Final Exam: Atomic and Molecular Physics 475
April 13, 2011

Duration 2.5 hours. Use of text book (Foot) permitted/required. Total 70 points. Individual values follow each question. Try and resist the temptation to look up all of the answers or you will run out of time. The written answers don't need to be very long or in elegant prose; they need to convince me that you know and understand the answer. Feel free to use formulas to support your statements.

1. The hydrogen atom is the simplest atomic system and forms the basis for our understanding of other atomic systems. (WARNING: this is the most computationally involved question)

   (a) Write the Schrödinger equation of the hydrogenic atom as separate angular and radial equations. (Even if you can't do this correctly you can find the right starting point in the book for the next parts of the question.) (5)

   (b) Write the appropriate angular function for the $n = 2$, $\ell = 1$, $m_\ell = 1$ state and show that it solves the angular equation with the appropriate quantum numbers. What does the angular probability function look like? (consider both θ and ϕ dependence). (8)

   (c) What is the normalization condition for a general radial function? (2)

2. We talked a lot about states with different J. Without going all the way to hyperfine coupling what kinds of internal atomic interaction are independent of J and what interactions are not? (just state the answer) In which category is the Zeeman effect (an external perturbation)? Out of all of these, roughly how do the J dependent effects change with Z? (Certain figures in the book should prove inspirational...) (8)

3. Suppose that we wanted to talk about the terms that result from a $3p^2$ configuration.

   (a) What does it mean if you have LS coupling? Give an example of a label for an LS coupled state versus a state that lacks this coupling. (i.e. I am asking for the good quantum numbers in the two cases). Don't worry about Pauli exclusion principle for this part of the question. (4)

   (b) Now I want you to think about Pauli exclusion. Consider the possibility of a ^3D term. Explain why we "could" get this term and then why we, in fact, don't. (Remember that you have $M_L = m_{\ell_1} + m_{\ell_2}$ and $M_S = m_{s_1} + m_{s_2}$ which can help determine quantum numbers for the electrons.) (4)

4. Hyperfine interactions come from the interaction of the magnetic moment of the nucleus with the electrons.

   (a) From a classical physics viewpoint briefly explain how the $\ell = 0$ electrons have a different hyperfine interaction than the $\ell \neq 0$ electrons (yes, eventually they both end up with an $\mathbf{I} \cdot \mathbf{J}$ but part of the interaction is quite different). Which interaction tends to be bigger? (5)

   (b) Suppose that you observe a set of hyperfine levels and label them $F = 0, 1, 2$. Explain how $I = 1$ and $J = 1$ is consistent with this labelling. (2)

   (c) Suppose that $A = 8 \times 10^{-6}$ eV. Give the relative energies of these levels in GHz (assuming the $F = 0$ level is the reference energy) and explain how the interval rule is manifest. (5)

(d) Give the g_F factor for the $F = 2$ hyperfine level. (2)

5. In our discussion of the alkalis we talked about the central field approximation. We modified the Bohr formula for the excited states to include a quantum defect term. Referring to Fig. 4.2 in the text and the accompanying text two of the quantum defects are listed as $\delta_s = 1.35$ and $\delta_p = 0.86$.

   (a) Physically, why are the quantum defects (and the predicted energies) different for s and p configurations. Keep in mind that they were degenerate in hydrogen and that has a central field. (4)

   (b) Predict the energies of the 3s ground state and 3p first excited state. (NOTE: Since this is a low n state it is more appropriate to use $\delta_s = 1.37$ for a better prediction.) What would the wavelength be? (6)

   (c) Use formula 4.13 to estimate fine structure splitting for this transition (eV or cm^{-1}). Which level is split or is it both? What quantum number is changing between the split levels? (5) (BTW it isn't cheating to read the book)

6. Briefly, how does the state selector of a hydrogen maser work. Include Fig. 6.10 as appropriate in your discussion. Why is the hydrogen maser important? Speculate what new frequency you might see if there was some deuterium mixed into the hydrogen beam (the deuteron has $I = 1$ and a $g_I = 0.88$ compared to the proton $I = 1/2$ and $g_I = 5.6$.) The discussion on the bottom of page 99 is quite useful. (10)