)

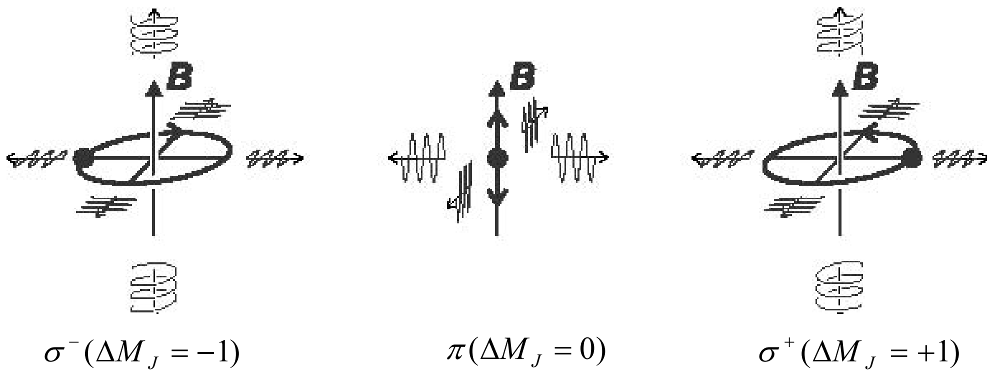

Fig. 3 Schematic representation of the polarization of the Zeeman components (ΔM_J:angular-momentum components of the emitted photons in the direction of the magnetic field)

Spectroscopy of the Zeeman components

The Zeeman effect enables spectroscopic separation of the differently polarized components. To demonstrate the shift. however, we require a spectral apparatus with extremely high resolution, as the two σ components of the red cadmium line are shifted e.g, at a magnetic flux density B = 1T by only Δf = 14GHz , respectively $\Delta\lambda$ = 0.02nm.

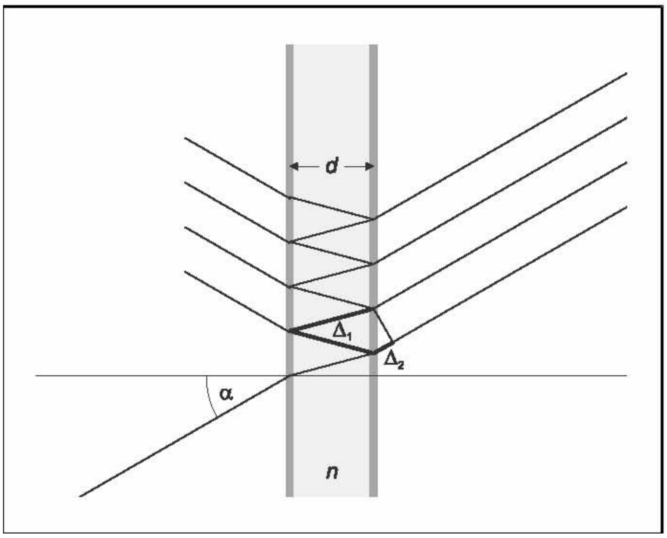

Fig. 4 Fabry-Perot etalon as an interference spectrometer. The ray path is drawn for an angle $\alpha > 0$ relative to the optical axis. The optical path difference between two adjacent emerging rays is $\Delta = n(\Delta_1 - \Delta_2)$

In the experiment a Fabry-Perot etalon is used. This is a glass plate which is plane parallel to a very high precision with both sides being aluminized. The slightly divergent light enters the etalon, which is aligned perpendicularly to the optical axis, and is reflected back and forth several times, whereby part of it emerges each time (see Fig. 4). Due to the aluminizing this emerging part is small, i.e., many emerging rays can interfere. Behind the etalon the emerging rays are focused by a lens on to the focal plane of the lens. There a concentric circular fringe pattern associated with a particular wavelength λ can be observed with an ocular. The aperture angle of a ring is identical with the angle of emergence α of the partial rays from the Fabry-Perot etalon.

The rays emerging at an angle of α_k interfere constructively with each other when two adjacent rays fulfil the condition for "curves of equal inclination" (see Fig. 4)

$$\Delta = 2d\sqrt{n^2 - \sin^2 \alpha_k} = k\lambda \tag{8}$$

Δ = optical path difference, d = thickness of the etalon, n = refractive index of the glass, k = order of interference.

