Example Questions: Atomic and Molecular Physics 475
April 21, 2006

1. Topic: Modern Physics
What was the main technique employed (and still employed) for studying energy levels and interactions in atoms? What is Bohr's formula for the energy levels in one electron atoms? If you observe a spectral line with $\lambda = 468.8$ nm coming from He^+ associated with the $n = 4$ to $n = 3$ transition what is the value of the Rydberg constant? What is the ionization energy of He^+?

2. Topic: Quantum Mechanics
   (a) In terms of operators what is the time-independent Schrödinger equation (SE)?
   (b) Give the Hamiltonian for a one-dimensional harmonic oscillator.
   (c) Convert SE for a SHO to dimensionless form. Consider the solution at large x to find the asymptotic form of ψ.
   (d) The full solutions to the simple harmonic oscillator may be classed as even and odd parity. Is this a surprise? Why or why not?
   (e) Show that $\psi_1(x)$ as given by 2.148 in your book solves SE for a SHO.

3. Topic: Perturbation Theory (and a bit of Fine Structure)
Consider an infinite square well that extends from $x = -L/2$ to $x = L/2$. What are the ground state and first excited state solutions? Now consider adding a "speed bump" to the middle of the well with height $V = \frac{\hbar^2\pi^2}{16mL^2}$ (i.e. one quarter of the ground state energy) with total width $L/5$.
   (a) Calculate the first order correction to the ground state energy.
   (b) Would you expect the absolute correction to the first excited state to be greater, less than, or equal to the ground state correction? Why?
   (c) If the well becomes really small then the ground state energy can become relativistic. Calculate the first order correction to the ground state energy coming from the relativistic correction to the kinetic energy, equation (5.5) in the text. (no speed bump now). For reasons mentioned in Chap.5 this should be easy. (I don't know if 5.5 is really appropriate for one dimension). Please give your answer in terms of α and other relevant scales. (I am guessing that there will be some kind of comparison between L and the "Compton bar" wavelength.)

4. Topic: Angular Momentum
Consider the problem of an infinite spherical well. Potential is zero inside but there is a hard wall at $r = R_0$. Write Schrödinger's equation, show how it separates, write the un-normalised eigenfunctions in terms of our usual suspects, and calculate in eV the ground state energy if an electron is contained in the well and $R_0 = a_0$.

5. Topic: One-electron atoms
Write the $n = 2$, $l = 1$, $m = 1$ solution for the hydrogenic wavefunction (neglecting electron spin) and show that it solves 3.5 with the appropriate eigenvalues. Find the energy of this state.

6. Topic: One-electron atoms
   Calculate $< \frac{1}{r^2} >$ for the above and show that it agrees with 3.77. (I haven't tried this but I *think* it is doable.)

7. Topic: Interaction with radiation

   (a) Under what circumstances do the selection rules $\Delta\ell = \pm 1$ apply? What about m?

   (b) What is the basis of the dipole approximation?

   (c) The book and my class notes have done a lot to evaluate the matrix element for spontaneous emission. This matrix element is also involved in absorption. Suppose that you use a moderately intense broad UV source (I don't know maybe 1 Watt per square centimeter evenly spread over the range from 50 nm to 300 nm). Calculate the transition rate from the hydrogen ground state.

8. Topic: Fine structure

   (a) What does fine structure mean? Where does it come from? By what percentage roughly do the energy levels shift? Calculate an example.

   (b) Explain how changing to states that are eigenstates of total angular momentum is useful for calculating the spin-orbit coupling. Isn't that strange? Why don't you use eigenstates of spin and orbital angular momentum if you want to calculate spin-orbit coupling?