Final Exam: Solid State Physics 476
Dec. 15, 2004

One cheat sheet, double sided. Point values are given with each question. Total exam is worth 90 points.

1. (a) How do you define a crystal structure? Are the choices you make unique? What is the difference between a primitive and non-primitive basis? (4)

   (b) What are the angles between the a, b, and c axes in an orthorhomic unit cell? (2)

   (c) Name three symmetry transformations that might apply to a crystal structure and state whether or not each one is a point group. (6)

   (d) Consider an n-glide in an orthrorhomic crystal where the plane is perpendicular to the a-axis and intersects it at $\frac{1}{4}$ of a lattice constant. The translation for the n-glide is in the diagonal $b-c$ direction and is half a lattice constant in each direction. Describe how a point at (x, y, z) would be transformed after the glide. (6)

   (e) Can you pick any translation distance for the glide or does it have to be some specific fraction of the unit cell distance? Why? (2)

2. (a) Contrast the Laue picture/conditions for elastic scattering to Bragg's Law. Provide a diagram and mention reciprocal lattice vectors and show how the two agree for scattering from the (100) planes in a simple cubic lattice. (10)

   (b) If the magnitude of the wavevector of the incident radiation is 6 Å^{-1} and you observe elastic scattering from the (100) planes at $2\theta = 18$ degrees give the plane spacing. (use whichever method you would like) (6)

   (c) Now suppose that the lattice is face-centred cubic. Why is the (100) reflection no longer observed? (2)

3. Consider a one-dimensional chain of atoms each with mass m, interaction constant k (or $m\omega_0^2$), and atomic separation a.

   (a) Give the equation of motion for the n-th atom of the chain. (you will need to include terms that include the displacement of its neighbours) (3)

   (b) If we assume that motion is harmonic then the displacement of any atom is proportional to $\exp(-i\omega t)$. If we assume periodic boundary conditions over a length L then there is a spatial modulation of $\exp(iqna)$ where n is the atom label number and $q = \frac{2\pi N}{L}$ where N is some integer (the mode number). So the assumed form of the motion for the n-th atom is

   $$x_n(t) = A\exp(i(qan - \omega t)). \tag{1}$$

   Substitute this form into the equation of motion and show that the dispersion of the phonon modes is

   $$\omega(q) = \sqrt{\frac{4k}{m}}|\sin(\frac{1}{2}qa)|. \tag{2}$$

   The identity

   $$\sin^2\theta = \frac{1-\cos 2\theta}{2} \tag{3}$$

   may be useful. (7 points)

(c) What is the spacing between the modes in q-space? (2)

(d) How does this disperion at low q compare to that of electrons? (2)

4. We spent a lot of time discussing the free electron gas model of metals. Consider a monovalent metal such as sodium near room temperature.

   (a) If each Na atom contributes one conduction electron and the lattice is body-centred cubic with $a = 4.23$ Å calculate the electron density n. (2)

   (b) The Drude model for electrical conduction says that

   $$\sigma = \frac{ne^2}{m}\tau \qquad (4)$$

   where σ is the electrical conductivity. Calculate the effective mean free time if $e = 1.6 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$, and $\sigma = 2.11 \times 10^7\ \Omega^{-1}\mathrm{m}^{-1}$. (2)

   (c) Calculate the Fermi energy at $T = 0$ K in electron volts. How does this compare to $k_B T$ at room temperature? (3) ($\hbar = 1.055 \times 10^{-34}$ J s, $k_B = 1.38 \times 10^{-23}$ J/K)

   (d) Give a sketch of the Fermi-Dirac distribution function for $T = 0$ K and $T > 0$ but still with E_F much greater than $k_B T$. (sketch them on the same set of axes) (3)

   (e) Sketch the density of states $g(E)$ for a 3-D electron gas. (2)

   (f) Formally what is $< E >$ the thermal average energy? (Give the integral expression and pay attention to limits.) Calculate $< E >$ at $T = 0$ K in terms of the Fermi energy. (6)

   (g) In words and with a diagram (or referring to earlier diagrams) explain why the specific heat of a degenerate Fermi gas is much lower than the classical prediction. You may refer to previous answers as appropriate. (4)

   (h) State the Wiedemann-Franz law. Make an estimate of κ for sodium at room temperature from your previous answer assuming the Lorenz constant is $2.45 \times 10^{-8}\ \mathrm{W} \cdot \Omega/\mathrm{K}^2$. Why isn't Umklapp or impurity scattering included in this calculation? (6)

5. Although we only briefly touched on band structure and semiconductors please answer one of the following two questions on the topic.

   (a) Does an electron with a wavevector equal to the Brillouin zone boundary propagate through the crystal? Why or why not? (10)

   (b) The basic expression for electrical conductivity is given by equation 4. In this context why is the electrical conductivity of pure semiconductor so low compared to that of a metal? (10)