A change in the wavelength by $\delta\lambda$ is seen as a change in the aperture angle by δa. Depending on the focal length of the lens, the aperture angle a corresponds to a radius r and the change in the angle δa to a change in the radius δr. If a spectral line contains several components with the distance $\delta\lambda$, each circular interference fringe is split into as many components with the radial distance δr. So a spectral line doublet is recognized by a doublet structure and a spectral line triplet by a triplet structure in the circular fringe pattern.

Part 1. Qualitative observation of the normal Zeeman effect

Setup

The complete experimental setup in transverse configuration is illustrated in Fig. 5.

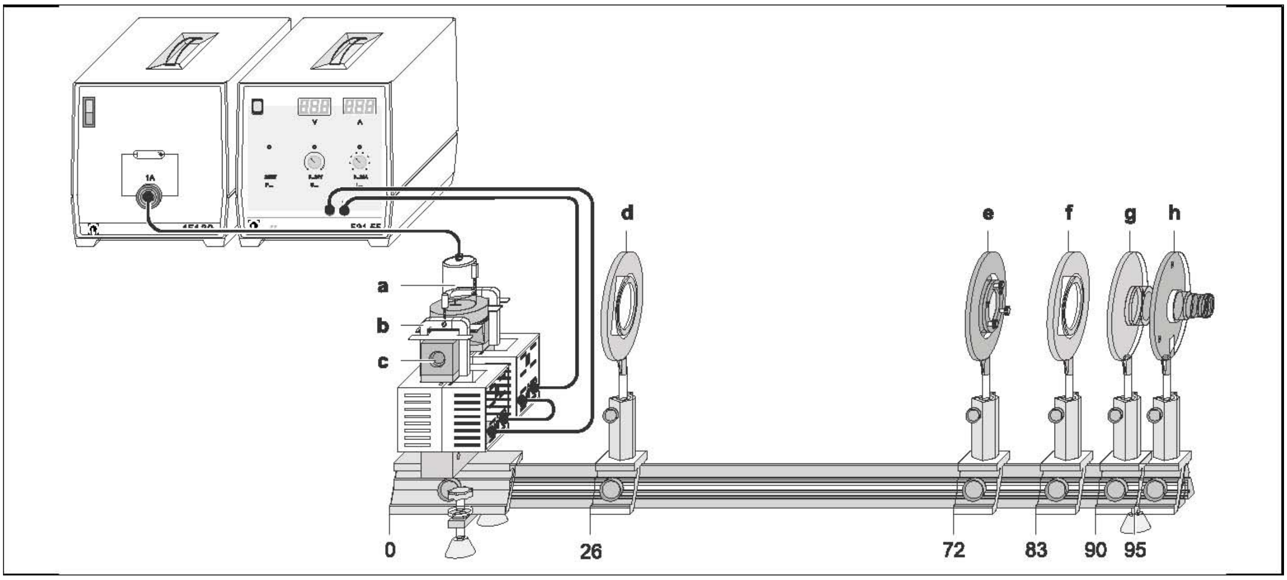

Fig. 5 Experimental setup for observing the Zeeman effect in transverse configuration. The position of the left edge of the optics riders is given in cm.

a Cadmium lamp with holding plate
b Clamps
c Pole pieces
d Positive lens, f = 150 mm (condenser lens)
e Fabry-Perot etalon
f Positive lens, f = 150 mm (imaging lens)
g Colour filter (red) in holder
h Ocular with line graduation

