Midterm Quiz: Atomic and Molecular Physics 475
March 9, 2006

2-sided formula sheet only. Total 36 + 2 bonus points. Individual values follow each question. I have put "hats" on the operators. If you don't in your answers that is okay.

1. What is the Bohr model? Describe how the assumptions that go into the Bohr model are related to deBroglie's hypothesis and the result of the Rutherford-Geiger-Marsden scattering experiment. What phenomenon/law was the Bohr model successful in describing? According to the Bohr model energy is quantized as follows:

$$E_n = -\frac{1}{2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{\mu}{\hbar^2}\frac{Z^2}{n^2} \tag{1}$$

   What is the wavelength of light corresponding to the $n_f = 4$ to $n_i = 3$ transition in hydrogen if we assume the reduced mass μ is approximately equal to the mass of the electron? (I will give you any constants that you need.) (10)

2. Do the quantum mechanical operators $\hat{x}$ and $\hat{p}_x$ commute? Is it possible to construct simultaneous eigenstates of these two quantities? What is the name of the general technique used to go from a $\Psi(x,t)$ representation to a $\Phi(p_x,t)$ representation of a wave function? We saw later on that this could also be referred to as a unitary transformation. Do the energy eigenvalues change after performing a unitary transformation? (6)

3. What functions are simultaneous eigenfunctions of $\hat{L}_z$ and $\hat{L}^2$? Show that

$$\psi(\theta,\phi) = 3A\sin\theta\cos\theta e^{i\phi} \tag{2}$$

   with A a constant has this property for

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi} \tag{3}$$

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]. \tag{4}$$

   Give the appropriate simple number eigenvalues for each operator (i.e. ℓ and m). (12)

4. Suppose that we have a Hamiltonian $\hat{H}_0$ that is perturbed by a second interaction $\lambda\hat{H}'$. If the eigenfunctions of $\hat{H}_0$ are $\psi_n^{(0)}$ give the formula for first order correction to the energy levels. Show how this satisfies the condition on λ to the first power terms in the full eigenvalue equation i.e.

$$\hat{H}'\psi_k^{(0)} + \hat{H}_0\psi_k^{(1)} = E_k^{(0)}\psi_k^{(1)} + E_k^{(1)}\psi_k^{(0)} \tag{5}$$

   You will probably find this easier if you "ket" up this equation and multiply by $< \psi_k^{(0)}|$. (8)

5. Bonus: Give one of the differences between Hermite polynomials and Legendre polynomials. (2)