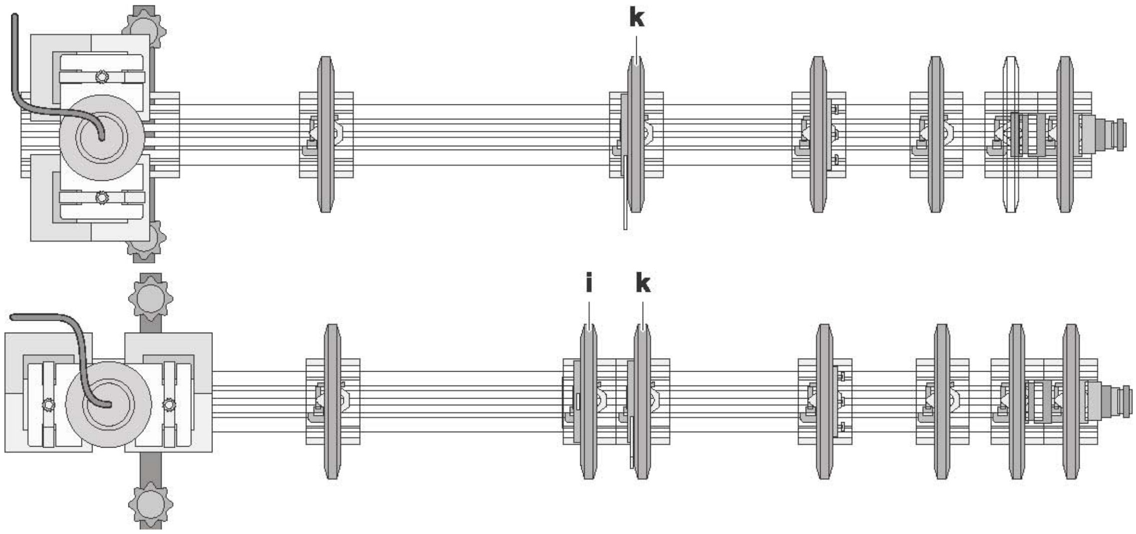

Fig. 6 Setup in transverse configuration (above) and in longitudinal configuration (below), as seen from above.

i Quarter-wavelength plate
k Polarization filter

Mechanical and optical setup:

- The cadmium lamp and magnet should be mounted on the rail when you arrive. Be very careful with moving the magnet assembly as the cadmium lamp is very fragile (and expensive!)
- Mount the optical components according to Fig. 5.

Electrical connection:

- Connect the cadmium lamp to the power supply; after switching on wait 5 min until the light emission is sufficiently strong.
- Connect the coils of the electromagnet in series and then to the high current power supply.

Adjusting the observing optics:

The optimum setup is achieved when the red circular fringe pattern is bright and contrasty with its centre on the line graduation. While adjusting do not yet insert the polarization filter and the quarter-wave plate so that the observed image is as bright as possible.

- Focus the ocular at the line graduation.
- Move the imaging lens until you observe a sharply defined image of the circular fringe pattern.
- Move the condenser lens until the observed image is illuminated as uniformly as possible.
- Shift the centre of the circular fringe pattern to the middle of the line graduation by slightly tipping the Fabry-Perot etalon with the adjusting screws.

If the adjustment range does not suffice:

- Rotate the Fabry-Perot etalon with its frame or adjust the height of the imaging lens and the ocular to each other.

Carrying out the experiment

a) Observing in transverse configuration:

- First observe the circular fringe pattern without magnetic field (I = 0 A).
- Slowly enhance the magnet current up to about I= 3 A until the split fringes are clearly separated.

To distinguish between π and σ components:

- Introduce the polarization filter into the ray path (see Fig. 6). and set it to 90° until the two outer components of the triplet structure disappear.
- Set the polarization filter to 0° until the (unshifted) component in the middle disappears.

b) Observing in longitudinal configuration:

- Carefully rotate the entire setup of the cadmium lamp with the magnet by 90°.
- First observe the circular fringe pattern without magnetic field (I = 0 A).
- Slowly increase the magnet current up to about I= 3 A until the split fringes are clearly separated.

To distinguish between σ^+ and σ^- components:

- Introduce a quarter-wavelength plate into the ray path between the cadmium lamp and the polarization filter (see Fig. 6), and set it to 0°.
- Set the polarization filter to +45° and –45°. In each case one of the two doublet components disappears.

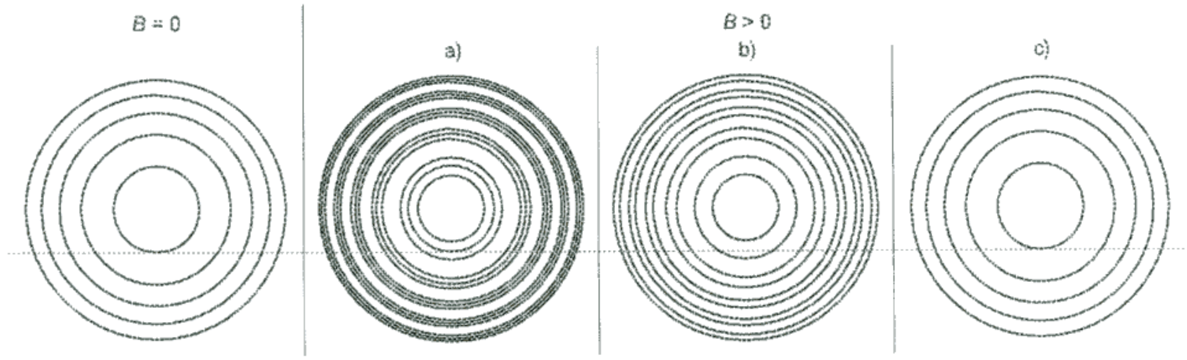

Fig. 7: Circular fringe pattern associated with the Zeeman effect in transverse configuration
a) without polarization filter, b) direction of polarization perpendicular to the magnetic field, c) direction of polarization parallel to the magnetic field

Measuring example and evaluation

a) Observing in transverse configuration: see Fig. 7
b) Observing in longitudinal configuration: see Fig. 8

Additional information
The total intensity of all Zeeman components is the same in all spatial directions. In transverse observation, the intensity of the π component is equal to the total intensity of the two σ components.

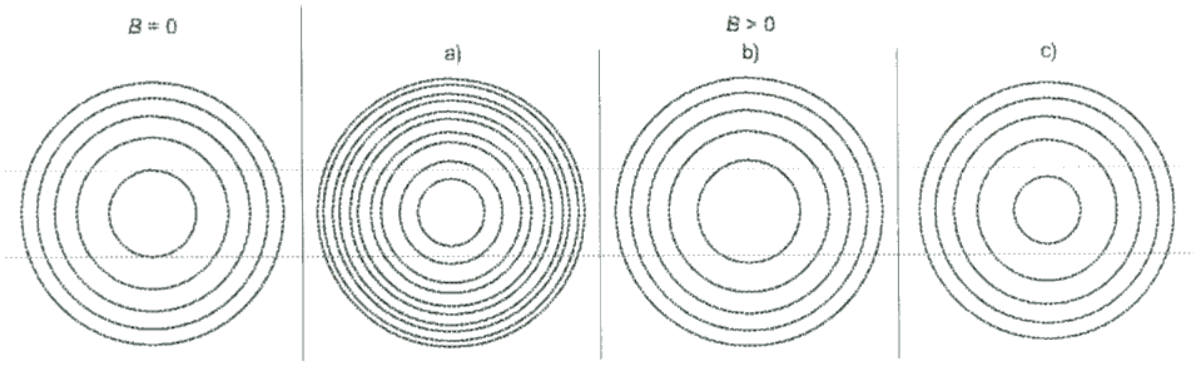

Fig. 8: Circular fringe pattern associated with the Zeeman effect in longitudinal configuration
a) without quarter-wavelength plate, b) and c) with quarter-wavelength plate and polarization filter for detecting circular polarization

Part 2. Quantitative measurements of the Zeeman effect

To perform quantitative measurements of the splitting of the red cadmium spectral line as a function of magnetic flux density the eyepiece, used for the first part of the experiment, is replaced with the linear CCD array and the intensity of a line through the ring system of the interferometer is displayed on the PC. To perform quantitative measurements the lamp is place in the transverse configuration and the polarizer is removed from the optical setup. The basic configuration should look like the schematic diagram shown in figure 9.

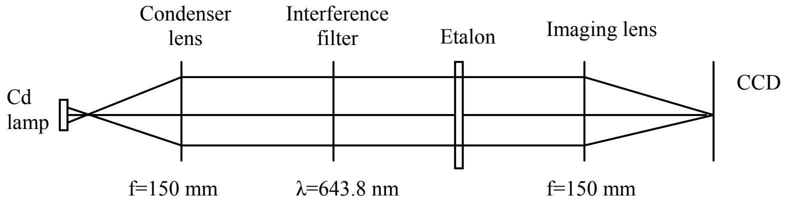

Figure 9. Schematic diagram of optical setup for quantitative measurements

Adjusting the setup

The software for the CCD camera can be used to adjust the camera settings to optimise data collection. For the initial setting up of the experiment set the camera to use only 256 pixels (button with a camera and I in the corner). This will allow the screen to be refreshed more quickly which aids in setting up the experiment. Once the setup is complete set the camera to 2048 pixels (button with a camera and II in the corner). It is also possible to adjust the exposure time (using the buttons with a magnifying glass) so that the peaks have an intensity of around 50 %.
To ensure that the CCD is in the focal plane of the imaging lens (f = 150 mm), move the imaging lens along the optical axis until the peaks of the observed curve are sharply imaged and show the maximum intensity. The centre of the ring system must then be imaged on the CCD line. For this, you can either move the VideoCom perpendicular to the optical axis or tilt the etalon interferometer slightly using the adjusting screws. You have found the centre of the ring system when further adjustment does not cause any more peaks to emerge and the two central peaks (left and right intersections of the innermost rings) are the maximum distance apart.
Move the condenser lens until you obtain the most uniform possible illumination of the entire CCD line. You should obtain a display similar to that shown in figure 10.

Diffraction Angle Calibration

Before you can determine the wavelength shift of the lines, the individual pixels of the CCD must be assigned to a diffraction α. For this to be possible, you must specify the effective focal length *f* with which the diffraction angle can be calculated to $\alpha = \arctan(x/f)$ where $x=(1024-p)*14$µm with p = pixel coordinate on the CCD (0 to 2047). This is performed in the software by simply entering the value for the focal length of the imaging lens used (f=150mm). Ask the demonstrators for help with this part of the software.
This calibration changes the horizontal axis to degrees and it is now necessary to set the correct zero position. Use the zoom and display coordinates functions to determine the angles of the two central peaks and use the mean of these as the zero point shift. Enter the negative value of this into the “shift zero point” box. This should set the centre of the ring system to 0° on the angular scale. Ask the demonstrators for help with using this software.

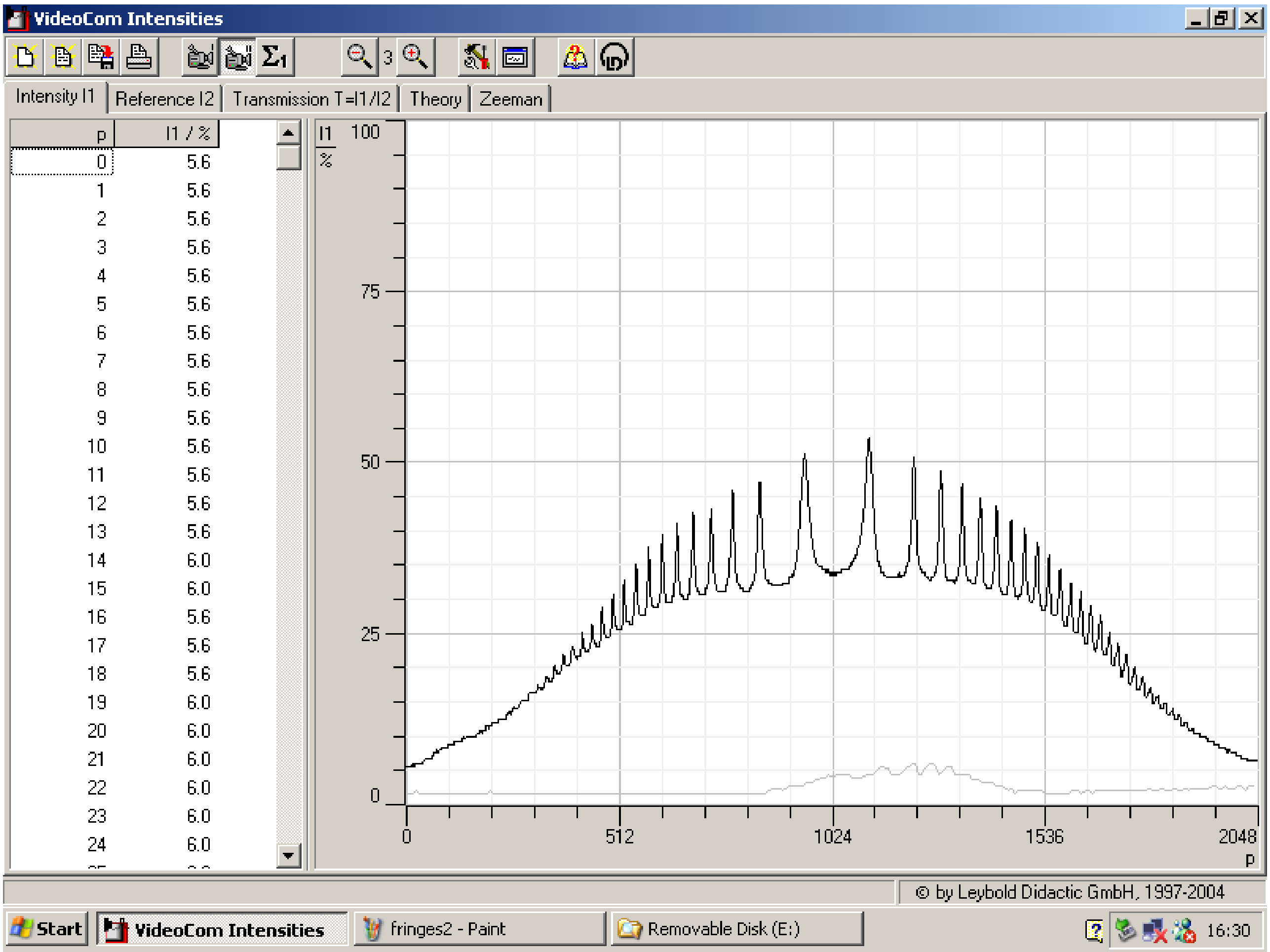

Figure 10. A screen capture of the fringes observed with no applied field

Quantitative evaluation

The intensity of the peaks should be around 50 % (adjust this if necessary as the luminance of the Cd lamp changes in the magnetic field). Use the Zoom and display coordinates function to determine the centre of one of the peaks with the magnetic flux density at 0T. Record the position of one of the peaks as the magnetic flux density is increased (use steps of 0.5A in the current through the magnet).

You should now have a table of angular position of the peak, α_1, as a function of magnetic flux density, B. The aim is to convert this into a shift in the energy of the peak so that equation 5 can be used to measure the value of the Bohr magnetron, μ_B. If one knew the interference order, k, for the peak you are measuring then knowing the angular position of the peak and the thickness and refractive index of the etalon, one would be able to determine the wavelength of the peak from equation 8. Unfortunately, it is not possible to know the interference order for any particular peak. However, each peak has a single value of k and as the peak moves with magnetic flux density this is due to the change in wavelength with k remaining constant. It is therefore possible to determine the relative shift in wavelength, $\Delta\lambda/\lambda$, from the shift in the angular position of a given peak.
If the angular position of the peak at zero magnetic field is α_0, then the relative shift in wavelength is given by,

$$\frac{\Delta\lambda}{\lambda} = \frac{\cos\beta_1}{\cos\beta_0} - 1 \qquad (9)$$

where

$$n = 1.46 = \frac{\sin\alpha}{\sin\beta} \qquad (10)$$

where α is the external angle a ray makes with the etalon and β is the internal angle the same ray makes. Equation 9 is derived from equation 8, (prove this for yourself) and equation 10 is Snell's Law.
Using these equations calculate $\Delta\lambda/\lambda$ as a function of magnetic flux density for your data. Finally you need to convert $\Delta\lambda/\lambda$ to ΔE. To do this use equation 11.

$$\Delta E = -\frac{\Delta\lambda}{\lambda} E = -hc\frac{\Delta\lambda}{\lambda^2} \qquad (11)$$

where λ=643.8 nm for the cadmium line. Use your data to calculate the value of the Bohr magnetron, μ_B.
The literature value is 57.9 μeV/T.

Repeat your measurements with other peaks.

Observing the polarization

In transverse configuration (magnetic field perpendicular to optical axis), place a polarization filter in the path of the light beam. By turning the polarization filter, it is possible to observe that the nondisplaced line is linearly polarized perpendicular to the two split lines. The measured intensity of the corresponding peak disappears at the respective positions 0° and 90° of the polarization filter.
In longitudinal configuration (magnetic field parallel to optical axis), place a quarter-wavelength plate (position e.g. 45°) and then a polarization filter behind it in the path of the light beam (proceeding from the Cd lamp). Depending on the position of the polarization filter (0° or 90°), only one or the other of the displaced lines can be observed. The other two lines are circularly polarized in opposition.

Derivation of equation 9

Any given peak has a value of k and as λ changes due to the magnetic field k remains constant, Therefore,

$$\frac{\Delta\lambda}{\lambda} = \frac{2d\sqrt{n^2 - \sin^2\alpha_1} - 2d\sqrt{n^2 - \sin^2\alpha_0}}{2d\sqrt{n^2 \sin^2\alpha_0}}$$

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\left(\frac{n^2 - \sin^2\alpha_1}{n^2 - \sin^2\alpha_0}\right)} - 1$$

From Snell's law

$$\sin\alpha = n\sin\beta$$

$$n^2\sin^2\alpha = n^2(1-\sin^2\beta) = n^2\cos^2\beta$$

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\left(\frac{n^2\cos^2\beta_1}{n^2\cos^2\beta_0}\right)} - 1 = \frac{\cos\beta_1}{\cos\beta_0} - 1$$